Feedback–feedforward control of offshore platforms under random waves

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SUMMARY

This study investigates the control of jacket-type offshore platforms. The deck displacement of jacket-type offshore platforms can be controlled using both passive and active control mechanisms. Among the passive control mechanisms, a tuned mass damper concept is studied in this paper. Active control mechanisms considered here include the active mass damper, the active tendon mechanism and the propeller thruster. An optimal frequency domain approach to active control of wave-excited platforms is used in which the $H_2$ norm of the transfer function from the external disturbance to the regulated output is minimized. In this study, the hydrodynamic drag force is evaluated using the JONSWAP wave spectrum. Unlike conventional linearization approaches, the influence of non-linearity in the drag force is retained in this scheme by expressing the non-linear force components in terms of higher-order convolutions of the water-particle velocities. To demonstrate the effectiveness of this scheme, the platform performance with and without control devices under different sea states is evaluated. It is demonstrated that the control devices are useful in reducing the displacement response of jacket-type offshore platforms, especially when the wave forces are concentrated at frequencies close to the natural frequencies of the platform. This becomes especially significant in deep waters because the natural frequencies of jacket-type platforms fall closer to the dominant wave frequencies in deep waters. Adding control devices to deep water platforms will ensure a reduction both in the global response of the platform and in localized effects, such as the fatigue of welded joints. Copyright © 2001 John Wiley & Sons, Ltd.

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INTRODUCTION

As the amount of oil near the shores decreases, the need to tap oil resources located in deep waters or hostile environments increases. Therefore, improvements in the design and

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construction practices of offshore drilling and production platforms are necessary [1]. It is essential that these platforms be able to withstand the action of wind, waves and earthquakes under both operating and extreme conditions.

In deep water, the natural frequencies of conventional gravity type platforms are within the range of everyday wave heights. This results in excessive fatigue-related damage to the platform. In the case of extreme waves in deep water offshore platforms can experience large motions. Excessive motions of the platforms can threaten their safety and operation [2]. To prevent fatigue damage and protect the operations and crew of the platform, a number of design modifications have been introduced to limit such impacts. These modifications include utilizing compliant platforms (e.g. guyed towers and tension leg platforms). Other possibilities involving implementation of control devices exist to reduce both fatigue effects and survival during extreme events. Despite the rich tradition of active control systems in the dynamic positioning of drilling ships, control devices have not been actively pursued by the offshore industry. This may be attributed to deck space being at a premium, needs for temperature control of energy-dissipating fluids and power and maintenance requirements of an active system. In civil engineering literature, few studies have been conducted using active control devices in offshore structures [2–9]. However, with recent success and burgeoning growth in the application of active systems in land-based structures such as tall buildings, TV towers and other structures, interest in these devices for possible application to offshore structures is receiving more attention [10–13].

There are two types of devices that can be used to control structural response, i.e. passive and active. Passive control devices utilize the dynamics of the structure itself to dissipate energy. One passive control device that can be used to control the structural response of offshore platforms is the tuned mass damper (TMD). The TMD is commonly used for the control of tall buildings under wind excitation [11, 12, 14]. The secondary mass of inertial type damping devices such as a TMD can be replaced by a water tank in which the sloshing of liquid mimics the actions of a TMD consisting of a mass damper and spring system. However, the ability of a passive device to control the response of a structure is limited, and the device can generally be tuned to only one of the structure’s natural frequencies.

The limitations that are inherent in passive devices can be overcome by using an active control device. Active control devices utilize an external power source to generate forces which control the structure. These generated forces are called control forces. Active control mechanisms that can be used in offshore structures include the active mass damper (AMD), the active tendon system and the propeller thruster. One type of active control device that has received some recent attention for offshore platforms is the thruster-assisted mooring system [8, 9]. Mooring systems help to maintain the general position of the platform. However, even with a mooring system, a platform may be susceptible to slow drift oscillations caused by currents and wind. To counteract such motion, thrusters can be added to the mooring system. Therefore, attention has been given to designing a thruster-assisted mooring system which can limit the motions caused by currents and wind. Another active control device that has been studied is active pneumatic control [2]. This device limits the deflections of a platform by individually controlling the air pressure inside air tanks located at the bottom of the platform.

In this paper, a jacket-type platform under random seas is used to demonstrate the effectiveness of various control devices. The analysis in this study is based on a frequency domain approach that minimizes the $H_2$ norm of the transfer function from the external disturbance to the regulated output [6, 15–17]. The displacement of the platform deck is controlled using
both passive and active control mechanisms. The approach presented in this paper includes a feedforward–feedback linkage which enhances performance of the controller.

THEORETICAL BACKGROUND

A state-space representation of wave load effects is presented in this section. This is followed by brief theoretical background concerning frequency-domain-based $H_2$ control, and its solution procedure. An example is then presented to illustrate the procedure and to evaluate the performance of different systems.

STATE–SPACE MODELLING OF WAVE LOAD EFFECTS—FEEDFORWARD LINK

In control theory, the equations of motion of a system are frequently represented using a state-space description. This general representation is readily applicable in numerical algorithms for analysis and synthesis. In structural control, the state vector is typically a $2n$-dimensional vector of displacements and velocities for an $n$-degree-of-freedom structure. Similarly, the external loading needs to be expressed in the state-space framework, which is then combined with the structural model for the control system design and analysis.

In many cases, the spectral properties of the environmental loadings on a structure are known and can be modelled. By including such models in the control design, better control of the structure can be obtained using little additional effort. The feedforward links can be formulated using two types of input. First, the feedforward link can be based on established spectral characteristics of an excitation such as earthquakes, winds or waves [16, 18, 19]. Second, the feedforward loops can be configured based on actual real-time measurements by fitting models such as the auto-regressive (AR) model to data [20, 21]. A feedback–feedforward control scheme includes a stochastic model of the external disturbance in the control design (e.g. References [16, 19]). In these studies, the external disturbance was an earthquake ground acceleration and wind velocity fluctuations, respectively. The equations of motion of the structural system were augmented with appropriate earthquake and wind excitation models obtained by filtering a Gaussian white noise process. The augmented equations of motion were then used to determine a control which utilized both feedback and feedforward compensation. The feedback loop incorporated measurements of the response of the structure into the control law. The information from both the structure and the excitation was utilized in the feedforward control law with an observer designed to estimate the states of the external excitation model based upon the base acceleration or mean wind speed information. Yamada and Kobori [20] and Mei et al. [21] utilized real-time measurements of earthquakes and fitted these in terms of AR models. These models were augmented with structural models for real-time control force calculation. The latter approach offers a more robust feedforward input as it is not based on a pre-defined spectral description. Rather, it adjusts to reflect the nature of actual measured input. In the following discussion, a state-space representation for wave load effects is presented.

Power spectral density functions of wave forces

The wave-induced loads on structures are given by the Morison equation [22]. This equation has an inertial force term and a drag term. The latter contains a non-linear term involving the
water-particle velocity. Historically, analyses of non-linear systems in the frequency domain have been based on the statistical linearization approach (e.g. References [23, 24]). However, the commonly used method of equivalent linearization fails to capture the higher-order statistics of the response, i.e. its non-Gaussian characteristics. In addition, response energy in frequency ranges outside that of the input spectrum is not observed using this technique.

In this study, an enhanced linearization approach is utilized which permits the loading spectral description to reveal spectral peaks outside the range of input frequencies that are undetectable using conventional linearization techniques. The enhanced approach involves casting spectral descriptions of the drag force by a series of convolutions involving water-particle velocity spectra [25–27].

Investigations of the dynamics of jacket-type platforms indicate that the quadratic dependence of the drag force on the water-particle velocity leads to significant excitation near one of the platform resonant frequencies, and thus results in an appreciable increase in the platform response — a phenomenon that is captured in this study.

Following the enhanced linearization scheme discussed above, the hydrodynamic wave loads acting on a typical offshore platform can be expressed as

\[
F_{\text{wave}}(t) = F_{\text{inertia}}(t) + F_{\text{linear drag}}(t) + F_{\text{current}}(t) + F_{\text{non-linear drag}}(t)
\]  

(1)

where \(F_{\text{inertia}}\) is the inertia force, \(F_{\text{linear drag}}\) is the linear part of the wave drag force, \(F_{\text{current}}\) is the force created by the current and \(F_{\text{non-linear drag}}\) is the non-linear part of the wave drag force. Each component of the force is expressed in terms of the wave height fluctuation. Accordingly, their corresponding spectral density functions can be related.

One of the most commonly used spectral density functions of wave surface elevations is the JONSWAP spectrum [22]. By introducing \(\hat{\omega} = (\omega/\omega_0)\) where \(\omega_0\) is the peak spectral frequency of the wave surface elevations, the JONSWAP spectrum can be written in a dimensionless form:

\[
\hat{S}_{\eta\eta}(\hat{\omega}) = \frac{S_{\eta\eta}(\omega)}{S_{\eta\eta}(\omega_0)} = \exp(1.25)\hat{\omega}^{-5} \exp(-1.25\hat{\omega}^{-4} \mu^{-1})
\]  

(2)

where \(\omega_0\) is a function of wind velocity and fetch length, \(\gamma\) is the sharpness magnification factor, \(\mu = \exp[-(1/2\sigma^2)(1 - \hat{\omega})^2]\) with \(\sigma = 0.07\) for \(\hat{\omega} \leq 1\) and \(\sigma = 0.09\) for \(\hat{\omega} > 1\). If \(\gamma\) is set equal to unity, the JONSWAP spectrum reduces to the other commonly used spectrum, the Pierson–Moskovitz spectrum. In Reference [28], the non-dimensional form of the JONSWAP spectrum was modelled by employing a cascade of two linear second-order filters as

\[
\hat{S}_{\eta\eta}(\hat{\omega}) \approx \hat{S}_d = \frac{G\hat{\omega}^4}{[(\hat{\omega}^2 - k_1)^2 + (c_1\hat{\omega})^2][(\hat{\omega}^2 - k_2)^2 + (c_2\hat{\omega})^2]}
\]  

(3)

where the parameters \(G, k_1, k_2, c_1\) and \(c_2\) are determined by a least-squares algorithm [28]. Note that Equation (3) represents a single-input, single-output (SISO) system. This form of the JONSWAP spectrum is utilized in this study because it can be conveniently expressed by a state-space model.

The spectral density function of the horizontal water-particle velocities at any water level \(z\) is related to the spectral density function of wave surface elevations via [22, 29, 30]

\[
S_{\eta\eta}(\omega, z) = |H_{\eta\eta}(j\omega, z)|^2 S_{\eta\eta}(\omega).
\]  

(4)

Following the linear wave theory, the magnitude of the transfer function from wave surface elevation to horizontal water-particle velocities is

$$|H_{eq}(j\omega, z)|^2 = \omega^2 \Lambda(\omega, z)(1 + \cos \kappa b(z))S_{eq}(\omega)$$

$$\Lambda(\omega, z) = \frac{\cosh^2(\kappa z)}{\sinh^2(\kappa d)}$$

where $j = \sqrt{-1}$, $\kappa = \omega^2 / g$ is the wave number, $g$ is the acceleration due to gravity, $b(z)$ is the horizontal distance between the two legs of the platform in the direction of wave propagation at level $z$ and $d$ is the water depth.

The relationship between the spectral density function of the wave forces per unit depth of the platform and the spectral density function of the horizontal water-particle velocities can be obtained from the linear wave theory [22, 30]. The cross-spectral density function of the wave forces per unit depth of the platform between water levels $z_m$ and $z_n$ is

$$S_{qm,qn}(\omega, z_m, z_n) = S_i(\omega, z_m, z_n) + S_{II}(\omega, z_m, z_n) + S_{III}(\omega, z_m, z_n)$$

where the spectral density function of the inertia force is

$$S_i(\omega, z_m, z_n) = k_D(z_m)k_D(z_n)S_{\text{inertia}}(\omega),$$

the spectral density function of the linear part of the wave drag force is

$$S_{II}(\omega, z_m, z_n) = \frac{8}{3}k_D(z_m)k_D(z_n)S_{\text{inertia}}(\omega),$$

the spectral density function of the non-linear wave force can be expressed in terms of a quadratic and cubic representation, $S_{III}(\omega)$ and $S_{IV}(\omega)$, given below

$$S_{III}(\omega, z_m, z_n) = 2k_D(z_m)k_D(z_n)\Phi(\gamma_c(z_m))\Phi(\gamma_c(z_n))S_{\text{inertia}}^2(\omega)$$

$$S_{IV}(\omega, z_m, z_n) = \frac{1}{3}k_D(z_m)k_D(z_n)\phi(\gamma_c(z_m))\phi(\gamma_c(z_n))S_{\text{inertia}}^3(\omega)$$

$$k_D(z) = \frac{\pi}{4} \rho C_M D^2(z)$$

$$k_D(z) = \frac{1}{2} \rho C_D D(z)$$

$$B(\gamma_c(z)) = 2\gamma_c(z)\Phi(\gamma_c(z)) + 4\phi(\gamma_c(z))$$

$$\gamma_c(z) = \frac{U_c}{\sigma_n(z)}$$

$$\Phi(\gamma_c(z)) = 2 \int_0^{\gamma_c(z)} \phi(t) \, dt$$

$$\phi(\gamma_c(z)) = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{\gamma_c^2(z)}{2} \right)$$

and the spectrum of wave particle acceleration is

$$S_{\text{inertia}}(\omega) = \omega^2 S_{\text{inertia}}(\omega)$$
where \( \rho \) is the density of the water, \( C_D \) is the drag coefficient, \( C_M \) is the inertia coefficient, \( D(z) \) is the characteristic diameter of the platform leg at water level \( z \), \( U_c \) is the uniform water current, \( \sigma_w(z) \) is the standard deviation of the wave particle velocity at water level \( z \), \( \sigma_{u_m} \) is the standard deviation of the horizontal wave velocities at water level \( z_m \) and the convolution \( S^{*n} \) is defined as

\[
S^{*n}(\omega) \hat{=} \int_{-\infty}^{\infty} S(g)^* (n-1) S(\omega - g) \, dg
\]  

(12)

It is the contributions of \( S_{II}(\omega) \) and \( S_{III}(\omega) \) which result in energy at frequencies both inside and outside the range of frequencies associated with the base process. The square and cube in the time domain are associated with the respective convolution in the frequency domain by the Fourier transform relationship between the two operations. The second-order convolution places energy at sums and differences frequencies representatives of the base process. The third-order convolution spreads energy about the central frequency of the base process at its third harmonic.

The spectral distortion effect of non-linearities is important because, via this mechanism, energy can creep into frequency ranges which are critical to a structural system being acted upon. If this effect is ignored, significant structural response energy may also go by the wayside, leading to non-conservative response estimates.

Notice that for a multi-input, multi-output (MIMO) wave force model, \( \sigma_{u_m} \) is an element of the covariance matrix of the horizontal water-particle velocities. In the presence of currents, the loading non-linearity is statistically asymmetric and the problem can essentially be treated with a quadratic representation. In the absence of current, i.e. a statistically symmetric non-linear case, the quadratic term drops out and the cubic term \( S_{III}(\omega) \) becomes more significant. In other cases, both terms contribute depending on the current amplitude [31].

Filter for \( S_I \). In modelling the wave forces, the spectral density matrix factorization approach will be used [32]. In this approach, a random process with a spectral density matrix \( S(\omega) \) is modelled as the output of a filter with a unit intensity white noise process as an input. In the Laplace domain (by substituting \( s \) for \( j\omega \) or \( -s^2 \) for \( \omega^2 \)), this spectral density matrix is factorized as

\[
S(s) = H(-s)H^*(s)
\]  

(13)

where \( H^* \) is the transpose of the complex conjugate of \( H \). Using the definition of the spectral density function, it can be shown that if the input of \( H(s) \) is a unit intensity white noise, then the output of \( H(s) \) will have a spectral density of \( S(s) \). \( H(s) \) is then expressed in the state-space format for the control design.

In the jacket-type offshore platform model used in this study, the wave forces will be acting at five levels. Therefore, a wave force model must be constructed that has one input, i.e. a unit intensity white noise, and five outputs, i.e. the wave forces on the platform at different depths. To simplify the procedure, this modelling will be done in three steps. The first step is to model the wave surface elevations. This is accomplished using a SISO model obtained from Equation (13) with a unit intensity white-noise process as an input and wave surface elevations as output. The second step is to model the transfer function from the wave surface elevation to the horizontal wave velocity. Five SISO models with wave elevations as input and horizontal water-particle velocities at five levels of the platform legs as output are obtained.
Figure 1. A full model of wave forces on offshore platforms.

separately. The third step is to use the results of the first two steps to obtain the final model. Note that the second step can be done in the same manner as the first step because

\[
|H_{\eta\eta}(jw)|^2 = H_{\eta\eta}(-jw)H_{\eta\eta}^*(jw)
\]

which is the same form as Equation (13). The third step, which transfers the five horizontal water-particle velocities into five wave forces on the legs of the platform, involves only constants.

The modelling of \( H_{\eta\eta} \), the transfer function between the input white noise and the wave height fluctuation, in the first step and of \( H_{\eta\eta} \) in the second step is equivalent to the modelling of the filter for \( S_{\eta\eta} \). By Equation (13),

\[
S_{\eta\eta}(s) = H_{\eta\eta}(-s)H_{\eta\eta}^*(s)
\]

By substituting \( H_{\eta\eta}(s) = H_{\eta\eta}(s)H_{\eta\eta}^*(s) \) into Equation (15), the following is obtained:

\[
S_{\eta\eta}(s) = H_{\eta\eta}(-s)H_{\eta\eta}^*(s)H_{\eta\eta}^*(s)
\]

The resulting full model that transfers a unit intensity white-noise process into wave forces on the platform is shown in Figure 1. In this figure, \( w \) is the unit intensity white-noise process, \( \eta \) is the wave surface elevations, \( v \) is the horizontal water-particle velocities and \( q \) is the wave forces per unit depth of the platform.

**Filters for \( S_1 \), \( S_{II} \), and \( S_{III} \).** The filters for \( S_1 \), \( S_{II} \) and \( S_{III} \) can be obtained in a similar manner as the filter for \( S_1 \). For the \( S_1 \) filter, the power spectral density function of the water-particle acceleration \( S_{auv} \) is needed. This function can be obtained from \( S_{auv} \) by Equation (11), which is the following in the Laplace domain:

\[
S_{auv}(s) = -s^2\cal{S}_{auv}(s)
\]

The filter for \( S_{auv} \) can then be obtained using the same procedure as the first step in obtaining the filter for \( S_{auv} \). From that, the filter for \( S_1 \) can be obtained following the second and third steps in obtaining \( S_1 \). For the filters of \( S_{II} \) and \( S_{III} \), the power spectral density functions \( S_{auv}^{2} \) and \( S_{auv}^{3} \) are needed. These functions can be numerically obtained from the convolution of \( S_{auv} \) as described in Equation (12). The results are pairs of numbers describing \( S_{auv}^{2} \) and \( S_{auv}^{3} \).
By some curve-fit processes, these numerical results are then presented as ratios of polynomials which can be factorized as in Equation (13). From these ratios of polynomials, the filters for $S_2^{\#2}$ and $S_3^{\#3}$ are obtained. Once these filters are obtained, the procedures for obtaining the $S_{II}$ and $S_{III}$ filters are straightforward.

**Frequency domain control**

Using a block diagram description, the structural control problem can be depicted as in Figure 2, where $y$ is the measured output vector, $z$ is the regulated output vector, $u$ is the control input vector and $d$ is the exogenous input vector. The regulated output vector $z$ may consist of any combination of states of the system and components of the control input vector $u$. By appropriately choosing elements of $z$, different control design objectives can be included in the problem formulation. Weighting functions can be added to elements of $z$ to determine the frequency range where each element of $z$ is to be minimized. The ‘system’ in Figure 2 can then contain the structure and the filters and weighting functions in the frequency domain.

The task here is to design a stabilizing controller in such a way that the norm or ‘size’ of the transfer function from disturbance $d$ to regulated output $z$ is minimized [15]. The control design method which minimizes this $H_2$ norm is termed the $H_2$ control. The freedom of the control designer to choose elements of $z$ and frequency dependent weighting functions makes this approach very flexible.

Let the ‘system’ in Figure 2 be denoted by $P$ and the ‘controller’ by $K$. Partitioning $P$ into its components,

$$P = \begin{bmatrix} P_{yd} & P_{yu} \\ P_{yd} & P_{yu} \end{bmatrix}$$  \hspace{1cm} (18)

It is shown that

$$y = P_{yd}d + P_{yu}u = P_{yd}d + P_{yu}Ky$$  \hspace{1cm} (19)

and

$$z = P_{yd}d + P_{yu}u$$  \hspace{1cm} (20)

Note that in Equation (18), a matrix with square brackets is used instead of a matrix with parentheses. This notation is used to emphasize that the elements of the matrix are not constant matrices, but transfer functions.
By rearranging Equation (19) and substituting it into Equation (20), the following is obtained:

\[ z = [P_{zd} + P_{zu}K(I - P_{yu}K)^{-1}P_{yd}]d \]

or

\[ z = H_{zd}d \]  

where

\[ H_{zd} = P_{zd} + P_{zu}K(I - P_{yu}K)^{-1}P_{yd} \]  

\( H_{zd} \) is the transfer function from \( d \) to \( z \) that needs to be minimized. One possible block diagram set-up for \( P \) is depicted in Figure 3.

In this figure, \( A, B \) and \( C \) are matrices that form the state-space representation of the Equations of motion of the platform. The block \( F \) represents the wave force model. \( C_z \) and \( C_y \) are matrices needed to extract parts of the system output to form regulated and measured output vectors. The input of \( F \) is a vector of unit-intensity white-noise processes, \( w \). The output of \( F \) is a vector of wave forces, \( w' \). The control force \( u \) and the wave forces \( w' \) generally enter the system differently. For example, the control force comes into the system via the active control device as indicated by the matrix \( B \), while wave forces produce loadings on every lumped mass of the platform. The external excitation \( d \) consists of two elements: \( w \) and \( v \), or

\[ d = \begin{pmatrix} w \\ v \end{pmatrix} \]

where \( w \) is a white-noise input vector to the wave force model filter and \( v \) is the measurement noise. The regulated output \( z \) consists of \( z_1 \) and \( z_2 \), i.e.

\[ z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \]

where \( z_1 \) is the weighted structural response and \( z_2 \) is the weighted control force.
The objective of the control design is to determine a controller $K$ such that a weighted norm of the transfer function from $d$ to $z$ is minimized. With this set-up, the matrix $P$ expressed in the form of Equation (18) is given by

$$P = \begin{bmatrix} P_{z_1w} & P_{z_1v} & P_{z_1u} \\ P_{z_2w} & P_{z_2v} & P_{z_2u} \\ P_{z_3w} & P_{z_3v} & P_{z_3u} \end{bmatrix} = \begin{bmatrix} z_1 W_1 C_G G' k & 0 & z_1 W_1 C_G \\ 0 & 0 & z_2 W_2 \\ C_G G' k & I & C_G \end{bmatrix}$$ (25)

In Figure 3 and Equation (25), $W_1$ and $W_2$ are frequency-dependent weighting functions, $z_1$ and $z_2$ are scalar multipliers for $W_1$ and $W_2$, respectively and $k$ is a scalar which expresses a preference in minimizing the transfer function from $d$ to $z$ versus minimizing the transfer function from $v$ to $z$. If $k$ is taken as a large value, then the effect of measurement noise is neglected in the control design. The transfer functions $G$ and $G'$ are defined as

$$G \triangleq C(sI - A)^{-1}B$$ (26)

and

$$G' \triangleq C(sI - A)^{-1}F$$ (27)

From Equation (25), the transfer function to be minimized, $H_{zd}$, can be computed by Equation (22). The result is

$$H_{zd} = \begin{bmatrix} H_{z_1w} & H_{z_1v} \\ H_{z_2w} & H_{z_2v} \end{bmatrix}$$ (28)

where

$$H_{zd}(11) = z_1 W_1(C_G G' k + C_G K(I - C_G G')^{-1} C_G)$$

$$H_{zd}(12) = z_1 W_1 C_G K(I - C_G G')^{-1}$$

$$H_{zd}(21) = z_2 W_2 K(I - C_G G')^{-1}(C_G G' k + F)$$

$$H_{zd}(22) = z_2 W_2 K(I - C_G G')^{-1}$$

Increasing $z_1 W_1$ in the frequency range while choosing a large value of $k$ causes the transfer function from the white-noise excitation vector $w$ to the weighted structural response $z_1$ to be minimized more. This will help improve the performance of the system at those frequencies. Increasing $z_2$ in the frequency range while choosing a large value of $k$ also causes the transfer function from the white-noise excitation $w$ to the weighted control force $z_2$ to be minimized more. This will help limit the control forces. Increasing one of the weightings in a frequency range decreases the importance of the other weighting in that frequency range. Therefore, one is faced with a tradeoff between different design objectives. The $H_2$ control strategy searches for a stabilizing controller while minimizing the $H_2$ norm of $H_{zd}$, $||H_{zd}||_2$. The $H_2$ solution procedure is given in Appendix A.

EXAMPLE

Modelling of the wave force spectrum

In the first part of this example, the wave force spectrum is modelled. The sea state is defined by the JONSWAP wave spectrum. The peak spectral frequency is $\omega_0 = 0.35 \text{ rad/s}$, which corresponds to a period of $18 \text{ s}$. The sharpness magnification factor is $\gamma = 3.78$. 

The modelling of the $S_1$, $S_{II}$, and $S_{III}$ filters will be described separately. These filters will then be added together to obtain the total wave force filter.

**Filter of $S_I$**

The following parameters are used in Equation (3): $G = 0.47$, $k_1 = 1$, $k_2 = 4$, $c_1 = 0.15$ and $c_2 = 3.5$ [28]. A significant wave height of 15 m was obtained by setting $G = 0.47$. The numerator and denominator of the spectral density function can be treated separately because this process involves a SISO system. In the Laplace domain, the numerator of $H_w$ in Equation (3) is

$$\frac{G}{\omega_0^4}s^4 = 3331.9s^4 \tag{29}$$

Let the model filter for $H_w$ be denoted as $\hat{H}_w$. The numerator of the filter $\hat{H}_w$ is then simply

$$\frac{\sqrt{G}}{\omega_0^2}s^2 = 57.723s^2 \tag{30}$$

The denominator of $\hat{H}_w$ is obtained by choosing the stable parts from the pairs in Equation (31), i.e. the denominator of the filter $\hat{H}_w$ is

$$66.639(s^2 + 1.2250s + 0.4900)(s^2 + 0.0525s + 0.1225) \tag{32}$$

Hence

$$H_w(s) = \frac{57.723s^2}{66.639(s^2 + 1.2250s + 0.4900)(s^2 + 0.0525s + 0.1225)} \tag{33}$$

It can be readily shown that

$$H_w(-s)H_w^*(s) = H_w(-s)H_w(s) = \hat{H}_w(s) \tag{34}$$

Recall from Equation (5) that the magnitude of the transfer function from wave surface elevations to horizontal water-particle velocities is

$$|H_{wp}(j\omega, z)|^2 = \omega^2 \Lambda(\omega, z)(1 + \cos \kappa b(z)) \tag{35}$$

with

$$\Lambda(\omega, z) = \frac{\cosh^2(\kappa z)}{\sinh^2(\kappa d)} = 1 + \frac{1}{6}(\kappa z)^2 + \frac{1}{25}(\kappa z)^4 + \frac{1}{720}(\kappa z)^6 + \cdots \tag{36}$$
Also
\[
(1 + \cos \kappa b) = 2 - \frac{1}{2}(\kappa b)^2 + \frac{1}{24}(\kappa b)^4 - \frac{1}{720}(\kappa b)^6 + \cdots \tag{37}
\]
Let the numerator of \(|H_v|^2\) be denoted by \(N_H\) and its denominator by \(D_H\). Substituting \(\kappa = \omega^2/g, \omega^2 = -s^2\), and truncating up to the sixth power of \(s\), we obtain
\[
N_H(s, z) = \frac{2z^2 - b^2}{2g^2} s^6 + 2s^2 \tag{38}
\]
and
\[
D_H(s) = \frac{d^3}{6g^3} s^6 - 1 \tag{39}
\]
Now, using the same procedure as in the first step, \(N_H(s, z)\) and \(D_H(s)\) can be easily factorized to obtain the numerator and denominator of the model filter \(H_v(s, z)\).

The third step is to ‘factorize’ a constant matrix \(S\) with individual element \((8/\pi)k_P(x_m)k_P(z_m) \propto_{\nu_1} \propto_{\nu_2}\) and to connect the result to \(H_v\) and \(H_w\). Note that \(\propto_{\nu_1}, \propto_{\nu_2}\) is an element of the covariance matrix of the horizontal water-particle velocities. The ‘factorization’ here is to find a matrix \(H_{qv}\) such that
\[
S = H_{qe} H_{qe}^T \tag{40}
\]
which simplifies into
\[
H_{qe} = S^{1/2} \tag{41}
\]
where \(H_{qe}\) is a constant matrix of real numbers due to the fact that \(S\) is symmetric. This operation can be done using various available matrix decomposition algorithms.

**Filter of \(S_1\)**

The filter of the wave particle acceleration spectrum is
\[
H_a(s) = s \frac{N_{H(w)}(s) N_{H(v)}(s)}{D_{H(w)}(s) D_{H(v)}(s)} \tag{42}
\]
This wave particle acceleration filter will then be used to calculate \(S_i\) in Equation (7) following the three steps described above.

**Filter of \(S_2\)**

The second-order convolution of the wave-particle velocity spectrum is calculated from the first-order convolution wave particle velocity spectrum [Equation (9)]. This spectrum is curve-fitted by the following function:
\[
S_u^{*2}(s) = \frac{N_S^{*2}(s)}{D_S^{*2}(s)} \tag{43}
\]
where $N_{S_{s}^2}(s) = 3.9700E^{-05}$ and

$$D_{S_{s}^2}(s) = s^{24} - (2.1185E + 00)s^{22} - (9.1675E + 00)s^{20} + (3.2935E - 01)s^{18} + (3.3090E + 01)s^{16}$$
$$+ (5.8484E + 01)s^{14} + (5.2266E + 01)s^{12} + (2.7996E + 01)s^{10} + (9.1206E + 00)s^{8}$$
$$+ (1.6584E + 00)s^{6} + (1.1996E - 01)s^{4} - (2.9456E - 03)s^{2} + (1.5707E - 05)$$

(44)

This second-order convolution wave particle velocity filter will then be used to calculate $S_{II}$ by Equation (9).
The third-order convolution of wave particle velocity spectrum is calculated by Equation (10) from the first- and second-order convolution wave particle velocity spectra. This spectrum is curve-fitted by functions that can be factorized according to Equation (13). For example, the third-order convolution of wave particle velocity spectrum for the wave with a period of 18 s that enters the platform at its fifth lumped mass is curve-fitted by a function

$$S_u^{(3)}(s) = \frac{N_{S_u}(s)}{D_{S_u}(s)}$$

(47)

with $N_{S_u}(s) = 0.01$ and

$$D_{S_u}(s) = s^{24} + 1.0476 \times 10^1 s^{22} + 4.9357 \times 10^1 s^{20} + 1.3717 \times 10^2 s^{18} + 2.4798 \times 10^2 s^{16}$$
$$+ 3.0324 \times 10^2 s^{14} + 2.5286 \times 10^2 s^{12} + 1.4173 \times 10^2 s^{10} + 5.1760 \times 10^1 s^8$$
$$+ 1.1923 \times 10^1 s^6 + 1.7372 s^4 + 1.4821 \times 10^{-1} s^2 + 6.1326 \times 10^{-3}$$

(48)

Both the exact and the curve-fitted spectra are plotted in Figure 5.

Based on the spectrum in Equation (47), the third-order convolution wave particle velocity filter is obtained as

$$F_{all}(s) = \frac{N_{F_{all}}(s)}{D_{F_{all}}(s)}$$

(49)
where $N_{\text{FIII}}(s) = 2.1794 \times 10^{-2}$ and

$$D_{\text{FIII}}(s) = s^{12} + 1.9656s^{11} + 7.1982s^{10} + 9.7126s^9 + 1.8081 \times 10^1 s^8$$
$$+ 1.7157 \times 10^1 s^7 + 2.0043 \times 10^1 s^6 + 1.3004 \times 10^1 s^5 + 9.7087s^4$$
$$+ 3.9011s^3 + 1.6588s^2 + 3.2358 \times 10^{-1} s + 7.0769 \times 10^{-2} \quad (50)$$

This third-order convolution wave particle velocity filter will then be used to calculate $S_{\text{III}}$ in Equation (10).

**NUMERICAL ANALYSIS OF OFFSHORE PLATFORM CONTROL**

**Platform characteristics**

In the second part of this example, the motion of two jacket-type offshore platforms is controlled utilizing both passive and active control strategies. The passive control device studied is the tuned mass damper. The active devices analysed here include the active mass damper, the active tendon mechanism and the propeller thruster. Each platform is individually modelled as six lumped masses, as shown in Figure 6. For each platform, five of these masses are located below the mean sea level.

**Platform A.** For the first platform, the depth of the sea is set at 700 ft. The lumped masses are $2 \times 10^3, 1.8 \times 10^3, 1.6 \times 10^3, 1.4 \times 10^3, 1.2 \times 10^3$ and $1.0 \times 10^3$ slugs, located at $z = 150, 280, 410, 540, 670$ and 800 ft above the sea bed, respectively. The distances between the legs of the platforms below the mean sea level at the locations of the lumped masses are $b(z) = 165, 148.75, 132.5, 116.25$ and 100 ft, respectively. The corresponding equivalent characteristic diameters of the legs are $D(z) = 45, 40, 35, 30$ and 25 ft, respectively. The equivalent characteristic diameter represents the total projected diameter of the platform legs at given elevations. The wave phase effect that results from the spatial separation between legs is accounted in the formulation [Equation (5)].
The first six natural frequencies of this platform are 0.2, 0.98171, 1.9527, 2.7209, 3.2802 and 3.8471 cycles/sec (Hz), respectively. The corresponding modal damping ratios are 0.02, 0.049704, 0.0866, 0.11579, 0.13705 and 0.15859, respectively.

Platform B. For the second platform, the depth of the sea is set at 1150 ft. The lumped masses are the same as those used for Platform A (i.e. \(2 \times 10^5\), \(1.8 \times 10^5\), \(1.6 \times 10^5\), \(1.4 \times 10^5\), \(1.2 \times 10^5\) and \(1.0 \times 10^6\) slugs). However, these masses are now located at \(z = 200, 400, 600, 800, 1000\) and \(1250\) ft above the sea bed, respectively. The distances between the legs of the platforms below the mean sea level at the locations of the lumped masses are \(b(z) = 226, 202, 178, 154\) and \(130\) ft, respectively.

The first six natural frequencies of Platform B are 0.16692, 0.81933, 1.6297, 2.2709, 2.7376 and 32.108, respectively. The corresponding modal damping ratios are 0.02, 0.049704, 0.0866, 0.11579, 0.13705 and 0.15859, respectively.

Sea states

Different sea states are applied for this study. The sea states are defined by the JONSWAP wave spectrum, using a sharpness magnification factor \(\gamma = 3.78\) and periods of 18, 14, 10 and 6 sec, respectively. It is noteworthy that the 18 sec dominant wave spectrum has considerable energy at or around the first natural periods of the two platforms studied here. This may result in large deck displacements even though the platform natural periods are removed from the dominant wave period in a sea state. Furthermore, higher-order loading component may add to the response at the platform natural period.

NUMERICAL RESULTS

The block diagram description for this control problem is given in Figure 3. In this figure, the matrix \(C_z\) is such that the regulated output \(z_1\) is the weighted displacement of the platform deck and \(C_y\) is such that the measured output \(y\) is the accelerations of all the lumped masses of the platform. The numerical results obtained using various control set-ups on platform A are given in Tables I–IV. The numerical results obtained using various control set-ups on platform B are given in Tables V–VIII. In these tables, \(\sigma_{d_z}\) is the standard deviation of the displacement of the top mass in cm, and \(\sigma_{d_{damp}}\) is the standard deviation of the displacement of the mass damper. The last entry in these tables, \(\sigma_u\), represents the standard deviations of the control force generated by the active control device. Various cases examined in these tables are listed below.

Case I represents the platform with no external control device. Case II represents the platform with a TMD installed on the platform deck. The design of the optimum TMD is based on the design formulae in Reference [33]. The properties of the TMD are: mass = 172 t (1 per cent of the generalized mass associated with the first mode of the platform), stiffness = 268 kN/m and damping coefficient = 20 t/s. The eigenfrequency of the damper is 0.199 Hz, which is 99.3 per cent of the first natural frequency of the platform.

Case III represents the platform with an AMD installed (1 per cent of the generalized mass associated with the first mode of the platform). The goal of structural control is to reduce the structural response in the frequency range where the structure is sensitive to disturbance, i.e., at
Table I. Standard deviations of displacements and control forces: platform A, wave period = 18 s.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\sigma_{d_h}$ (cm) (deck)</th>
<th>% reduction</th>
<th>$\sigma_{d_{da}}$ (cm) (damper)</th>
<th>$\sigma_u$ (kN) (control force)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.9236E1</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>II</td>
<td>1.7151E1</td>
<td>10.8</td>
<td>7.3883E1</td>
<td>3.2291E2</td>
</tr>
<tr>
<td>III</td>
<td>1.4345E1</td>
<td>25.4</td>
<td>2.3101E2</td>
<td>3.9293E3</td>
</tr>
<tr>
<td>IV</td>
<td>1.3507E1</td>
<td>29.8</td>
<td>--</td>
<td>7.6260E2</td>
</tr>
<tr>
<td>V</td>
<td>1.4000E1</td>
<td>27.2</td>
<td>--</td>
<td>3.8310E1</td>
</tr>
<tr>
<td>VI</td>
<td>1.3685E1</td>
<td>28.9</td>
<td>6.0375E1</td>
<td>9.2715E2</td>
</tr>
<tr>
<td>VII</td>
<td>1.3637E1</td>
<td>29.1</td>
<td>5.9009E1</td>
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</tr>
</tbody>
</table>

Table II. Standard deviations of displacements and control forces: platform A, wave period = 14 s.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\sigma_{d_h}$ (cm) (deck)</th>
<th>% reduction</th>
<th>$\sigma_{d_{da}}$ (cm) (damper)</th>
<th>$\sigma_u$ (kN) (control force)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.3643E1</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>II</td>
<td>1.0133E1</td>
<td>25.7</td>
<td>5.9812E1</td>
<td>1.8482E2</td>
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<tr>
<td>III</td>
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<td>47.3</td>
<td>1.4431E2</td>
<td>1.7870E3</td>
</tr>
<tr>
<td>IV</td>
<td>7.1896E0</td>
<td>47.3</td>
<td>--</td>
<td>3.5478E2</td>
</tr>
<tr>
<td>V</td>
<td>7.3082E0</td>
<td>46.4</td>
<td>--</td>
<td>1.6174E3</td>
</tr>
<tr>
<td>VI</td>
<td>7.2164E0</td>
<td>47.1</td>
<td>4.5818E1</td>
<td></td>
</tr>
<tr>
<td>VII</td>
<td>7.1780E0</td>
<td>47.4</td>
<td>4.4596E1</td>
<td>3.7588E2</td>
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</table>

Table III. Standard deviations of displacements and control forces: platform A, wave period = 10 s.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\sigma_{d_h}$ (cm) (deck)</th>
<th>% reduction</th>
<th>$\sigma_{d_{da}}$ (cm) (damper)</th>
<th>$\sigma_u$ (kN) (control force)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>7.6417E0</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>II</td>
<td>5.6128E0</td>
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<td>3.5066E1</td>
<td>1.0482E2</td>
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<tr>
<td>III</td>
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<td>8.4810E1</td>
<td>6.8261E2</td>
</tr>
<tr>
<td>IV</td>
<td>3.9415E0</td>
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<td>--</td>
<td>1.3913E2</td>
</tr>
<tr>
<td>V</td>
<td>3.9889E0</td>
<td>47.8</td>
<td>--</td>
<td>6.5825E2</td>
</tr>
<tr>
<td>VI</td>
<td>3.9123E0</td>
<td>48.8</td>
<td>3.0947E1</td>
<td></td>
</tr>
<tr>
<td>VII</td>
<td>3.8965E0</td>
<td>49.0</td>
<td>2.8710E1</td>
<td>1.3641E2</td>
</tr>
</tbody>
</table>
Table IV. Standard deviations of displacements and control forces:
platform A, wave period = 6 s.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\sigma_{d_e}$ (cm)</th>
<th>% reduction</th>
<th>$\sigma_{d_{d0}}$ (cm)</th>
<th>$\sigma_u$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>6.4129E0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>II</td>
<td>4.7863E0</td>
<td>25.4</td>
<td>3.1443E1</td>
<td>9.3222E1</td>
</tr>
<tr>
<td>III</td>
<td>2.0849E0</td>
<td>67.5</td>
<td>9.1357E1</td>
<td>8.1844E2</td>
</tr>
<tr>
<td>IV</td>
<td>2.0653E0</td>
<td>67.8</td>
<td>-</td>
<td>1.7679E2</td>
</tr>
<tr>
<td>V</td>
<td>2.1355E0</td>
<td>66.7</td>
<td>-</td>
<td>1.7679E2</td>
</tr>
<tr>
<td>VI</td>
<td>2.0532E0</td>
<td>68.0</td>
<td>7.2941E1</td>
<td>1.7679E2</td>
</tr>
<tr>
<td>VII</td>
<td>1.9528E0</td>
<td>69.5</td>
<td>1.6447E1</td>
<td>1.8552E2</td>
</tr>
</tbody>
</table>

Table V. Standard deviations of displacements and control forces:
platform B, wave period = 18 s.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\sigma_{d_e}$ (cm)</th>
<th>% reduction</th>
<th>$\sigma_{d_{d0}}$ (cm)</th>
<th>$\sigma_u$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>5.9848E1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>II</td>
<td>4.1441E1</td>
<td>30.8</td>
<td>2.4776E2</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>2.7636E1</td>
<td>53.8</td>
<td>8.1167E2</td>
<td>1.0496E3</td>
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<tr>
<td>IV</td>
<td>2.7332E1</td>
<td>54.3</td>
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<td>6.8849E3</td>
</tr>
<tr>
<td>V</td>
<td>2.6653E1</td>
<td>55.5</td>
<td>-</td>
<td>1.8602E3</td>
</tr>
<tr>
<td>VI</td>
<td>2.7070E1</td>
<td>54.8</td>
<td>1.7053E2</td>
<td>6.6043E3</td>
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<tr>
<td>VII</td>
<td>2.6541E1</td>
<td>55.7</td>
<td>1.0340E2</td>
<td>1.3607E3</td>
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</table>

Table VI. Standard deviations of displacements and control forces:
platform B, wave period = 14 s.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\sigma_{d_e}$ (cm)</th>
<th>% reduction</th>
<th>$\sigma_{d_{d0}}$ (cm)</th>
<th>$\sigma_u$ (kN)</th>
</tr>
</thead>
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<tr>
<td>I</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>II</td>
<td>2.2761E1</td>
<td>24.9</td>
<td>1.3083E2</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>1.5079E1</td>
<td>50.2</td>
<td>5.1214E2</td>
<td>6.4906E2</td>
</tr>
<tr>
<td>IV</td>
<td>1.4540E1</td>
<td>52.0</td>
<td>-</td>
<td>2.9217E3</td>
</tr>
<tr>
<td>V</td>
<td>1.4294E1</td>
<td>52.8</td>
<td>-</td>
<td>6.9490E2</td>
</tr>
<tr>
<td>VI</td>
<td>1.4695E1</td>
<td>51.5</td>
<td>8.8036E1</td>
<td>2.6991E3</td>
</tr>
<tr>
<td>VII</td>
<td>1.4791E1</td>
<td>51.2</td>
<td>8.6345E1</td>
<td>6.7750E2</td>
</tr>
</tbody>
</table>

larger $n$ yields a steeper slope. For this example, the weighting function is

$$W_i = \frac{a}{s + a}$$

(51)

The corner frequency $a = 4 \text{ rad/s}$ is used throughout these numerical studies. To minimize control forces, a weighting function which has a small magnitude at low frequencies and
Table VII. Standard deviations of displacements and control forces: platform B, wave period = 10 s.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \sigma_{d_s} ) (cm) (deck)</th>
<th>% reduction</th>
<th>( \sigma_{d_{MD}} ) (cm) (damper)</th>
<th>( \sigma_c ) (kN) (control force)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>2.172E1</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>II</td>
<td>1.592E1</td>
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<td>9.702E1</td>
<td>4.308E2</td>
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<tr>
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<td>61.1</td>
<td>3.849E2</td>
<td>2.346E3</td>
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<tr>
<td>IV</td>
<td>8.431E0</td>
<td>61.2</td>
<td>—</td>
<td>5.560E2</td>
</tr>
<tr>
<td>V</td>
<td>8.463E0</td>
<td>61.0</td>
<td>—</td>
<td>1.906E3</td>
</tr>
<tr>
<td>VI</td>
<td>8.453E0</td>
<td>61.1</td>
<td>5.951E1</td>
<td>4.577E2</td>
</tr>
<tr>
<td>VII</td>
<td>8.498E0</td>
<td>60.9</td>
<td>5.838E1</td>
<td>4.577E2</td>
</tr>
</tbody>
</table>

Table VIII. Standard deviations of displacements and control forces: platform B, wave period = 6 s.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \sigma_{d_s} ) (cm) (deck)</th>
<th>% reduction</th>
<th>( \sigma_{d_{MD}} ) (cm) (damper)</th>
<th>( \sigma_c ) (kN) (control force)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>2.925E1</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>II</td>
<td>1.814E1</td>
<td>38.0</td>
<td>1.371E2</td>
<td>1.647E2</td>
</tr>
<tr>
<td>III</td>
<td>3.268E0</td>
<td>88.8</td>
<td>2.854E2</td>
<td>2.524E3</td>
</tr>
<tr>
<td>IV</td>
<td>3.248E0</td>
<td>88.9</td>
<td>—</td>
<td>5.898E2</td>
</tr>
<tr>
<td>V</td>
<td>3.240E0</td>
<td>88.9</td>
<td>—</td>
<td>2.462E3</td>
</tr>
<tr>
<td>VI</td>
<td>3.243E0</td>
<td>88.9</td>
<td>5.180E1</td>
<td>5.494E2</td>
</tr>
<tr>
<td>VII</td>
<td>3.220E0</td>
<td>89.0</td>
<td>5.174E1</td>
<td>5.494E2</td>
</tr>
</tbody>
</table>

Increases at high frequencies is needed. For this example, the following is used:

\[ W_2 = \frac{s + 0.1}{1 \times 10^{-6}s + 0.1} \]  \hspace{1cm} (52)

These weighting functions are plotted in Figure 7.

Case IV represents the platform with a tendon mechanism located between its fifth and sixth lumped masses. The active tendon system consists of a diagonal set of tendons/pulley with an actuator which transmits its force by altering tension in tendons, thus the system characteristics.

Case V represents the platform with a propeller thruster located on its fifth lumped mass. Case VI represents the platform with a combination of a TMD and a tendon mechanism located between its fifth and sixth lumped masses. Case VII represents the platform with a combination of a TMD and a propeller thruster located on its fifth lumped mass.

The above tables show that the response of the platform is reduced by the application of the TMD, and that adding an AMD reduces it further. However, for some cases which include an AMD, the displacements of the mass damper are too large to be practical for the control of jacket-type offshore platforms. These displacements can be reduced using a larger mass for the mass damper. However, this solution might not be practical. When only a tendon is used below the deck of the platform, similar reductions in the platform displacements are achieved with a significantly larger control force. When only a propeller thruster is used just below...
the sea water level, the same reductions in the platform displacements are achieved using a significantly smaller control force. The control force produced by the propeller thruster is generally smaller than the control force generated by the tendon control. When a TMD and a tendon below the deck of the platform are used, the control force is smaller than when only a tendon is used, and the displacement of the damper mass is smaller than the displacement of the AMD. Hence it can be seen here that the tendon mechanism helps to reduce the travel distance of the TMD. Therefore, a combination of a TMD and tendon mechanism may allow a smaller mass ratio for the TMD. When a TMD and a propeller thruster just below the sea water level are used, the control force is even smaller than in the previous case. Therefore, this combination is probably the best for controlling the jacket-type offshore platforms considered here.

The dominant periods of ocean waves considered here signify typical waves experienced during the everyday life of the platform and during extreme storm conditions, e.g. the longer the period it represents stormy conditions. It is noted that when the fundamental platform period is separated from the wave dominant period, the waves have considerable background energy around the platform natural periods, thus resulting in large deck displacements. In the case of 18 sec waves, the non-linear drag component also introduces loading at or near third harmonic which falls close or at the natural periods of the platforms under study here. In the cases where the two periods are close, the platform response is dominated by the resonant component. However, in this case the sea state is characterized by lower wave heights, which results in not very large deck displacements. It is in these latter cases that the effectiveness of the control devices is enhanced, which results in more significant reductions in response.

CONCLUSIONS

This paper examines the control of jacket-type offshore platforms using passive/active auxiliary devices. An optimal frequency domain control method based on the minimization of the $H_2$ norm of the system transfer function is used. The JONSWAP wave spectra has been employed to model the wave surface elevation, from which the wave force spectra was obtained,
including contributions of the nonlinear effects. This wave force spectra was then factorized to give a linear filter for expressing wave load effects in a state-space format which is essential for the control procedure utilized here. A numerical study was conducted in which the displacements of the decks of platforms were controlled by passive and active control mechanisms. The passive control mechanism consisted of a TMD, and the active control devices included an AMD, active tendon mechanism, propeller thrusters, and a combination of these.

The size of the control forces and the reduction in the platform response during different sea states were the primary factors taken into consideration in this study. This study suggests that the combined TMD and propeller thruster mechanism was the most effective means for controlling the jacket-type offshore platforms studied here. For the sea states in which the wave forces were concentrated at frequencies near the dominant frequencies of the platform, the response of the platform was significantly reduced. This study suggests that by including a single or a combination of auxiliary active/passive damping devices, the platform response can be reduced significantly, thus improving both serviceability performance and fatigue life of offshore platforms. The methodology presented here can readily compensate for system time delays and actuator–structure interactions.

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APPENDIX A: $H_2$ CONTROL SOLUTION PROCEDURE

As suggested by the name, $H_2$ control design is a control design which searches for a stabilizing controller while minimizing the $H_2$ norm of $H_{sd}$, $\|H_{sd}\|_2$. The $H_2$ solution procedure is based on the state-space realization of the system $P$ in Figure 2 in the form

$$\begin{align*}
\dot{x} &= Ax + B_1d + B_2u \\
z &= C_1x + D_{12}u \\
y &= C_2x + D_{21}d + D_{22}u
\end{align*}$$

(A1) \hspace{2cm} (A2) \hspace{2cm} (A3)

According to Chiang and Safonov [34], the $H_2$ norm optimal control problem is equivalent to a conventional LQG optimal control problem of the system

$$\begin{align*}
\dot{x}(t) &= Ax(t) + B_2u(t) + w(t) \\
y(t) &= C_2x(t) + v(t)
\end{align*}$$

(A4) \hspace{2cm} (A5)
with a performance index

\[
J = E \left[ \int_0^{\infty} \begin{pmatrix} x(t) & u(t) \end{pmatrix} \begin{pmatrix} Q & X \end{pmatrix} \begin{pmatrix} x(t) \\ u(t) \end{pmatrix} dt \right]
\]

(A6)

\[
= E \left[ \int_0^{\infty} \begin{pmatrix} x^T & u^T \end{pmatrix} \begin{pmatrix} C_1^T \\ D_{12}^T \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} dt \right]
\]

(A7)

and the correlated white plant noise \( w \) and white measurement noise \( v \) have the joint correlation function

\[
E \left[ \begin{pmatrix} w(t) \\ v(t) \end{pmatrix} \begin{pmatrix} w(\tau) \\ v(\tau) \end{pmatrix}^T \right] = \begin{pmatrix} S_w & X_f \\ X_f^T & S_v \end{pmatrix} \delta(t-\tau) = \begin{pmatrix} B_1B_1^T & B_1D_{21} \\ D_{21}B_1^T & I \end{pmatrix} \delta(t-\tau)
\]

(A8)

The final \( H_2 \) optimal controller is the transfer function

\[
\dot{x}_{cp} = A_{cp}x_{cp} + B_{cp}y, \quad u = C_{cp}x_{cp} + D_{cp}y
\]

(A9)

where

\[
A_{cp} = A - K_cC_2 - B_2K_c + K_cD_{22}K_c
\]

(A10)

\[
B_{cp} = K_f
\]

(A11)

\[
C_{cp} = K_c
\]

(A12)

and

\[
D_{cp} = 0
\]

(A13)

where \( K_c \) is the control gain matrix and \( K_f \) is the Kalman–Bucy filter gain matrix. Both are obtained by solving respective Riccati equations [6].

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