Efficacy of the Implied Approximation in the Identification of Flutter Derivatives

Xinzhong Chen[1] and Ahsan Kareem[2]

Abstract: Structural motion induced aerodynamic forces on bridges are customarily characterized in terms of flutter derivatives. Considerable effort has been extended to refine the procedure to identify flutter derivatives of bridge decks using spring-suspended two-degree-of-freedom bridge deck section models in wind tunnels. In this context, techniques and implied approximations employed in the literature to identify flutter derivatives from section model studies are highlighted. Through a parametric study, this Technical Note assesses the efficacy of a customarily used identification procedure which provides an improved insight and better understanding of the identification technique for flutter derivatives.

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Introduction

Aerodynamic forces on bridges are conventionally separated into the self-excited and buffeting force components. The self-excited forces modify bridge frequencies and damping ratios and may result in coupling among modal response components. An accurate modeling of the self-excited forces is essential for a reliable prediction of the buffeting and flutter response of bridges (e.g., Katsuchi et al. 1999; Chen et al. 2000). The lift and pitching moment components of the self-excited forces on a unit length of the bridge deck are commonly described in terms of flutter derivatives as (Scanlan and Tomko 1971; Sarkar et al. 1994)

$$L_{se}(t) = \frac{1}{2}\rho U^2(2b)\left(kH_1^*(k)\frac{\dot{h}}{U} + kH_2^*(k)\frac{b\dot{\alpha}}{U} + k^2H_3^*(k)\alpha + k^2H_4^*(k)\frac{h}{b}\right) \quad (1)$$

$$M_{se}(t) = \frac{1}{2}\rho U^2(2b^2)\left(kA_1^*(k)\frac{\dot{h}}{U} + kA_2^*(k)\frac{b\dot{\alpha}}{U} + k^2A_3^*(k)\alpha + k^2A_4^*(k)\frac{h}{b}\right) \quad (2)$$

where ρ=air density; U=mean wind velocity; $B=2b$=bridge deck width; $k=\omega b/U$=reduced frequency; ω=circular frequency of oscillation; H_i^* and A_i^* (i=1,2,3,4)=flutter derivatives, which are functions of reduced frequency or reduced wind velocity and depend on the geometric configuration of the bridge deck; h and α=displacement of the bridge deck section in the vertical and torsional directions; and each dot denotes a derivative with respect to time.

Despite recent advances in computational fluid dynamics (CFD), wind tunnel testing using scaled bridge section models has remained as a most reliable means of quantifying the flutter derivatives. There are two main experimental techniques used in conjunction with system identification schemes to extract the flutter derivatives. The first is based on a forced vibration test in which a bridge section model is rigidly supported by a mechanism which can oscillate in precise motions while time histories of either instantaneous pressure distribution or total integrated aerodynamic forces are measured (e.g., Matsumoto 1996; Haan et al. 2000). The second technique involves a free-vibration experiment in which a model is spring-suspended to allow oscillations in the vertical or torsional direction or in both vertical and torsional directions (e.g., Scanlan and Tomko 1971; Sarkar et al. 1994). The time histories of the free vibration displacement are monitored for further analysis. Although the forced vibration approach involves a relatively more complicated driving mechanism and measurements, it provides very reliable estimates of flutter derivatives. In addition, since the angle of incidence and the amplitude of oscillations can be easily controlled, this technique facilitates the measurements at different test conditions for investigating potential nonlinearities in self-excited forces. The free vibration technique is more widely utilized because of its simplicity and convenience in measurements.

This Technical Note addresses the identification techniques for

[1]Assistant Professor, Wind Science and Engineering Research Center, Department of Civil Engineering, Texas Tech University, Lubbock, TX 79409. E-mail: xinzhong.chen@coe.ttu.edu

[2]Robert M. Moran Professor, Nathaz Modeling Laboratory, Dept. of Civil Engineering and Geological Sciences, Univ. of Notre Dame, Notre Dame, IN 46556. E-mail: kareem@nd.edu

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flutter derivatives based on the free vibration of the two-degree-of-freedom (2DOF) tests. The approximation concerning the self-excited forces implied in this approach is highlighted, and a parametric study is conducted to assess its validity. The discussion presented in this Technical Note provides an improved insight and better understanding of the procedure used to identify flutter derivatives.

Equations of Motion

Consider a bridge section model suspended by springs in a wind tunnel to allow oscillations in the vertical and torsional directions. At a given wind velocity, the free vibration displacements of the 2DOF bridge model can be expressed in terms of the complex modal properties as

$$h(t) = h_1(t) + h_2(t) = \sum_{j=1}^{2} (A_{0j}A_{hj}e^{\lambda_j t} + A_{0j}^* A_{hj}^* e^{\lambda_j^* t}) \tag{3}$$

$$\alpha(t) = \alpha_1(t) + \alpha_2(t) = \sum_{j=1}^{2} (A_{0j}A_{\alpha j}e^{\lambda_j t} + A_{0j}^* A_{\alpha j}^* e^{\lambda_j^* t}) \tag{4}$$

where

$$\lambda_j = -\xi_j\omega_j + i\omega_j\sqrt{1-\xi_j^2};$$

$$\mathbf{\Phi}_j = [A_{hj}\ A_{\alpha j}]^T \quad (j = 1,2) \tag{5}$$

are the eigenvalues and eigenvectors of the two complex modes; ξ_j and $\omega_j = j$th complex modal damping ratio and frequency; $h_j(t)$ and $\alpha_j(t)$=vertical and torsional displacements associated with two complex modes; A_{0j}=constant coefficients which are dependent on the initial free-vibration conditions; and the asterisk denotes the complex conjugate operator.

Accordingly, the associated self-excited forces acting on a unit length of the bridge section model are given by

$$\begin{aligned} L_{se}(t) = L_{se1}(t) + L_{se2}(t) &= \frac{1}{2}\rho U^2(2b)\left(k_1H_1^*(k_1)\frac{\dot{h}_1}{U} + k_1H_2^*(k_1)\frac{b\dot{\alpha}_1}{U}\right. \\ &\left. + k_1^2H_3^*(k_1)\alpha_1 + k_1^2H_4^*(k_1)\frac{h_1}{b}\right) + \frac{1}{2}\rho U^2(2b)\left(k_2H_1^*(k_2)\frac{\dot{h}_2}{U}\right. \\ &\left. + k_2H_2^*(k_2)\frac{b\dot{\alpha}_2}{U} + k_2^2H_3^*(k_2)\alpha_2 + k_2^2H_4^*(k_2)\frac{h_2}{b}\right) \end{aligned} \tag{6}$$

$$\begin{aligned} M_{se}(t) = M_{se1}(t) + M_{se2}(t) &= \frac{1}{2}\rho U^2(2b^2)\left(k_1A_1^*(k_1)\frac{\dot{h}_1}{U}\right. \\ &\left. + k_1A_2^*(k_1)\frac{b\dot{\alpha}_1}{U} + k_1^2A_3^*(k_1)\alpha_1 + k_1^2A_4^*(k_1)\frac{h_1}{b}\right) + \frac{1}{2}\rho U^2(2b^2) \\ &\times\left(k_2A_1^*(k_2)\frac{\dot{h}_2}{U} + k_2A_2^*(k_2)\frac{b\dot{\alpha}_2}{U} + k_2^2A_3^*(k_2)\alpha_2\right. \\ &\left. + k_2^2A_4^*(k_2)\frac{h_2}{b}\right) \end{aligned} \tag{7}$$

where $k_1 = \omega_1 b/U$ and $k_2 = \omega_2 b/U$=reduced frequencies corresponding to the two modes.

The governing equations of the bridge section model are given by

$$\mathbf{M}\ddot{\mathbf{X}}_1 + \mathbf{C}(k_1)\dot{\mathbf{X}}_1 + \mathbf{K}(k_1)\mathbf{X}_1 = \mathbf{0} \tag{8}$$

$$\mathbf{M}\ddot{\mathbf{X}}_2 + \mathbf{C}(k_2)\dot{\mathbf{X}}_2 + \mathbf{K}(k_2)\mathbf{X}_2 = \mathbf{0} \tag{9}$$

where

$$\mathbf{M} = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix}; \quad \mathbf{X}_1 = \begin{bmatrix} h_1 \\ \alpha_1 \end{bmatrix}; \quad \mathbf{X}_2 = \begin{bmatrix} h_2 \\ \alpha_2 \end{bmatrix} \tag{10}$$

$$\mathbf{C}(k_1) = \begin{bmatrix} 2m\xi_h\omega_h - \rho\omega_1 b^2H_1^*(k_1) & -\rho\omega_1 b^3H_2^*(k_1) \\ -\rho\omega_1 b^3A_1^*(k_1) & 2I\xi_\alpha\omega_\alpha - \rho\omega_1 b^4A_2^*(k_1) \end{bmatrix} \tag{11}$$

$$\mathbf{K}(k_1) = \begin{bmatrix} m\omega_h^2 - \rho\omega_1^2 b^2H_4^*(k_1) & -\rho\omega_1^2 b^3H_3^*(k_1) \\ -\rho\omega_1^2 b^3A_4^*(k_1) & I\omega_\alpha^2 - \rho\omega_1^2 b^4A_3^*(k_1) \end{bmatrix} \tag{12}$$

$$\mathbf{C}(k_2) = \begin{bmatrix} 2m\xi_h\omega_h - \rho\omega_2 b^2H_1^*(k_2) & -\rho\omega_2 b^3H_2^*(k_2) \\ -\rho\omega_2 b^3A_1^*(k_2) & 2I\xi_\alpha\omega_\alpha - \rho\omega_2 b^4A_2^*(k_2) \end{bmatrix} \tag{13}$$

$$\mathbf{K}(k_2) = \begin{bmatrix} m\omega_h^2 - \rho\omega_2^2 b^2H_4^*(k_2) & -\rho\omega_2^2 b^3H_3^*(k_2) \\ -\rho\omega_2^2 b^3A_4^*(k_2) & I\omega_\alpha^2 - \rho\omega_2^2 b^4A_3^*(k_2) \end{bmatrix} \tag{14}$$

where m and I=mass and mass moment of inertia per unit length of the bridge deck; ω_h, ω_α, and ξ_h, ξ_α=mechanical circular frequencies and damping ratios in the vertical and torsional directions, respectively.

It is noteworthy that the free vibrations associated with two distinct modes correspond to two sets of distinct aeroelastic stiffness and damping matrices expressed in terms of two sets of eight frequency dependent flutter derivatives. This feature is well demonstrated by the flutter analysis, in which an iterative procedure for calculating the complex eigenvalues has to be carried out for each complex mode until the assumed frequency to evaluate the self-excited forces agrees with the imaginary part of the target modal frequency (e.g., Chen et al. 2000).

Identification of Flutter Derivatives

An indirect method was proposed to extract the flutter derivatives based on the free vibration response data in Scanlan and Tomko (1971) and Kumarasena et al. (1992). This technique requires the vertical and torsional single-degree-of-freedom (SDOF) tests to extract the direct flutter derivatives, i.e., H_1^*, H_4^*, A_2^*, and A_3^*, which is followed by the vertical and torsional coupled 2DOF test to extract the coupled terms, i.e., H_2^*, H_3^*, A_1^*, and A_4^*. This approach requires that the two SDOF tests and the 2DOF test must be carried out at the same reduced frequency. Although this procedure provides assessment of all flutter derivatives, it lacks efficiency and also leaves doubt concerning its reliability.

Recent efforts have been focused on the development of more efficient procedures by only utilizing a 2DOF test (Poulsen et al. 1992; Yamada et al. 1992; Sarkar et al. 1994; Iwamoto and Fujino 1995; Jakobsen and Hansen 1995; Gu et al. 2000). Although different schemes were employed in these studies, it is noted that the approximation concerning the aeroelastic stiffness and damping matrices implied in each study was the same, i.e.,

$$\mathbf{C}(k_1) \approx \mathbf{C}(k_2) \approx \mathbf{C}_{ef}; \quad \mathbf{K}(k_1) \approx \mathbf{K}(k_2) \approx \mathbf{K}_{ef} \tag{15}$$

where $\mathbf{C}_{ef}$ and $\mathbf{K}_{ef}$=effective aeroelastic stiffness and damping matrices

$$\mathbf{C}_{\mathrm{ef}}=\begin{bmatrix} 2m\xi_h\omega_h-\rho\omega_1 b^2H_1^*(k_1) & -\rho\omega_2 b^3 H_2^*(k_2) \\ -\rho\omega_1 b^3 A_1^*(k_1) & 2I\xi_\alpha\omega_\alpha-\rho\omega_2 b^4 A_2^*(k_2) \end{bmatrix} \tag{16}$$

$$\mathbf{K}_{\mathrm{ef}}=\begin{bmatrix} m\omega_h^2-\rho\omega_1^2 b^2H_4^*(k_1) & -\rho\omega_2^2 b^3 H_3^*(k_2) \\ -\rho\omega_1^2 b^3 A_4^*(k_1) & I\omega_\alpha^2-\rho\omega_2^2 b^4 A_3^*(k_2) \end{bmatrix} \tag{17}$$

Accordingly, at a given wind velocity, the governing equations of motion of the 2DOF bridge deck model [Eqs. (8) and (9)] can be approximately expressed as

$$\mathbf{M}\ddot{\mathbf{X}}+\mathbf{C}_{\mathrm{ef}}\dot{\mathbf{X}}+\mathbf{K}_{\mathrm{ef}}\mathbf{X}=\mathbf{0} \tag{18}$$

where $\mathbf{X}=\mathbf{X}_1$ or $\mathbf{X}_2$.

Therefore the identification of flutter derivatives at a given wind velocity can be carried out as follows: first, the effective aeroelastic stiffness and damping matrices, $\mathbf{K}_{\mathrm{ef}}$ and $\mathbf{C}_{\mathrm{ef}}$, are identified either based on the modal properties of the two complex modes, or directly based on the time histories of model displacements, akin to the system identification of other dynamic systems. This is followed by calculating the flutter derivatives $H_1^*(k_1)$, $H_4^*(k_1)$, $A_1^*(k_1)$, and $A_4^*(k_1)$, and $H_2^*(k_2)$, $H_3^*(k_2)$, $A_2^*(k_2)$, and $A_3^*(k_2)$ based on the changes in the effective aeroelastic stiffness and damping matrices from the structural stiffness and damping matrices (at zero wind velocity).

The identification of the effective aeroelastic stiffness and damping matrices as well as the eight flutter derivatives, which correspond to two different reduced frequencies for a given wind velocity, can be uniquely performed as long as both complex modes are sufficiently excited. However, difficulty is experienced at higher wind velocity close to the flutter velocity because the time histories of response involving the contribution of high damping mode are too short to obtain a reliable estimate of the modal properties. At lower wind velocities, it is also difficult to obtain reliable estimates of the coupled terms of flutter derivatives because the coupled motion is too weak. These difficulties can be eliminated by changing the mechanical dynamic characteristics of the bridge section model so that reliable results can be identified in a range of reduced wind velocities of interest. In Yamada et al. (1992), the identification of eight flutter derivatives has been performed at high wind velocities close to the critical flutter velocity. Since at these higher wind velocities only one mode, generally the mode dominated by torsional motion, is actually manifested in the time histories of vertical and torsional displacements, therefore the accuracy of the identification results comes into question due to the lack of information required to uniquely determine eight flutter derivatives.

It is obvious that without the approximation involving the frequency dependent aeroelastic stiffness and damping matrices, the flutter derivatives cannot be uniquely determined only based on the 2DOF free-vibration response data. This data at a given wind velocity can only provide eight independent conditions, i.e., modal frequencies and damping ratios and two complex modal shapes, therefore it fails to uniquely determine two sets of eight unknown flutter derivatives associated with two distinct reduced frequencies at the same wind velocity.

Instead of making the preceding approximation concerning the self-excited forces, Yamada and Miyata (1997) proposed a procedure to extract the eight flutter derivatives associated with the same reduced frequency. In their approach, the modal information identified at different wind velocities was used to extract a set of pairs of modal information, which corresponds to the same reduced frequencies. Accordingly, for each given reduced velocity, the eight flutter derivatives can be uniquely determined based on the modal information of two complex modes. The disadvantage of this approach is that an interpolation from the modal information at a set of wind velocities has to be carried out to predict the modal information associated with a set of reduced frequencies, which adds uncertainty to these estimates.

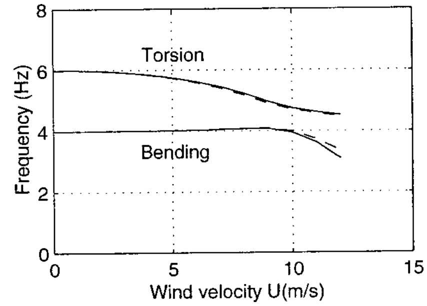

Fig. 1. Frequency versus wind velocity (airfoil)

Validation of the Approximation in Self-Excited Forces

It is clear that approximation concerning the self-excited forces must be made in order to uniquely identify the flutter derivatives based on the 2DOF free vibration technique. Although this assumption is widely invoked, it has not been well recognized and its validity has not been clearly discussed to date.

To validate this approximation, several different configurations of bridge deck sections are considered here including airfoil, twin-box section, and rectangular sections with a width-height ratio of 10. Their flutter derivatives were obtained from the Theodorsen function and wind tunnel tests using the forced vibration technique (Matsumoto 1996). Using these flutter derivatives, the complex eigenvalue analysis was conducted for a 2DOF spring-suspended section model to calculate the modal information based on Eqs. (8) and (9), at different wind velocities. At a given wind velocity, the effective aeroelastic stiffness and damping matrices are calculated by the following equation for known modal information:

$$\begin{bmatrix}\mathbf{K}_{\mathrm{ef}} & \mathbf{C}_{\mathrm{ef}}\end{bmatrix}\begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_2 \\ \lambda_1\mathbf{\Phi}_2 & \lambda_2\mathbf{\Phi}_2 \end{bmatrix}=-\mathbf{M}\begin{bmatrix}\lambda_1^2\mathbf{\Phi}_1 & \lambda_2^2\mathbf{\Phi}_2\end{bmatrix} \tag{19}$$

$\mathbf{K}_{\mathrm{ef}}$ and $\mathbf{C}_{\mathrm{ef}}$ are taken as the output of the free vibration identification techniques and used to calculate the flutter derivatives based on Eqs. (16) and (17). These calculated flutter derivatives are then compared to the correct (target) flutter derivatives used in the complex mode analysis. The use of theoretical values of

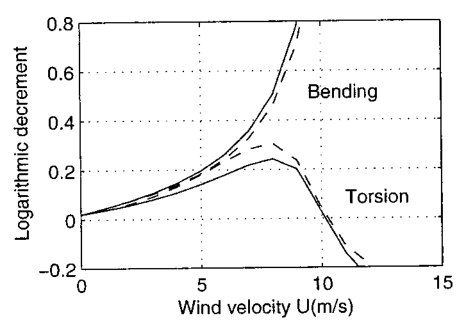

Fig. 2. Damping ratio versus wind velocity (airfoil)

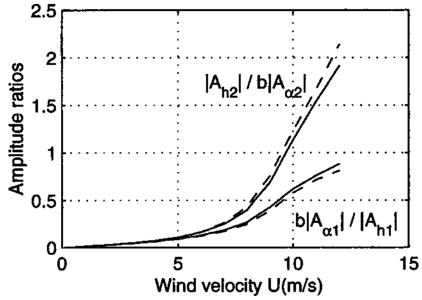

Fig. 3. Amplitude ratio versus wind velocity (airfoil)

modal information, calculated by the complex eigenvalue analysis, rather than the use of identified values based on the time histories of responses helps to put the following discussion in the proper context. Clearly, it only reflects the influence of the approximation in the self-excited forces on the identified flutter derivatives, and does not include the influences of other uncertainties involved in the identification of the effective aeroelastic stiffness and damping matrices that depend on individual system identification techniques. The dynamic parameters of the spring suspended section model system used in the flutter analysis are $b=0.1$ m; $m=2$ kg; $I=0.005$ kg$\cdot m^2$; $f_h=\omega_h/(2\pi)=4$ Hz; $f_\alpha=\omega_\alpha/(2\pi)=6$ Hz; $\xi_h=\xi_\alpha=0.32\%$.

Figs. 1–4 show the variation in frequencies, damping ratios, amplitude ratios, and phase differences with increasing wind velocities for the airfoil section. These are calculated by complex eigenvalue analyses with an iterative procedure. The solid lines are the correct results which are calculated based on the flutter derivatives defined at their individual frequencies. For the purpose of comparison, the dashed lines are the results calculated by using the flutter derivatives defined at another modal frequency, i.e., for the calculation of modal property of the first mode, $\mathbf{K}(k_2)$ and $\mathbf{C}(k_2)$ were used rather than using $\mathbf{K}(k_1)$ and $\mathbf{C}(k_1)$. These do not have a physical meaning but provide some insight to the sensitivity of modal properties with respect to the variation in the flutter derivatives due to changes in the reduced frequency. It is emphasized that the correct estimation of the aeroelastic modal properties are based on two sets of eight flutter derivatives.

Figs. 5–7 show the comparison of the identified flutter derivatives with their target values for different bridge deck sections, indicated by circles and solid lines, respectively. For all sections considered, excellent agreement was obtained despite the approximation in the self-excited forces being made. This suggests that the modification of the system dynamic matrices due to the aeroelastic terms, i.e., $\rho\omega b^2H_1^*$, $\rho\omega b^3H_2^*$, $\rho\omega^2b^3H_3^*$, $\rho\omega^2b^2H_4^*$, $\rho\omega b^3A_1^*$, $\rho\omega b^4A_2^*$, $\rho\omega^2b^4A_3^*$, and $\rho\omega^2b^3A_4^*$, are not very sensitive to the reduced frequency. The results of the parametric study support the approximation customarily implied in the identification of flutter derivatives using free vibration techniques.

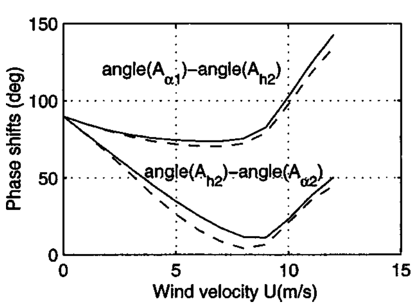

Fig. 4. Phase shifts versus wind velocity (airfoil)

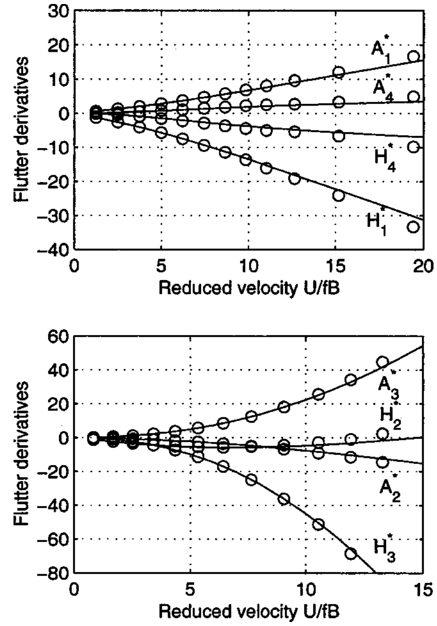

Fig. 5. Comparison of flutter derivatives (airfoil;—target and *o* identification)

Conclusion

Based on the definition of frequency dependent flutter derivatives, it was pointed out in this Technical Note that the free-vibration response of a 2DOF bridge section model at a given wind velocity did not provide sufficient information required to uniquely identify the two sets of eight flutter derivatives associated with two different modal frequencies. To ameliorate this difficulty, an ap-

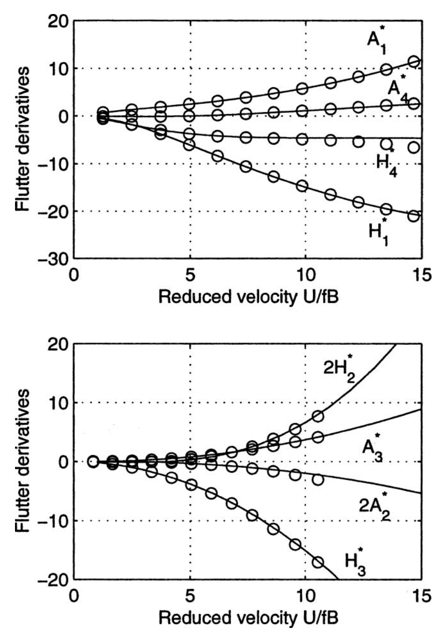

Fig. 6. Comparison of flutter derivatives (twin-box section;—target and *o* identification)

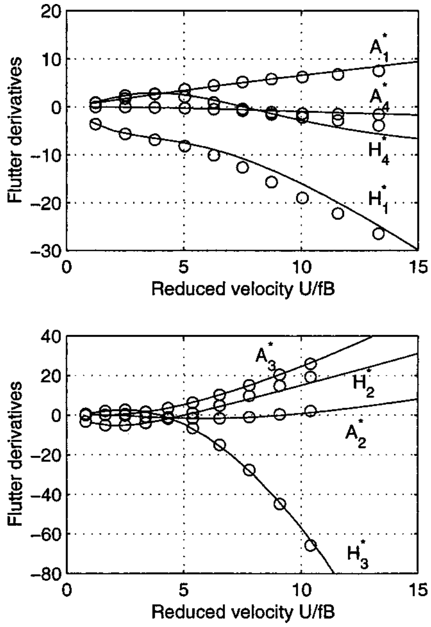

Fig. 7. Comparison of flutter derivatives (rectangular section $B/D = 10$;—target and o identification)

proximation in the self-excited forces is customarily invoked in order to uniquely identify all flutter derivatives. A parametric study involving a host of bridge deck sections lent support to this approximation customarily implied in the wind tunnel practice.

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