Analysis and Simulation Tools For Wind Engineering

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This paper examines state-of-the-art analysis and simulation tools for applications to wind engineering, introduces improvements recently developed by the authors, and directions for future work. While the scope of application extends to a variety of environmental loads (e.g. ocean waves and earthquake motions), particular reference is made to the analysis and simulation of non-Gaussian features as they appear in wind pressure fluctuations under separated flow regions and non-stationary characteristics of wind velocity fluctuations during a gust front, a thunderstorm or a hurricane. A particular measured non-Gaussian pressure trace is used as a focal point to connect the various related topics herein.

Various methods of non-linear system modeling are first considered. Techniques are then presented for modeling the probability density function of non-Gaussian processes. These include maximizing the entropy functional subject to constraints derived from moment information, Hermite transformation models, and the use of the Kac–Siegel approach based on Volterra kernels. The implications of non-Gaussian local wind loads on the prediction of fatigue damage are examined, as well as new developments concerning gust factor representation of non-Gaussian wind loads. The simulation of non-Gaussian processes is addressed in terms of correlation-distortion methods and application of higher-order spectral analysis. Also included is a discussion of preferred phasing, and concepts for conditional simulation in a non-Gaussian context. The wavelet transform is used to decompose random processes into localized orthogonal basis functions, providing a convenient format for the modeling, analysis, and simulation of non-stationary processes. The work in these areas continues to improve our understanding and modeling of complex phenomena in wind related problems. The presentation here is for introductory purposes and many topics require additional research. It is hoped that introduction of these powerful tools will aid in improving the general understanding of wind effects on structures and will lead to subsequent application in design practice. Copyright © 1996 Elsevier Science Ltd.

BACKGROUND

Over the last few decades, our understanding of wind–structure interactions and resulting load effects has significantly improved, yet a need remains for further examination of a host of issues. Many of the studies encompassing analysis and modeling of wind effects on structures have tacitly assumed that the involved random processes are Gaussian. This assumption has been invoked primarily for the convenience in analysis, since information concerning statistics of Gaussian processes is abundant. This assumption is quite valid for loads that involve integral effects of the random pressure field over large areas. Nonetheless, regions of structures under separated flows experience strong non-Gaussian effects in the pressure distribution characterized by high skewness and kurtosis. The non-Gaussian effects in pressure result in non-Gaussian local loads, and give way to increased expected damage in glass panels and higher fatigue effects on other components of cladding.

The probabilistic analysis of pressure fields has been of interest to those involved in wind tunnel studies. Peterka and Cermak¹ and Kareem² demonstrated that in pressure regions where the mean pressure was below –0.25, the pressure probability density functions (pdf) are skewed such that the probabilities for large negative fluctuations are much higher than those for Gaussian processes. Similar observations have also been reported by others. It was also noted that due to non-linear
relationships between wind and pressure fluctuations the
pdf of pressure under high turbulence may be non-
Gaussian. Low-rise structures immersed in the highly
turbulent lower part of the boundary layer, whose
structure is further invigorated by the presence of
roughness elements in the surroundings, may experi-
ence non-Gaussian pressure fluctuations even on their
windward faces. These non-Gaussian effects may be
amplified further as the approaching wind fluctuations
depart from a Gaussian process. Similar effects are
observed in wave effects on structures. Holmes and
Kuwait utilizing quasi-steady and strip theories evalu-
ated the derived pdf of pressure. The resulting
distribution showed good agreement with measured
data on the surfaces with attached flows. However, as
expected, the derived pdf of pressures in the separated
regions is not predicted by the quasi-steady theory as the
wind—structure interactions at several scales of turbu-
elence may introduce additional components. This
observation is again corroborated by Letchford et al.
utilizing full-scale data. In an attempt to identify
admittance functions for wind pressures, Thomas et al.
have noted that the quasi-steady theory fails to
model spectral descriptions of pressures under separated
regions despite the inclusion of the square of the
fluctuating velocity term. Similar comments are offered
by Tieleman and Haji based on their analysis of full-
scale data. In summary, the quasi-steady theory offers
reliable estimates of load effects when the dominant
mode of loading is attributed to buffeting, e.g., surface
pressures responding to large-scale low frequency turbu-
elence. However, the pressures resulting from wind—
structure interaction effects cannot be predicted from
the quasi-steady theory. A departure from the quasi-
steady theory is reflected in the non-Gaussian field.

In light of the established inability of quasi-steady
theory to predict the dynamics and probabilistic
structure of pressure fluctuations in the separated
regions, some thoughts on the modeling of non-
Gaussian processes are presented. This approach holds
promise for providing answers and perhaps models for
situations in which the quasi-steady theory has failed to
do so because pressure fluctuations are a result of a non-
linear dynamic interaction.

The analysis of a non-stationary processes such as
transient wind gusts in short, measured wind records
have been limited due to shortcomings in the Fourier
analysis. Here, we apply a set of basis functions local in
time and frequency to decompose the signal into
octave-banded constitutive parts. The wavelet transform
is useful in the location of energy transfer in time, and in
the simulation of non-stationary processes.

MODELING OF NON-GAUSSIAN PROCESSES

In the study of physical systems, the relationship
between the input and the system output is often
sought to model the system response. For linear
systems, e.g., in the formulation of gust loading factors
such a relationship is used for the prediction of extreme
response. In many instances in wind engineering,
however, the input and output are not related by a
linear transfer function due to non-linear characteristics,

Vogler systems

In the Volterra series formulation, the input—output
relationship may be expressed in terms of a hierarchy of
linear, quadratic and higher-order transfer functions or
impulse response functions (e.g. Refs 10–13). These
transfer functions can be determined from experimental
data or from theoretical considerations. For example, a
non-linear system modeled by Volterra's stochastic
series expansion is described by

\[ y(t) = \int h_1(\tau)x(t-\tau)d\tau + \int h_2(\tau_1,\tau_2)x(t-\tau_1) \]

\[ \times x(t-\tau_2)d\tau_1d\tau_2 + \ldots \]

(1)

where \( h_1(\tau) \) and \( h_2(\tau_1,\tau_2) \) are the first and second-order
impulse response functions.

The Fourier transform of the Volterra series expan-
sion up to second order (retaining two terms on the right
hand side) in eqn (1) gives the response in the frequency
domain as

\[ Y(f) = H_1(f)X(f) + \sum_{f_1+f_2=f} H_2(f_1,f_2)X(f_1)X(f_2) \]

(2)

For linear systems, the first term on the right hand
side of eqn (2) is all that is needed to describe the
relationship between input and output. This linear
model assumes that the Fourier components at
different frequencies are uncoupled. In the first (linear)
term on the right hand side of eqn (2) the response \( Y(f_i) \)
at frequency \( f_i \) is dependent only on input and the
transfer function at frequency \( f_i \).

In the case where the system is non-linear, the Fourier
components are coupled, and additional terms are
needed to capture this interaction. The second term on
the right hand side of eqn (2) couples the response \( Y(f_i) \)
at frequency \( f_i \) with pairs of input components at
frequencies whose sum or difference is \( f_i \) through the
quadratic transfer function (QTF) \( H_2(f_i,f_2) \). Equation
(2) describes a system whose non-linear component
is non-symmetric with respect to the probability density
function (e.g. an even powered polynomial non-linearity). A third-order system captures the behavior of systems with both symmetric and non-symmetric non-linearities (e.g. polynomial non-linearities with odd and even powers).

In the case when input \( x(n) \) and output \( y(n) \) of a system is available, the information can be used to estimate the Volterra kernels in eqn (2) directly. The linear transfer function is given by

\[
H_1(f_1) = \frac{\langle X^*(f_1)Y(f_1) \rangle}{\langle |X(f_1)|^2 \rangle}
\]

(3)

where \( \langle \cdot \rangle \) is the expected value operator. Here, the numerator is the cross-power spectrum of the input \( x(n) \) and output \( y(n) \) in terms of their Fourier transforms \( Y(f) \) and \( X(f) \), and the denominator is the auto-power spectrum of the input.

Just as \( H_1(f_1) \) is derived from the cross-power spectrum, the QTF is derived from a higher-order cross-spectrum. The higher-order cross-spectrum between the input \( x(n) \) and the output \( y(n) \) needed to estimate the QTF is called the cross-bispectrum, denoted \( B_{xyy}(f_1,f_2) \). Analogous to the cross-power spectrum in the numerator of eqn (3), the cross-bispectrum can be expressed in terms of the expected value of input and output Fourier components as

\[
B_{xyy}(f_1,f_2) = \langle X^*(f_1)X^*(f_2)Y(f_1 + f_2) \rangle
\]

(4)

The QTF is given by

\[
H_2(f_1,f_2) = \frac{1}{2} \frac{B_{xyy}(f_1,f_2)}{\langle |X(f_1)|X(f_2)|^2 \rangle}
\]

(5)

If no phase coupling exists between \( Y(f_1 + f_2) \) and \( X(f_1) \) and \( X(f_2) \), then their phases will be random and independent, thus the net expected value of the cross-bispectrum will be zero. The formulation for the QTF given in eqn (5) is valid for a Gaussian input process \( X(f) \). The linear and quadratic transfer functions can also be estimated for a general random input, i.e. without assuming particular statistics of the input (e.g. Ref. 14).

Equation (2) addresses a second-order Volterra system, which assumes the non-linearity is asymmetric. More generally, higher-order spectral analysis may be applied to non-linear system identification via the higher-order transfer function, which may then be used with a Volterra series similar to eqn (2) with additional higher-order terms to model the non-linear system.

The analogies between the power spectrum and higher-order spectra may be extended to glean some insight into their physical meaning. The significance of the power spectrum \( S_{xx}(f) \) is well understood to be the decomposition of the signal variance \( \langle x^2(t) \rangle \) as a function of frequency. Similarly, the bispectrum \( B_{xxx}(f_1,f_2) \) may be viewed as the decomposition of skewness \( \langle x^3(t) \rangle \) as a function of two frequencies, and the trispectrum \( T_{xxx}(f_1,f_2,f_3) \) as the decomposition of kurtosis \( \langle x^4(t) \rangle \) as a function of three frequencies. The volumes under the bispectrum and trispectrum yield the third and fourth central moments, respectively. When viewed in this light, it is apparent that the existence of higher-order spectra indicates a deviation from Gaussian.

An estimated bispectrum for an experimentally measured wind pressure record is shown in Fig. 1. This non-zero bispectrum indicates a deviation from Gaussian due to interaction between low frequency

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**Fig. 1.** Estimated bispectrum of a measured wind pressure record.
components. For a quadratic non-linear process that is a 
square of a narrow-banded linear process, the bispect-
rum contains peaks where components of the linear 
process interact at their sum and difference frequencies, 
impacting energy at those frequencies to the resulting 
non-linear process. In this case, the bispectrum does not 
consist only of sharp peaks, indicating that the pressure 
record is not the result of the square of a narrow-banded 
process, but more likely the output of an at least 
partially quadratic system with a wide-banded input. 
The input process in this case is in fact a wide-banded 
wind velocity process. Were it the case that pressure was 
the result of a cubic non-linearity acting on the wind 
velocity, the bispectrum would not exist, and the 
trispectrum would reveal the symmetrically non-linear 
relation of the pressure to velocity.

Alternatives to Volterra systems

Several researchers have addressed the modeling of non-
linear systems by means other than a Volterra series 
expansion and application of higher-order spectra. For 
example, Bendat\(^{15}\) replaced the higher-order frequency 
domain contribution in eqn (5) by a zero-memory 
squared term in series with a linear term. Non-linear 
pressure on a building from Gaussian wind velocity 
input is modeled in this fashion to improve upon the 
modified quasi-steady theory by using a multiple 
admittance function.\(^{7}\) Here, the pressure autospectrum 
is expressed in terms of the spectra of the horizontal and 
vertical fluctuating wind components, the spectra of 
these components squared, and transfer functions in 
terms of the cross-spectra of the inputs with measured 
pressure output. Bendat’s model replaces the second-
order Volterra kernel by a linear kernel based on the 
assumption that the QTF is constant along lines normal 
to the diagonal as in

\[
H_2(f_1,f_2) = A(f_1 + f_2) \tag{6}
\]

This is equivalent to all frequency pairs for a particular 
sum or difference frequency containing the same level of 
phase coupling. This assumption conveniently reduces, 
for example, a single input/single output second-order 
Volterra model to a two input/single output linear 
model. The efficiency of the analysis is advantageous, 
and retains limited memory. The error associated with 
this representation is lumped into a noise or residual 
spectrum which is minimized with respect to the transfer 
functions describing the linear systems in parallel. The 
non-linearity is represented, but the assumption of its 
form may be restrictive for some systems. The model 
may be modified to facilitate the input of non-Gaussian 
wind velocity.\(^{15}\)

Neural networks

Another recently developed approach to non-linear 
system modeling is the application of neural networks.

A multi-layered set of processing elements receives input 
information and uses the desired final output informa-
tion to adjust a weighting factor between each of the 
elements. Figure 2 shows such a network with three 
weighting layers \(W_g(m), m = 1, \ldots, 3\), where 
\(i = 1 \ldots N_m, j = 1, \ldots, N_{m-1}\), and \(N_m\) and \(N_{m-1}\) are the 
number of elements in the \(m\)th and the \(m - 1\)th layers, 
respectively. The network in Fig. 2 has two hidden 
element layers \(a_1(1)\) and \(a_2(2)\) between the input and 
output layers \(a_0(0)\) and \(a_3(3)\). In this example, the input 
layer consists of the input occurring at the same time 
as the current output from \(a_3(3)\), and two delayed inputs. 
\(W_g(m)\) then represents the weighting of the output from 
the element \(a_j(m - 1)\) before its input to element \(a_i(m)\). 
The output of each element \(b_i(m)\) is a non-linear 
function of the weighted linear sum of the output from 
each of the elements in the previous layer as in

\[
b_i(m) = \sum_{j=1}^{N_{m-1}} W_{g}(m) a_j(m-1) + \theta_i(m) \tag{7}
\]

\[
a_i(m) = f(b_i(m)) \quad 1 \leq i \leq N_m \quad 1 \leq m \leq 3
\]

where \(\theta_i(m)\) is a threshold value fixed for each \(a_i(m)\). 
Various non-linear functions may be applied at the 
elements. One commonly applied function is the sigmoid 
function

\[
f(b_i) = \frac{1}{1 + e^{-b_i/\sigma}} \tag{8}
\]

where \(\sigma\) is a parameter to control the shape of \(f(b_i)\). 

The element weights in the neural network are 
adjusted iteratively to minimize the error between the 
resulting and desired final output. This is the training 
phase, in which the optimum model parameters \(W_g(m), 
m = 1, \ldots, M\) are identified, where \(M\) is the number 
of network layers, and \(M = 3\) for the example in Fig. 2.\(^{16}\)

An example application is shown in Figs 3 and 4. In 
Fig. 3, the input is a simulated Gaussian wind velocity 
\((U + u(t))\) sampled at 100 Hz for 10s, and the output is 
the resulting force on a unit area using 
\(F = \rho C_d A (U + u(t))^2 / 2\). The neural network has two

\[
W_g(3) \quad a_1(3) \quad output layer
\]

\[
W_g(2) \quad a_1(2) \quad a_2(2) \quad a_3(2) \quad hidden layer
\]

\[
W_g(1) \quad a_1(1) \quad a_2(1) \quad a_3(1) \quad hidden layer
\]

\[
a_0(0) \quad a_1(0) \quad a_2(0) \quad a_3(0) \quad input layer
\]

Fig. 2. Multilayer neural network with three weighting layers 
and two hidden layers (adapted from Kung\(^{15}\)).
MODELING OF PROBABILITY DENSITY FUNCTION

The modeling of the probabilistic structure of non-Gaussian pressure fluctuations is essential for a wide range of applications in wind engineering, e.g., accurate determination of design wind pressure for glass panels. Large skewness results in probabilities for negative pressure fluctuations much higher than those for Gaussian processes. Series distribution methods, including Gram–Charlier, Edgeworth, and Longuet–Higgins, based on Hermite polynomials, have been commonly used (e.g., Ref. 19), but tend to exhibit oscillating and negative tail behavior. For extreme response, alternative means are considered.

The non-Gaussian distribution derived based on the non-linear relationship between wind and pressure fluctuations with the assumption of Gaussian velocity is valid mostly for windward and leeward faces. This has been shown to fail in separated regions over surfaces parallel to wind flow, where quasi-steady theory breaks down. The lognormal distribution has been used in the literature to model pressure data as the tail of the distribution is higher than for the normal distribution. This often provides values close to the observations, but still fails to predict the occurrence of values far from the mean. Calderone et al.\textsuperscript{20} recently noted that the lognormal distribution does not fit the pressure data perfectly.

In view of the preceding shortcomings, three alternative approaches to modeling the pdf of non-Gaussian pressure fluctuations in separated regions are considered here. The maximum entropy method (MEM), Hermite transformation-based models and Kac–Siebert expansion-based models are applied to the pressure records through their statistical moments or quadratic transfer function to determine the parameters of the respective estimation models. Applications of these methods are illustrated by way of two examples.

Maximum entropy method

An approach to approximate the pdf of non-linear systems is the maximum entropy method (MEM), in which the Shannon entropy functional is maximized subject to constraints in the form of moment information. In the limiting case of infinite moment information, a unique pdf is defined. In reality, we will always have a finite amount of moment information for which an infinite number of probability density functions are admissible. The pdf which maximizes the entropy functional is the least biased estimate for the given moment information. The Lagrange multiplier method is applied to solve this variational problem, and provides the joint pdf of higher-order systems directly. A brief outline of MEM is presented here. Complete details can be found in Sobczyk and Trebicki\textsuperscript{21}, Kareem and Zhao\textsuperscript{13} and Kapur.\textsuperscript{22}
The available information for a process \( y(t) \) can be expressed as the process joint moments

\[
E[y_1 y_2 \ldots y_n] = \int \ldots \int y_1 y_2 \ldots y_n p(y) \, dy = m_{r_1 \ldots r_n},
\]

where \( n \) is the order of the system, \( r_i = 0, 1, 2, \ldots, M \), where \( M \) is the maximum order or correlation moment, \( p(y) \) is the joint pdf, and \( m_{r_1 \ldots r_n} \) is the value of the joint moment. The integral is multifold to \( n \). One possible pdf of the process \( y(t) \) is that which maximizes the entropy functional

\[
H = -\int p(y) \ln p(y) \, dy
\]

subject to constraints from the moment information.

After application of the Lagrange multiplier method, the resulting description of \( p(y) \) for an \( n \)-dimensional case is

\[
p(y) = \exp(-\lambda_0 - 1) \exp\left(-\sum_{r_1=0}^{M} \lambda_{r_1 \ldots r_n} y_1^{r_1} \ldots y_n^{r_n}\right)
\]

Substitution of eqn (11) into the moment constraints and an additional normalization constraint \( \int p(y) \, dy = 1 \) gives the following system of equations:

\[
\int y_1^{r_1} \ldots y_n^{r_n} \exp\left(-\sum_{r_1=0}^{M} \lambda_{r_1 \ldots r_n} y_1^{r_1} \ldots y_n^{r_n}\right) \, dy = m_{r_1 \ldots r_n}
\]

(12)

\[
\int \exp\left(-\sum_{r_1=0}^{M} \lambda_{r_1 \ldots r_n} y_1^{r_1} \ldots y_n^{r_n}\right) \, dy = \exp(\lambda_0 + 1)
\]

(13)

This system of non-linear integral equations is solved numerically and the results yield the least biased estimate of the system joint pdf under the given moment constraints using eqn (11).\(^\text{13,21}\) The moment information which constrains the maximum entropy functional may be in the form of moment equations rather than moment values as presented above. Details are omitted here, interested readers may refer to Sobczyk and Trebicki.\(^\text{21}\)

**Moment-based Hermite transformation model**

This approach is based on a functional transformation of a standardized non-Gaussian process, \( x(t) \), to a standard Gaussian process, \( u(t) \), (e.g. Ref. 23)

\[
x(t) = (X(t) - \bar{X})/\sigma_X = g(u(t))
\]

(14)

Several choices of \( g(u) \) are possible to preserve only the first four moments. A cubic model of \( g(u) \) offers a convenient and fairly accurate representation.\(^\text{24}\) Accordingly, the pdf of \( x(t) \) is given by\(^\text{23,24}\)

\[
p_x(x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{u^2(x)}{2}\right] \frac{du(x)}{dx}
\]

(15)

\[
u(x) = \left[\sqrt{\xi^2(x) + c + \xi(x)}\right]^{1/3}
\]

- \[\left[\sqrt{\xi^2(x) + c - \xi(x)}\right]^{1/3} - a\]

(16)

where \( \xi(x) = 1.5b(a + \frac{e}{3}) - a^3 \)

\[
a = \frac{h_3}{3h_4}, \quad b = \frac{1}{3h_4}, \quad c = (b - 1 - a^2)^3
\]

\[
h_3 = \frac{\Gamma_3}{4 + 2\sqrt{1 + 1.5\gamma_4}} \quad h_4 \sqrt{1 + 1.5\gamma_4 - 1} \quad \frac{18}{18}
\]

and \( \gamma_3 \) and \( \gamma_4 \) are the skewness and kurtosis of the fluctuating process, which reduce to zero for Gaussian. An improvement to this model is suggested here by using the expressions for \( h_3 \) and \( h_4 \) which are approximations as initial conditions for solving the following pair of non-linear algebraic equations:

\[
\gamma_3 = \alpha^3(8\hat{h}_3 + 108\hat{h}_3\hat{h}_4 + 36\hat{h}_3 \hat{h}_4 + 6\hat{h}_4)
\]

(17)

\[
\gamma_4 + 3 = \alpha^4(60\hat{h}_4^4 + 3348\hat{h}_4^4 + 2233\hat{h}_4^4 + 2 \hat{h}_4 + 292\hat{h}_4 + 1296\hat{h}_4^3 + 576\hat{h}_4^3 + 24\hat{h}_4 + 3)
\]

(18)

These equations have been derived for use in this study by setting the third- and fourth-order central moments of \( g[u(t)] \) equal to the known central moments of \( x(t) \). This yields new coefficient values which exactly match the statistics up to the fourth order of the modeled non-Gaussian process. This is referred to herein as the modified Hermite method.

The transformation above is for the case when \( x(t) \) is a softening process, for example, the response of a linear system subjected to non-linear viscous drag force. Winterstein\(^\text{24}\) also outlines a transformation for the case in which \( x(t) \) is a hardening process. This development is not repeated here, but such a situation could arise, for example, if the structural system is characterized by a non-linear stiffness.

**Kac–Siebert approach**

Once a system has been modeled in terms of a Volterra series (eqn (1)), different approaches are available to estimate the pdf of the process. One such approach is that of Kac and Siegert.\(^\text{25}\) In this approach, the system
output is expressed in terms of the sum of standardized normal random variates \( X_j \) and their squares as described below (e.g. Refs 13, 26, 27)

\[
y(t) = \sum_{j=1}^{2N} (B_j X_j + \lambda_j X_j^2)
\]

(19)

The parameter \( B_j \) is related to the eigenvectors \( \psi_j \), and \( \lambda_j \) are the eigenvalues of the Fredholm integral equation of the second kind given by

\[
K(\omega_1, \omega_2)\psi_j(\omega_2)d\omega_2 = \lambda_j \psi_j(\omega_1)
\]

(20)

\[
K(\omega_1, \omega_2) = H(\omega_1, \omega_2)\sqrt{S(\omega_1)S(\omega_2)}
\]

where \( H(\omega_1, \omega_2) \) is the QTF discussed earlier, and the frequency domain counterpart of \( h_2(\tau, \tau_2) \), and \( S(\omega) \) is a two-sided spectrum of the underlying linear process.

The characteristic equation of \( x \) is now expressed as

\[
M(\theta) = \prod_{j=1}^{2N} M_j(\theta)
\]

where

\[
M_j(\theta) = (1 - 2i\lambda_j)^{-1/2} \exp \left[ -\frac{B_j^2 \theta^2}{2(1 - 2i\lambda_j \theta)} \right]
\]

(21)

The pdf of the process \( x \) is the Fourier transform of the characteristic function

\[
p(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ix\theta} M(\theta) d\theta
\]

(22)

In general, eqn (22) cannot be solved in closed form, and must be numerically estimated. This representation of a non-Gaussian pdf is most appropriate for non-Gaussian systems resulting from a quadratic transformation. Any other transformation must be recast in a quadratic form to obtain the best results.

The \( n \)th cumulant \( k_n \) of a random process \( y \) is defined in terms of the characteristic function and after appropriate substitutions is given by

\[
k_n = \sum_{j=1}^{2N} \frac{1}{2(2\lambda_j)^n} \left\{ (n - 1)! + \left( \frac{B_j}{2\lambda_j} \right)^n n! \delta_{n1} \right\}
\]

(23)

where \( \delta_{n1} \) is zero for \( n = 1 \), and unity otherwise. These cumulants may be used as coefficients in a series of expansion of the pdf, or as constraints in the maximum entropy method.

**Examples**

An application of the MEM, Hermite moment, Kac–Siegent, and Gram–Charlier polynomial series methods of pdf estimation are shown along with a Gaussian model in Fig. 5, where the process is the non-linear response of an offshore platform to random wind loads.\(^{13}\) The inset is a view of the tail of the pdf on a logarithmic scale, in which the higher probability of extreme response in a softening non-linear system can be observed. In applying the Kac–Siegent technique, 128 discrete frequencies are used resulting in 256 and 128 values of \( \lambda \) and \( B \), respectively, to form the characteristic function in eqn (21). Discretization finer than this does not noticeably improve results in this example. Equation (23) is used to produce the cumulant values used in the MEM, Hermite moment, and Gram–Charlier Hermite polynomial methods. In this example, the skewness and kurtosis are 0.3829 and 0.2066, respectively. Deviation from Gaussian is not large in this case. All methods give approximately the same prediction.

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**Fig. 5.** Kac–Siegent, Hermite moment, MEM, Gram–Charlier and Gaussian pdf estimates of nonlinear response.
in the mean region, while the Kac–Siebert and Hermite moment tend to provide more conservative estimates in the tail region (inset).

The second example concerns the measured pressure fluctuations on a full-scale low-rise building. The pressure data, histogram, a fitted Hermite moment model, and a lognormal fit are given in Figs 6 and 7. The skewness and kurtosis are reported in Table 1 along with the parameters used in the Hermite transformation of eqn (16). In this case, the deviation from Gaussian is large, and the modified Hermite moment model fits the histogram well. The negative tail region is shown in the inset, where the close fit of the extremes of the data by the Hermite moment model contrasts that of the Gaussian tail. A lognormal fit utilized by Calderone et al. is also shown on this figure and while it predicts the tail region more accurately than the Gaussian distribution, it falls well short of equalling the effectiveness of the moment-based Hermite transformation in quantifying the extremes of the distribution. A Kolmogorov–Smirnov goodness-of-fit test also confirms the superior fit of the Hermite moment model.

APPLICATIONS OF PDF MODELS

Fatigue/glass damage

Typically, the fatigue damage caused by wind-induced fluctuations due to non-Gaussian gusts or pressures under separated flow regions is assessed based on the assumption that the underlying process is Gaussian. These results may considerably underestimate fatigue damage for some regions on a building envelope. Based on the models involving Volterra series, or information on the skewness and kurtosis of a process, a systematic correction factor can be added to the Gaussian fatigue damage estimate that reflects the effects of non-Gaussianity. A simple measure of the influence of non-Gaussian effects on fatigue damage accumulation is the ratio of fatigue damage under non-Gaussian loading to that under Gaussian loading

$$\lambda = \frac{E[D_{\text{ng}}]}{E[D_{\text{g}}]}$$

Table 1. Coefficients used in eqn (16) for Hermite moment fit of pressure data

<table>
<thead>
<tr>
<th>Skewness</th>
<th>Kurtosis</th>
<th>$\gamma_3$</th>
<th>$\gamma_4$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$\hat{h}_3$</th>
<th>$\hat{h}_4$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.9869$</td>
<td>$2.3278$</td>
<td>$-1.358$</td>
<td>$9.660$</td>
<td>$317.4$</td>
<td>$-0.1403$</td>
<td>$0.0345$</td>
<td>$0.9775$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
This ratio, based on Hermite moment transformation models for the non-Gaussian narrow-banded processes, is given by Winterstein as

$$\lambda = \left( \frac{\sqrt{\pi} k}{2V^2} \right)^{m\lambda} \left( \frac{m V^2}{V} \right)^{\lambda \mu} \quad \lambda = \frac{4}{\pi} (1 + h_4 + h_6) - 1$$

(25)

where $h_4 = \gamma_4 / 24$, $k = (1 + 2h_3^2 + 6h_4^2)^{-1/2}$, and $h_3$ and $h_4$ have been defined in eqn (16). Utilizing the non-Gaussian pressure fluctuation data in Fig. 6, $\lambda$ is equal to 1.6. This could potentially enhance fatigue of cladding components by 60 percent. The assumption of normality may lead to unconservative fatigue life prediction when the actual response is non-Gaussian with a kurtosis value greater than zero. However, conservative estimates are expected for non-Gaussian cases with a kurtosis value less than zero.

The importance of non-Gaussian local pressures on cladding glass has been addressed by, among others, Holmes, Reed, and Calderone and Melbourne. Holmes and Reed utilized a non-Gaussian distribution of pressure fluctuations based on the relationship between Gaussian wind velocity and non-Gaussian pressure variations. This relationship is based on quasi-steady theory which has its limits but works well for stagnation face pressure. Numerical simulations involving wind tunnel data have revealed that non-Gaussian fluctuations result in greater glass damage. The cumulative damage criterion is used to determine the effect of fluctuating pressure through an equivalent constant pressure. Beside the non-linear relationship between the pressure and resulting stress in glass, the glass size, and its geometry, the non-Gaussian features of pressure fluctuations play an important role in determining the cumulative damage. The equivalent constant pressure is given by

$$P_E = \left( \frac{1}{T_E} \sum_{i=1}^{l} p_{\text{local}} t_i \right)^{1/2} = \left[ T_s \left( \frac{1}{T_E} \int_{-\infty}^{\infty} p_{\text{local}} f_{\text{pdf}}(p) dp \right) \right]^{1/2}$$

(26)

where $T_E$ is the duration of the equivalent pressure (60 s in the U.S.), $p_i$ is the pressure at a particular instant, $l$ is the total number of instants being accumulated, $t_i$ is the time duration of pressure $p_i$, $s$ is the slope of the straight line on a log-log scale of the plot between pressure and surface tensile strength of glass, and $n$ is dependent on the type of glass. In the second expression for $P_E$, $T_s$ is the length of the sample considered and $f_{\text{pdf}}(p)$ is the pdf of the pressure process. Using a histogram of a sample of data ($T_s = 900$ s) to represent the pdf of the data, the ratio of $P_E$ for the non-Gaussian model to Gaussian is equal to 1.84. Using the Hermite moment model, this ratio is 1.76 and using a lognormal model the ratio is 1.88. For comparison's sake, the limits of the integration have been taken to be the end points of the range for which pressure data is available in the realization of data considered. This illustrates the significance of non-Gaussian effects in the evaluation of $P_E$ and more importantly the effectiveness of the moment-based Hermite transformation in capturing this significance.

**Gust factors**

The use of gust factors to account for the dynamics of wind fluctuations is accepted worldwide. The concept, based on the original formulation by Davenport, relies on the assumption of a Gaussian process. For dynamic pressures resulting from the square of wind velocity the Gaussian assumption may break down. Soize and Kareem and Zhao have extended Davenport's Gaussian model to include non-Gaussian effects, which are more pronounced for relatively stiffer structures. The non-Gaussian contribution also increases for high levels of turbulence. The gust factor, $G$, relates the mean of the extremes of a process to the mean of the parent process as follows:

$$X_{\text{ex}} = X + g \sigma = X \left( 1 + \frac{g^2}{X} \right) = G X$$

(27)

where $g$ is the peak factor. When $X = 0$, we simply have $X_{\text{ex}} = g \sigma$ and we need merely to compute the peak factor. The peak factor used in the non-Gaussian gust factor formulation employs the moment-based Hermite transformation which has been shown to be more accurate in representing the tail regions of the pdf of a non-Gaussian process than the Edgeworth series employed by Soize. Treating the standardized non-Gaussian random variable as a non-linear function of a Gaussian random variable as in eqn (14), the probability density function of the process $X$ may be readily derived (e.g. Ref. 32).

According to Cartwright and Longuet-Higgins, the distribution of all maxima (positive and negative) of the standardized Gaussian process are given as

$$p_{\text{max}}(u) = \frac{1}{\sqrt{2\pi}} \left[ \varepsilon \exp \left( -\frac{u^2}{2\varepsilon^2} \right) ight. \\
+ \sqrt{1 - \varepsilon^2} u \exp \left( -\frac{u^2}{2} \right) \\
\times \int_{-\infty}^{\varepsilon u} \exp \left( -\frac{\varepsilon^2 x^2}{2} \right) dx$$

(28)

where $\varepsilon = (1 - m_2 / (m_0 m_4))$ is a descriptor of the bandwidth of the parent Gaussian process and $m_2$ can be described in terms of moments of the one-sided spectral density of the process, $m_2 = \int_0^\infty n S(n) dn$, where $n$ is frequency in Hertz. For a narrow-banded process, $\varepsilon = 0$, and eqn (28) yields the Rayleigh distribution.

The cumulative distribution of the extremes of the
parent Gaussian process is (e.g. Refs 9, 34)
\[ P_{U_\alpha}(u) = \exp[-N(1 - P_{U_{\text{max}}}(u))] \]  
(29)
where \( N = \sqrt{m_2/m_4}T \) is the expected number of maxima during an interval of length, \( T \) (e.g. Ref. 35), and from eqn (28) assuming that \( u \) is large, we may approximate \( 33 \)
\[ 1 - P_{U_{\text{max}}} \approx \sqrt{1 - e^{-\frac{u^2}{2}}} + O \left( \frac{1}{u} \exp \left( -\frac{u^2}{2e^2} \right) \right) \approx \frac{m_2}{\sqrt{m_2 m_4}} \exp \left( -\frac{u^2}{2} \right) \]  
(30)
which leaves
\[ P_{U_{\alpha}}(u) = \exp \left[ -\sqrt{m_2/m_0}T \exp \left( -\frac{u^2}{2} \right) \right] \]  
(31)
So, we have for the extremes of \( X \)
\[ P_{X\text{ex}}(x) = \exp \left[ -\sqrt{m_2/m_0}T \exp \left( -\frac{u^2(x)}{2} \right) \right] \]  
(32)
The peak factor, \( g_{\text{ng}} \), relates the mean of the positive extreme values of \( X \) to its standard deviation. To compute it, we must first determine \( dP_{X_{\text{ex}}}(x) \). Making the assumption in eqn (30), we have for the case when \( X \) is zero mean
\[ dP_{X_{\text{ex}}}(x) = \exp(-\psi) d\psi, \quad \psi = \sqrt{m_2/m_0} \exp[ -u^2(x)/2 ] \]  
(33)
This gives
\[ X_{\text{ex}} = \int_0^\infty x(\psi) \exp(-\psi) d\psi = g_{\text{ng}} \sigma_x \]  
(34)
In order to evaluate eqn (34), we must develop the form of \( x(\psi) \). This is accomplished by first solving for \( u(x) \), according to
\[ u(x) = \sqrt{2\ln\psi} - 2\ln\psi \]  
(35)
Since the time, \( T \), is usually very large, an asymptotic expansion for eqn (35) is
\[ u(x) = \beta - \ln\psi - \frac{(\ln\psi)^2}{2\beta^2} + \ldots \]  
(36)
where \( \beta = \sqrt{2\ln\psi}/T \). Substituting eqn (36) into the moment-based Hermite transformation model and retaining terms of \( O(\beta^{-1}) \) and greater, yields
\[ g_{\text{ng}} = k \left\{ \left( \beta + \frac{\gamma}{\beta} \right) + \hat{h}_3 (\beta^3 + 2\beta - 1) + \hat{h}_4 \times \left[ \beta^3 + 3\beta(\gamma - 1) + \frac{3}{\beta} \left( \frac{2}{12} - \gamma + \frac{\gamma^2}{2} \right) \right] \right\} \]  
(37)
where \( \gamma = 0.5772 \) (Euler’s constant), \( \beta = \sqrt{2\ln(\nu_0 T)} \) and \( \nu_0 \) is the zero-upcrossing rate, \( k, \hat{h}_3, \hat{h}_4 \) are functions of skewness and kurtosis. For Gaussian processes, \( \hat{h}_3 = 0 \) and \( \hat{h}_4 = 0 \) which reduces eqn (37) to the standard Gaussian form given by Davenport. \( 35 \) For the 900 s realization of data in Fig. 6, \( \nu_0 = 1.334 \). The peak factor based on eqn (37) using statistical information derived from the data is equal to -7.4, whereas the corresponding value for the Gaussian case is -3.9. It is important to note that eqn (37) is for positive extremes and for the negative extremes which we have considered here, the opposite of the skewness value must be used.

By comparison, Cheong \( 37 \) treats exceedence of a threshold which lies two standard deviations below the mean as a separate random variable. By choosing a low threshold, successive exceedences may be considered independent. The occurrence of exceedences of the threshold is modeled as a Poisson process and the distribution of exceedences is modeled by an exponential distribution. From this model, the distribution of the largest exceedence, \( S \), for a duration, \( t \), is derived as
\[ P_S(t) = e^{-\theta t} \]  
(38)
where \( \mu \) and \( \theta \) may be obtained from data as the reciprocals of the mean exceedence and the mean inter-exceedence interval, respectively. In terms of the distribution parameters, the expected value and the most probable value of the maximum exceedence are given by
\[ E[S(t)] = \ln(\theta t) + 0.5772 \]  
(39)
Thus, the expected minimum \( C_p \) and its most probable minimum are
\[ E[C_p(t)] = C_p - 2\sigma_c - E[S(t)] \]  
(40)
\[ C_{p_{\text{mp}}} = C_p - 2\sigma_c - S_{\text{mp}} \]
For the data considered above, \( E[C_p(t)] = -7.019 \), \( C_{p_{\text{mp}}} = -6.562 \). These values approach the peak factor attained by using the moment-based Hermite transformation model, but are somewhat less conservative in their estimate of the extreme value. This can be noted in Figs 6 and 7.

In a recent study, Krayer and Marshall \( 38 \) have pointed out that the linear gust factor based on extratropical storms may underestimate the gust factor for hurricane conditions. It is quite clear that the hurricane wind field comprised of turbulence due to convective processes superimposed upon a large, coherent vertical structure results in non-Gaussian fluctuations. Near the ground, a change in energy distribution with respect to frequency may result from non-linear interactions between different frequency components. Changes in the pdf and the power spectral density would certainly introduce changes in the statistics of velocity fluctuations, leading to a larger gust factor. A closer examination using a theory based
on the preceding comments is being pursued to model the observed data.

SIMULATION OF NON-GAUSSIAN PROCESSES

Among a host of approximate analytical techniques developed for the analysis and prediction of non-linear system response, simulation methods are becoming more attractive due to the increasing ability of high speed computers. For implementation of time domain schemes, the time histories of loading functions are generated in accordance with desired statistical and spectral characteristics. The simulation procedures for Gaussian random processes are well established. However, progress in the simulation of non-Gaussian processes has been elusive. A recent book on non-Gaussian processes provides an excellent overview of current methods of non-Gaussian simulation. Several promising methods currently being pursued by the authors are presented here.

Correlation-distortion method

An approach used by Yamazaki and Shinozuka for the simulation of non-Gaussian processes begins with the simulation of a Gaussian process which is then transformed to the desired non-Gaussian process through the following mapping:

\[ X(t) = F^{-1}_x \{ \phi(y) \} \]  

(41)

An iterative procedure is necessary to match the desired target spectrum since the non-linear transformation in eqn (41) also modifies the spectral contents.

The necessity for an iterative procedure may be eliminated if one begins with the target spectrum or auto-correlation of the non-Gaussian process and transforms it to the underlying correlation of the Gaussian process. Then, a simulation based on the schematic shown in Fig. 8 would eliminate the spectral distortion caused by the non-linear transformation. This approach is referred to as the correlation-distortion method in stochastic system literature. For a given static single-valued nonlinearity \( x = g(u) \), where \( u \) is a standard normal Gaussian process, the desired autocorrelation of \( x \) in terms of \( y \) can be expressed as

\[ R_{xx}(\tau) = \sum_{k=0}^{\infty} a_k^2 \rho_{xx}(\tau) \]

\[ a_k = \frac{1}{\sqrt{2\pi k!}} \int_{-\infty}^{\infty} g(\sigma u) \exp \left( -\frac{u^2}{2} \right) H_k(u) du \]  

(42)

where \( \rho_{xx} \) is the normalized autocorrelation of the non-Gaussian process, and \( H_k(u) \) is the kth Hermite polynomial given by

\[ H_k(u) = (-1)^k \frac{d^k}{du^k} \exp \left( -\frac{u^2}{2} \right) \]  

(43)

An alternative to the preceding approach is to express \( x \) as a function of a polynomial whose coefficients are determined by a minimization procedure (e.g. Ref. 49). Another approach uses translational models involving the Hermite moment transformation described earlier. In this study, we utilize a Hermite model. The simulation algorithm is as follows: (i) estimate the auto-correlation of the mean-removed normalized parent non-Gaussian process to be simulated; (ii) transform to the auto-correlation of the underlying Gaussian process by solving

\[ R_{xx}(\tau) = \alpha^2 \left[ R_{uu}(\tau) + 2\beta \frac{d}{d\tau} R_{uu}(\tau) + 6\beta^2 R_{uu}(\tau) \right] \]  

(44)

for \( R_{uu}(\tau) \), where eqn (44) is a truncated infinite series, and the unspecified parameters are defined in eqn (16); (iii) simulate a Gaussian process using the spectrum associated with \( R_{uu}(\tau) \); (iv) transform this process back to a non-Gaussian process using

\[ x = \alpha [u + \bar{h}_3 (u^3 - 1)] + \bar{h}_4 (u^3 - 3u) \]  

(45)

(v) replace the mean and variance of the original parent process. A sample realization of a simulated process consistent with the data in Fig. 6 is given in Fig. 9. The comparison of simulated and target pdfs and spectral characteristics is excellent and is shown in Fig. 10, where the simulation results are an ensemble average of 100 realizations.

Fig. 8. Schematic of the correlation-distortion method of non-Gaussian process simulation.
Direct transformation methods

Another application of the correlation-based approach concerns the simulation of a process consistent with a sample of a non-Gaussian time history. The non-Gaussian sample process, $x(t)$, is transformed to its Gaussian underlying form, $u(x)$, through eqn (16). Subsequently, linear simulations created through standard techniques based on the target spectrum of the Gaussian process are transformed back to the non-Gaussian parent form through eqn (45). The shortcoming of this direct transformation technique is the distortion of the frequency distribution of signal energy. The resulting simulated non-Gaussian signal power spectrum does not match the parent non-Gaussian spectrum to a satisfactory degree. This distortion may stem from the inability of the three-term truncated Hermite moment transformation in eqn (16) to produce a Gaussian signal for cases when the parent signal is highly non-Gaussian. The linear simulation is then based on a target spectrum derived from a process which is assumed Gaussian, but is not. It is at this point where the frequency information is distorted, and results in poor simulations. One option for improved results is to add terms to the Hermite series until a Gaussian transformation is achieved. This may require a different number of terms to achieve accuracy for varying input sample signals.

A simple correction has been suggested to remove this distortion in the direct transformation method.45
Referring to eqn (16), it can be seen that the governing parameters \( \tilde{h}_3, \tilde{h}_4, a, b, c, \alpha \) and thus \( u(x) \) are dependent on the skewness and kurtosis. \( \gamma_3 \) and \( \gamma_4 \) may be treated as adjustable input parameters in order to force the transformed process, \( u(x) \), to be Gaussian. Optimization of these two parameters is based on the minimization of the function

\[
\min(\gamma_3^2 + \gamma_4^2)
\]

where \( \gamma_3 \) and \( \gamma_4 \) are the skewness and kurtosis of the inverse Hermite transformed process \( u(x) \). The optimized input parameters \( \gamma_3 \) and \( \gamma_4 \) now provide a Gaussian process, and the linear simulations do not contain distortion. The same parameters are used to transform back to a non-Gaussian simulation whose pdf and power spectral density closely match those of the parent process. This correction is essentially a quantification of the error in truncating the Hermite series after the third order.

An example is given in Fig. 11, where the simulated and target pdf are shown along with a sample realization of the measured pressure in Fig. 6. The pdf is an ensemble average of the pdfs from 100 realizations of the same length as the original record. The comparison of target and simulation power spectral density is similar to that in Fig. 10, and is not shown here.

**Simulation based on higher-order transfer functions**

The Volterra series model in the frequency domain (eqn (2)) lends itself to the simulation of non-linear processes for which the Volterra kernels are available or may be estimated. A second-order non-Gaussian signal resulting from a quadratic transformation of a Gaussian process may be simulated by the addition of second-order contributions to the complex spectral amplitude components at the appropriate sum and difference frequencies before inverse Fourier transforming the sequence to the time domain. These second-order contributions are formed from the products of pairs of linear Fourier components with the QTF in the frequency domain, and correlate the phase between various frequency components to a degree weighted by the QTF. The memory retained by convolution with the QTF facilitates the simulation of processes that are able to match not only the power spectrum and pdf of the parent process, but the bispectrum as well.

This approach requires information concerning the QTF of the desired process. In the case of non-Gaussian waves and their loads on offshore structures, the QTFs can be derived computationally. However, in the case of wind effects this is not possible due to the complexity of non-linear interactions that take place as turbulent wind encounters a structure. In the absence of the necessary information, it is possible to estimate a QTF based on the desired process and an extracted underlying Gaussian process. Such an attempt is made here by using the pressure data in Fig. 6. Due to the presence of higher-order non-linearities, an estimated QTF may not completely model the non-Gaussian features. This concept in principle may easily be extended to the simulation of non-linear processes beyond the second order, where higher-order information, e.g. trispectrum, may improve the model.

A simulation based on a QTF extracted from pressure data is shown in Fig. 12, along with a sample of the measured data, and indeed reflects a lack of completeness in the second-order Volterra model. The model fails to properly reflect the skewness and kurtosis of the measured record. The use of higher-order transfer functions is currently being researched to improve upon the simulation. A second example is shown in Fig. 13, where the data being simulated is mean-removed.
measured wind velocity from a hurricane. In this case the higher-order statistics of the measured and simulated processes match up well, and the visual comparison is much closer to the target than the previous example. The second-order Volterra model is appropriate for the severity and type of non-linearity in this case.

The role of phase tailoring

It is also interesting to examine the role of phase tailoring. It will be shown here that certain constraints on the envelope of the time series may be accommodated by controlling the phase. The widely used simulation approach for generating processes with a prescribed spectral density function uses the inverse Fourier transform of a complex amplitude spectrum whose components possess deterministically chosen amplitudes and random phases. For a phase angle of zero, the simulated time history represents the impulse response of a system. For a linearly varying phase spectrum, the resulting time series is the same as a delayed zero phase. Accordingly, a random phase results in a random signal. This is in direct contrast to the distinctly transient, impulse response-like signal which is generated when smooth and deterministic phase variation with frequency is introduced. An assumed description of the phase spectrum does not alter the spectral characteristics of the simulated time history. It is thus possible to introduce spiky features or grouping effects in a simulated time history by tailoring the phase spectrum before inverse Fourier transformation. Such deterministic signals cannot be utilized for statistical analysis, but may be quite useful for deterministic applications such as input to a system under test loading. The importance of phase information in the Fourier representation of non-Gaussian pressure records is used as the basis for a simulation method which combines an autoregressive model with Fourier transformation. Kobayashi et al. have implemented the concept of phase tailoring in a wind-tunnel simulation of wind gusts observed at a site utilizing oscillating vanes.

The phase spectrum contains no relevant information under the assumption of a Gaussian random process. However, attention has recently been given to the identification of phase information for non-Gaussian processes. Bispectral analysis of measured processes, e.g. ocean wave records, indicates the presence of phase coupling among the various component wave frequencies in many cases. This coupling results in a non-Gaussian signal which is capable of dramatically altering the response statistics of a system thought to be subject to Gaussian input. The removal of the coupled phase information from the record returns a Gaussian signal with no significant bispectral characteristics, but identical autospectra. This demonstrates the potential importance of the phase information in

Fig. 12. Non-Gaussian simulated pressure realization using eqn (2) (left), and a measured sample of pressure data (right).

Fig. 13. Non-Gaussian simulated hurricane wind velocity realization using eqn (2) (left), and a measured sample of hurricane wind velocity data (right).
identifying non-Gaussian signals, and is the basis of the simulation of non-Gaussian signals in the previous section, where the second-order contributions to the linear complex spectral amplitudes result in phase coupling weighted by QTF. The interpretation of information from the phase spectrum of non-Gaussian signals is still not well established and is an area of current research in wave mechanics, e.g. Ref. 52. Higher-order spectral analysis offers a convenient format which has provided significant insight into phase information, and is currently being used as a tool to identify distinctive phase characteristics of non-Gaussian wind pressure data.

In a less complicated application of phase tailoring, the injection of constant phase over a small frequency range of otherwise random phase results in a signal with characteristics often desired in the simulation and analysis of system response to particular types of grouped input. This concept may be applied to simulating concentrated groups of turbulent gusts during a thunderstorm in synthetic wind records. Further work is needed to identify and quantify the existence of such groups of gusts in thunderstorms.

### Conditional simulation

Simulation of random velocity and pressure signals at uninstrumented locations of a structure conditioned on measured records are often needed in wind engineering. For example, malfunctioning instruments may leave a hole in a data set or information may be lacking due to a limited number of sensors. This concept is similar to conditional sampling in experiments or numerical simulations. This field has matured significantly in the last few years (e.g. Refs 54–58). Fundamentally, two approaches have been introduced in which the simulation is either based on a linear estimation or kriging, or on a conditional probability density function. Following Borgman’s work on ocean waves,54 Murulidharan and Kareem59 have developed schemes for conditional simulation of Gaussian wind fields utilizing both frequency and time domain conditioning. The conditional simulation permits generation of time histories at new locations when one or more time series for the full length interval are given, and extension of existing records beyond the sampling time for cases where conditioning time series are limited to a small subinterval of the full length. Consider a pair of correlated Gaussian random vectors $V_1$ and $V_2$. Let the bivariate normal distribution of these variables be denoted

$$p(V) = p\left(\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \right) = N\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}\right)$$

(47)

where $\mu_1$ is the mean value, and $C_{ij}$ is the auto or cross-covariance between the variables. If a sample of $V_1$ is measured and denoted as $v_1$, then it is the conditional simulation of $V_2$ based on the measured record that is desired. The conditional pdf for $V_2$ given the information on $V_1$ is expressed as

$$p(V_2|V_1 = v_1) = N(\mu_2 + C_{12}C_{11}^{-1}(v_1 - \mu_1), C_{22} - C_{12}C_{11}^{-1}C_{12})$$

(48)

and a conditional simulation is provided by

$$V_2|V_1 = v_1 = C_{12}C_{11}^{-1}(v_1 - V_1) + V_2$$

(49)

Derivations of the covariance matrices $C_{11}$ and $C_{12}$ in the time and frequency domains then provide all that is needed for conditional simulation. Details concerning these matrices for wind simulation may be found in Murulidharan and Kareem.59 In a conditionally simulated field, fluctuations at intermediate points will follow the fluctuations of the surrounding locations provided the scale of fluctuations at surrounding locations. An interesting application of conditional simulation concerns generation of wind velocity fluctuation at a large number of grid points as an upward boundary condition for a computational study conditioned on measurements at a limited number of locations in a wind tunnel.59

An example application of Gaussian conditional simulation is shown in Figs 14 and 17. These examples are based on measured correlated wind velocity records at four elevations on a full-scale tower with the mean removed. Figure 14 shows three of these records at stations 1 through 3 and a frequency domain conditional simulation of the fourth location based on information from the other three known records. Here, eqn (49) is used for a uni-dimensional multivariate conditional simulation. Figure 15 is a comparison of the target cospectrum with the cospectrum between the measured records at station 1 and 4, and the cospectrum between the conditionally simulated record at station 4 and the measured record at station 1. The jaggedness of the cospectra in the figure arises from the variance inherent in individual realizations. An ensemble average of many simulations results in a smooth cospectrum which lies along the target cospectrum. Figure 16 is a comparison of the target power spectral density with that from the measured and conditionally simulated records at station 4. Figure 17 shows a measured record up to 2500 s in the top figure. It is assumed here that the record is only available up to 980 s, indicated by the darker portion of the signal. A time domain conditional simulation of the record from 980–2500 s is shown as the lighter portion in the bottom figure, based on information from the first 980 s. The lighter part of the top figure indicates the portion of this signal which is not known when generating the bottom figure. Both examples demonstrate the effectiveness and utility of conditional simulation.

In cases involving non-Gaussian processes, the conditional simulation schemes suffer impediments like their Gaussian counterparts. In Elishakoff et al.,38
iterative schemes have been utilized to simulate non-Gaussian processes. The authors sought to combine the techniques developed for unconditional simulation of non-Gaussian processes and the procedure of conditional Gaussian processes. The non-Gaussian known processes are mapped into underlying Gaussian processes, where conditional simulation is done. These simulated time histories are then mapped back into the non-Gaussian domain.

**WAVELET TRANSFORMS**

The inability of conventional Fourier analysis to preserve the time dependence and describe the evolutionary spectral characteristics of non-stationary processes requires tools which allow time and frequency localization beyond customary Fourier analysis. The short-term Fourier transform (STFT) provides time and frequency localization to establish a local spectrum for

**Fig. 14.** Measured wind velocity at 40, 60 and 80 m, frequency domain conditional simulation at 100 m based on records at lower three stations.

**Fig. 15.** Conditional, measured, and target cospectral density between stations 1 and 4.
any time instant. The problem is that high resolution cannot be obtained in both time and frequency domains simultaneously. The moving window must be chosen for locating sharp peaks or low frequency features, because of the inverse relation between window length and the corresponding frequency bandwidth.

This drawback can be alleviated if one has the flexibility to allow the resolution in time and frequency to vary in the time–frequency plane to reach a multi-resolution representation of the process. This is possible if the analysis is viewed as a filter bank consisting of band-pass filters with constant relative bandwidths. One type of local transform is the recently developed wavelet transform (WT) which decomposes a signal using

**Fig. 16.** Target, measured, and conditionally simulated power spectral density at station 4.

**Fig. 17.** Measured wind velocity (top figure) and time domain conditional simulation (light portion of bottom figure).
wavelet functions. Fourier methods of signal decomposition use infinite sines and cosines as basis functions, whereas the wavelet transform uses a set of orthogonal basis functions which are local. Various dilations and translations of a parent wavelet are joined to form the family of basis functions. This allows the retention of local transient signal characteristics beyond the capabilities of the harmonic basis functions. The wavelet transform allows a multi-resolution representation of a process and provides a flexible time–frequency window which narrows to observe high frequency energy content, and broadens to capture low frequency phenomena.

**Brief wavelet overview**

Development of the parent wavelet begins with the solution of a dilation equation to determine a scaling function $\phi(n)$, dependent on certain restrictions. The scaling function is used to define the parent wavelet function, $\psi(n)$. The basis functions used to represent the signal are defined by translations and dilations of the parent wavelet. The shape of the parent wavelet is not a single unique shape, but depends on the desired wavelet order.

The signal being decomposed must consist of $2^M$ samples, where $M$ is an integer. Wavelet analysis decomposes the signal into $M + 1$ levels, where the level is denoted as $i$, and the levels are numbered $i = 1, 0, 1, \ldots, M - 1$. Each level of $i$ consists of $j = 2^i$ translated and partially overlapping wavelets equally spaced $2^M/j$ intervals apart. The $j = 2^i$ wavelets at level $i$ are dilated such that an individual wavelet spans $N - 1$ of that levels intervals, where $N$ is the order of the wavelet being applied. Each of the $j = 2^i$ wavelets at level $i$ is scaled by a coefficient $a_{ij}$ determined by the forward wavelet transform, a convolution of the signal with the wavelet. The notation is such that $i$ corresponds to the wavelet dilation, and $j$ is the wavelet translation in level $i$. $a_{ij}$ is often written as a vector $a_{2^i,j}$, where $j = 0, 1, \ldots, i - 1$. There are as many wavelet coefficients as signal samples. The level $i = -1$ is the signal mean value. A variety of packages are available to perform discrete wavelet transform (DWT) analysis (e.g. Ref. 62).

**Applications to wind engineering**

The present research concerns the use of wavelets to aid in the analysis and simulation of non-stationary data. Multi-scale decomposition of processes utilizing wavelets reveals events otherwise hidden in the original time history. Wavelet coefficients may be used to derive an estimate of the power spectrum. These estimates may be extended to multi-variate, e.g. cospectral, estimation. The wavelet coefficients provide the scalogram, which describes the signal energy on a time-scale domain over a range of logarithmically-spaced frequency bands. This facilitates identification of time-varying energy flux and spectral evolution. The property of accurate energy representation lends itself well to signal reconstruction and simulation. A stochastic manipulation of the wavelet coefficients leads to a simulation which is statistically similar to the original signal.

**Wavelet filterbank**

Figure 18 presents the time history of the response of a large floating structure to wind and wave loads, and the resulting band-passed time histories using a wavelet-based filterbank. The summation of the band-passed histories returns the original time history. This figure unfolds the response time history into a very revealing display of the time-scale representation. The top left block is the mean-removed original signal, the blocks following column-wise downward are the band-pass filtered signal in order of decreasing frequency, and the lower right block is the low pass channel or mean of the signal. Note the different scales on the plots for the filtered processes, indicating relative contribution in that frequency band. The power spectral density of the signal in Fig. 18 is shown in Fig. 19, in which the frequency bands 1 through 7 of the filtered process are marked. The higher relative magnitudes of bands 3 and 4 correspond to the right peak in the spectrum, and are due to first-order wave effects. The high relative magnitude in bands 6 and 7 corresponds to structural resonance due to wind and second-order wave effects. The wavelet-based filter bank has helped to identify, e.g. high frequency spikes and their time of occurrence, associated with waves slamming the structure, observed in bands 1 and 2. These transient events in the response of the structure exposed to wind and wave fields are not clearly discernible in the time history where large excursions may be due to either occasional slamming, or large but not slamming waves. The improved efficiency over FFT and other filtering techniques, e.g. multi-filtering with simple oscillators (e.g. Ref. 63), renders wavelet filterbanks a quick and convenient time-scale decomposition method.

**Signal analysis with spectral methods and wavelets/time-scale decomposition**

A wavelet power spectrum is estimated by plotting the summed coefficients with respect to the scale axis only. Small changes in frequency within an octave band are not easily resolvable. This is a larger problem for the high frequency range where the octave spans half the total frequency range. This may be alleviated by several methods which allow intra-octave wavelet coefficient estimation. One option is to apply a number of slightly dilated parent wavelets recursively to the data, creating a denser sampling grid than the octave-by-octave grid used by the original parent wavelet. Another method
used in this study is the application of zoom techniques to the filtered data.

An example of octave band and intra-octave band wavelet spectral estimations and an FFT-based estimation are shown in Fig. 20. The left figure is the power spectrum of the response of an offshore platform to wind loads, and the right figure is the cospectrum between the measured wind speed input and resulting platform response. The areas under the wavelet spectral and cospectral estimates represent the true variance and covariance from the time histories almost exactly, while the area under the FFT estimate does not. Further, smoothing of the FFT estimate with segment averaging renders its resolution inferior to that of the wavelet estimate at low frequencies. The FFT spectral estimates are the average of eight segments, while the wavelet estimate is based on the entire data record.

Wavelength coefficients in an octave band represent the energy at time intervals equally spaced over the duration of the signal, and may be used to analyze non-stationary events for transient and evolutionary phenomena. Accordingly, the transfer of energy from one octave band to the next may be observed along the time-scale in the scalogram. Two example applications of the scalogram are shown in Figs 21 and 22. In Fig. 21 the analyzed signal in the top plot is a sine wave of constant amplitude whose frequency is steadily increased in time.

The transfer of energy from lower to higher frequencies in time is clearly demonstrated as the dark region. In Fig. 22, the signal is a hurricane velocity record measured after the hurricane eye has passed the instrument. In the scalogram the light region shows the band of frequency content of the record remains relatively constant, while the magnitude at earlier times is larger than at later time, suggesting non-stationary features.

The concept of applying a modulated stationary process centered at narrow-banded frequencies to model ground motion has been extensively used (e.g. Refs 65, 66). In
this representation each component process, \( s_j(t) \), is modulated by a different modulating function \( m_j(t) \)

\[
x(t) = \sum_j m_j(t)s_j(t)
\]

(50)

There are different approaches to modeling \( m_j \) and \( s_j \) to describe \( x \).

The retention of both time and frequency information makes wavelets a useful tool for the simulation of non-stationary signals. This can be done given either a parent non-stationary signal, or a target spectrum and modulation function for each octave. Given a parent non-stationary signal, e.g. a local wind velocity record, an ensemble of signals may be simulated whose average statistics closely resemble those of the parent process. The parent signal is discrete wavelet transformed (DWT), and the coefficients multiplied by a Gaussian white noise of unit variance \( \omega(n) \). The inverse wavelet transform (IWT) then produces a simulation statistically similar to the parent process.

Given a target spectrum and modulator functions for each octave, the simulation is done by first finding the

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**Fig. 19.** Power spectral density of measured offshore structural response seen in Fig. 18.

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**Fig. 20.** Left — Power spectrum estimates of offshore platform response using FFT, octave band wavelet, and intra-octave band wavelet estimation techniques. Right — cospectrum estimates between wind velocity input and resulting platform response.
When a parent signal is used to determine the modulator function, the measured wavelet coefficients $a_{ij}$ and target spectrum are used as

$$m_{ij} = A_i \sqrt{2^{i+2-2^j} \frac{|d_{ij}|}{S_i}}$$ (52)

where $A_i$ is the level-dependent amplitude constant and $S_i$ is the energy corresponding to the $i$th octave from the target power spectrum. Figure 23 shows a measured non-stationary wind velocity record, and a simulated process using the wavelet transform. Both statistical and visual comparisons between the target and simulated records are good.

CONCLUDING REMARKS

Progress in quantifying and simulating the non-Gaussian and non-stationary effects of wind on structures has been elusive due to the limitations of traditional analytical tools. Here, an overview of techniques is presented with examples which aid in the efficient modeling, simulation, and pdf estimation of non-Gaussian processes. The estimation of Volterra kernels from system identification is addressed, as well as other representations for non-linear systems. The estimated pdf of non-linear system response is presented via several methods with examples which involve the use of Volterra kernels in the Kac–Siebert approach, the use of joint moment information as constraints on system entropy, and the use of moment-based Hermite transformation models. Several techniques for simulating non-Gaussian signals, including convolving Fourier amplitude pairs with higher-order Volterra kernels, non-linear mapping, as well as the concept of conditional
simulation, are discussed. The implications of non-Gaussian winds and their load effects on fatigue damage and gust factor representation of dynamic wind loads are illustrated. The analysis and simulation of non-stationary processes is accomplished by the application of localized basis functions via the wavelet transform. Applications and examples are given which pertain to non-stationary effects of wind on structures.

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