Model Predictive Control of Wind-Excited Building: Benchmark Study

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Abstract: In this paper, a ‘‘third generation’’ benchmark problem that focuses on the control of wind excited response of a tall building, using the Model Predictive Control (MPC) scheme, is presented. A 76 story, 306 m tall concrete office tower proposed for the city of Melbourne, Australia, is being used to demonstrate the effectiveness of MPC. The MPC scheme is based on an explicit use of a prediction model of the system response to obtain the control actions by minimizing an objective function. Optimization objectives in MPC include minimization of the difference between the predicted and desired response trajectories, and the control effort subjected to prescribed constraints. By incorporating input/output hard constraints, the MPC scheme provides an optimal control force that satisfies the prescribed constraints.

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Introduction

The field of structural control is becoming increasingly important in civil engineering as we continue to build tall and long span structures that are sensitive to wind and earthquake excitations. In the past decade, a number of control design schemes and control devices have been proposed and some have been actually implemented in buildings and bridges to control their motions (e.g., Soong 1990; Houser et al. 1997; Kareem et al. 1999). However, it is very difficult to evaluate their relative effectiveness because each represents a different structure with a different control device and design criterion. In 1995, the ASCE Committee on Structural Control initiated a benchmark study in structural control. The benchmark study proposed evaluating the performance of different control strategies and devices with the prescribed design objectives. The first generation benchmark study involved a scaled model of a three-story building subjected to ground motion, which was controlled by employing an active mass driver and an active tendon. In 1998, the ‘‘second generation’’ benchmark studies were developed at the Second World Conference of Structural Control. One of these related to an earthquake excited building by Spencer et al. (1998), the other concerned a wind-excited building (Yang et al. 1998). Additional research led to some modifications related to these benchmark problems and were launched as ‘‘third generation’’ benchmark problems. One of these is an earthquake-excited nonlinear building (Ohtori et al. 2004) and the other is a wind-excited tall building (Yang et al. 2000).

This paper investigates the ‘‘third generation’’ benchmark problem concerning wind-excited response of a tall building using the model predictive control (MPC) scheme. MPC belongs to a class of algorithms that compute a sequence of manipulated variable adjustment in order to optimize the future behavior of a plant. A system model is used to predict the open-loop future behavior of the system over a finite time horizon from present states. The predicted behavior is then used to find a finite sequence of control actions which minimize a particular performance index within pre-specified constraints.

The MPC scheme has been commonly used for the control of chemical processes, and applications to automotive and aerospace industries (Ricker 1990; Morari et al. 1994; Qin and Badgwell 1996; Camacho and Bordons 1999). Researchers Rodellar et al. (1987) and Lopez-Almansa et al. (1994a,b) applied a special case of MPC which is a predictive control scheme in civil engineering studies. In their approach the objective function was expressed in terms of the predicted trajectory and control force for one time step only. This may result in control force equal to zero, which is not a viable control design. This problem does not exist in the MPC scheme which is the focus of this study, since the objective function is expressed in terms of the predicted trajectory and control force over the prediction horizon. Recent applications of MPC to the control of civil engineering structures have been demonstrated in Mei et al. (1998, 2001, 2002).

The MPC scheme is based on an explicit use of a prediction model of the system response to obtain the control actions by minimizing an objective function. Optimization objectives include minimization of the difference between the predicted and reference response, and the control effort subjected to prescribed constraints. The effectiveness of MPC is demonstrated to be equivalent to the optimal control (Mei et al. 2001). It displays its main strength in its computational expediency, real-time applications, intrinsic compensation for time delays, treatment of constraints, and potential for future extensions of the methodology.

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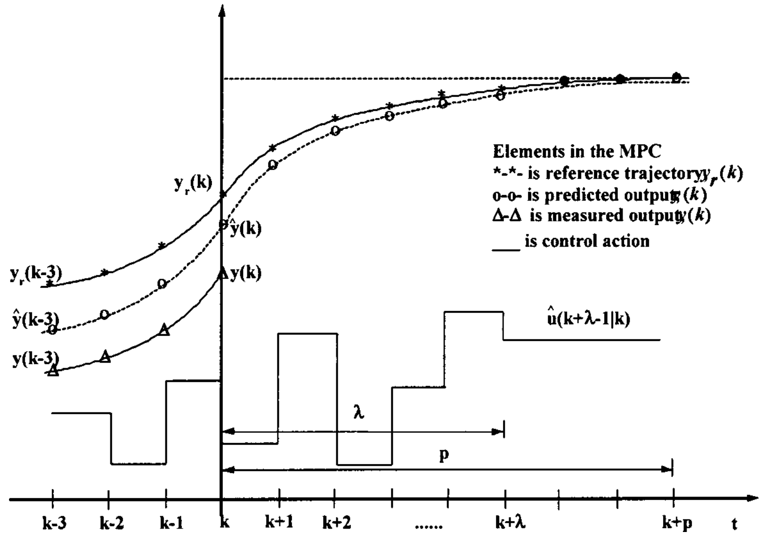

Fig. 1. Basic scheme model predictive control

In this version of the benchmark problem, the wind loading time history was obtained from a wind tunnel study in Sydney University, Australia to facilitate the time domain analysis. A reduced order model of the 76-story building is controlled by MPC using an active tuned mass damper (ATMD). The MPC provides an alternative control scheme, e.g., linear quadratic Gaussian (LQG), with the added attractive feature that permits handling of prescribed constraints concerning the actuator and building response level at a given height (Mei et al. 2000). Two cases of handling the constraints are considered here. The first case involves the MPC in which the prescribed physical constraints in the objective function are not considered. Rather, the constraints are satisfied by choosing appropriate weighting matrices. The second case concerns the MPC scheme in which the physical constraints are introduced in the objective function and an optimal solution is sought in the constrained space. Accordingly, the inequality constraints on the maximum control force and mass damper displacement are included in the optimization of the objective function. At each time step, MPC reduces to an optimization problem subjected to inequality constraints. A quadratic programming algorithm is used to obtain the optimal control force. Based on this scheme, the control force and mass damper displacement reside within the prescribed constrained space.

Problem Description

The benchmark problem in Yang et al. (2000) involves a 76-story, 306-m tall office tower subjected to along-wind or across-wind loads. The building motion is controlled by an active tuned mass damper (ATMD) installed on the top floor. An evaluation model with 48 states was obtained through a model reduction scheme. The equations of motion were expressed in a state space form

$$\dot{x}=Ax+Bu+Ew$$
$$z=C_z x+D_z u+F_z W \qquad (1)$$
$$y=C_y x+D_y u+F_y W+v$$

where $x=[\bar{x},\dot{\bar{x}}]^T$ is the 48-dimensional state vector, $\bar{x}=[x_3,\cdots,x_i]$, $i=6$, 10, 13, 16, 20, 23, 26, 30, 33, 36, 40, 43, 46, 50, 53, 56, 60, 63, 66, 70, 73, 76, m; u=scalar control force; and W=wind excitation vector of dimension 24; $z=[\bar{z},\dot{\bar{z}},\ddot{\bar{z}}]^T$=control output vector, and $y=[\dot{\bar{z}},\ddot{\bar{z}}]^T$=measured output vector of the evaluation model, in which $\bar{z}=[x_1,\cdots,x_i]$, $i=30$, 50, 55, 60, 65, 70, 75, 76, m; v=a vector of measured noise; and x_m=the relative displacement of the mass damper with respect to the top floor. Matrices A, B, E, C_z, D_z, F_z, C_y, D_y, and F_y were provided in Yang et al. (2000).

In the benchmark problem, the wind force data acting on the benchmark building were determined from wind tunnel tests. Twelve evaluation criteria for the time domain response analysis have been defined in terms of the RMS and the peak response values of the 76-story building. The actuator capacity constraints include the following: the maximum control force $\max|u(t)| \leqslant 300$ KN and the maximum stroke $\max|x_m(t)| \leqslant 95$ cm. Ten design requirements for ATMD are imposed on the proposed control design. Additional details are available in Yang et al. (2000).

Model Predictive Control Scheme

The MPC scheme is based on an explicit use of a prediction model of the system response to obtain control actions by minimizing an objective function. The optimization objectives include minimization of the difference between the predicted and desired response and the control effort subject to prescribed constraints such as limits on the magnitude of the control force. In the MPC scheme, first a reference response trajectory $y_r(k)$ is specified. The reference trajectory is the desired target trajectory of the structural response. This is followed by an appropriate prediction model which is then used to estimate the future building response $y(k)$. The prediction is made over a preestablished extended time horizon using the current time as the prediction origin. For a discrete time model, this means predicting $\hat{y}(k+1)$, $\hat{y}(k+2),\ldots,\hat{y}(k+i)$ for i sample times in the future. This prediction is based on the past control inputs $u(k)$, $u(k-1),\ldots,u(k-j)$ and on the sequence of future control efforts determined using the prediction model that are needed to satisfy a prescribed optimization objective. The control signals that were determined using the prediction model are then applied to the structure, and the actual structural system output $y(k)$ is found. Finally, the actual measurement $y(k)$ is compared to the model prediction $\hat{y}(k)$ and the

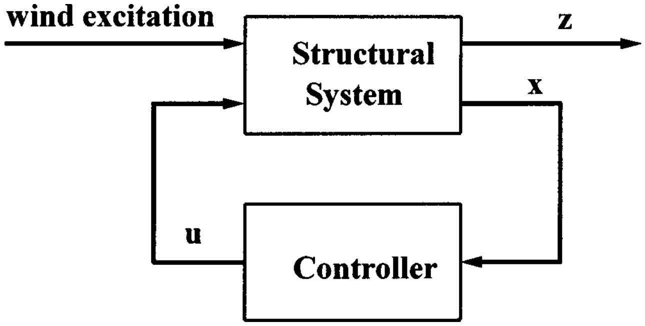

Fig. 2. Control diagram for state feedback

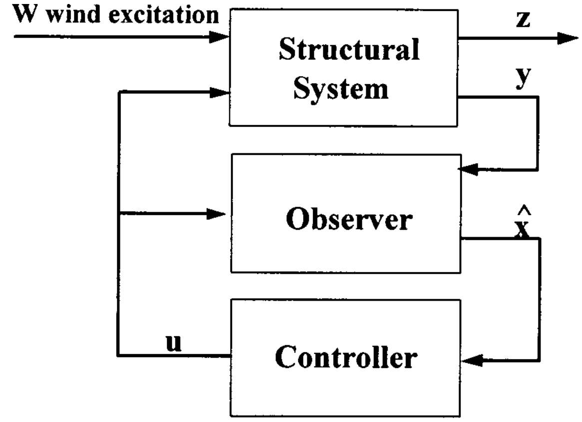

Fig. 3. Control diagram for acceleration feedback

prediction error $[\hat{e}(k)=y(k)-\hat{y}(k)]$ is utilized to update future predictions. Fig. 1 describes schematically the basic MPC scheme.

In the general formulation of the model predictive control, the discrete-time state-space equations of the system are used to estimate the future states of the system

$$\hat{x}(k+1|k)=\Phi\hat{x}(k|k-1)+\Gamma_u\hat{u}(k|k)+\Gamma_e\hat{e}(k|k)$$

$$\hat{z}(k|k-1)=C_z\hat{x}(k|k-1)+D_z\hat{u}(k|k) \quad (2)$$

$$\hat{y}(k|k-1)=C_y\hat{x}(k|k-1)+D_y\hat{u}(k|k)$$

where $\hat{x}(k+1|k)$=estimator of the states at a future time step $k+1$ based on the information available at k and $\hat{z}(k|k-1)$=predicted controlled output vector and is used in the objective funciton. $\hat{y}(k|k-1)$=structural system output estimator at time step k based on the information at time step $k-1$; Γ_e=Kalman-Bucy estimator gain matrix; and $\hat{e}(k|k)$=estimated error: $\hat{e}(k|k)=y(k)-\hat{y}(k|k-1)$. If $C_y=I$ and $D_y=0$, state feedback is used as shown in Fig. 2.

The structural system output predicted at the kth step and the subsequent time steps $k+j$, $j=1,...,p$ can be expressed as a function of $\hat{x}(k|k-1)$ and control vector $\mathbf{u}(k)=[\hat{u}^T(k|k)\cdots\hat{u}^T(k+\lambda-1|k)]^T$ shown below.

$$\Psi(k)=H\mathbf{u}(k-1)+Y_z\hat{x}(k|k-1)+Y_e\hat{e}(k|k) \quad (3)$$

where $\Psi(k)=[\hat{z}^T(k+1|k)\cdots\hat{z}^T(k+p|k)]^T$; p=prediction horizon; and λ=control horizon and is not greater than p.

The objective function is chosen as

$$J=\frac{1}{2}[\Psi(k)-\Psi_r(k)]^T\overline{Q}\Psi(k)-\Psi_r(k)+\frac{1}{2}\mathbf{u}^T(k)\overline{R}\mathbf{u}(k) \quad (4)$$

where $\Psi_r(k)=[z_r^T(k+1|k)\cdots z_r^T(k+p|k)]^T$=reference output which is chosen as a zero vector here.

Model Predictive Control with No Constraints

By minimizing the objective function, the control force for the MPC with no constraints case can be explicitly written as

$$\mathbf{u}=-[H^T\overline{Q}H+\overline{R}]^{-1}H^T\overline{Q}[Y_z\hat{x}(k|k-1)+Y_e\hat{e}(k|k)] \quad (5)$$

in which H, $\overline{Q}$, $\overline{R}$, Y_z, and Y_e are defined as

$$H=\begin{bmatrix} H_1+D_z & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ H_\lambda & H_{\lambda-1} & \cdots & H_1+D_z \\ H_{\lambda+1} & H_\lambda & \cdots & H_1+H_2 \\ \cdots & \cdots & \cdots & \cdots \\ H_p & H_{p-1} & \cdots & H_1+\cdots+H_{p-\lambda} \end{bmatrix}, \quad \text{where}$$

$$H_k=C_z\Phi^{k-1}\Gamma_u \quad (6)$$

$$Y_z=[(C_z\Phi)^T\ (C_z\Phi^2)^T\ \cdots\ (C_z\Phi^p)^T]^T, \quad (7)$$

$$Y_e=\left[(C_z\Gamma_e)^T\ (C_z(I+\Phi)\Gamma_e)^T\ \cdots\ \left[C_z\sum_{k-1}^{p}(\Phi^{k-1})\Gamma_e\right]^T\right]^T, \quad (8)$$

$$\overline{Q}=\begin{bmatrix} Q & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & Q \end{bmatrix}, \quad \overline{R}=\begin{bmatrix} R & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & R \end{bmatrix}. \quad (9)$$

Model Predictive Control with Constraints

For the MPC with constraints case, the objective function is expressed as

$$J=\frac{1}{2}[\Psi(k)-\Psi_r(k)]^T\overline{Q}[\Psi(k)-\Psi_r(k)]+\frac{1}{2}\mathbf{u}^T(k)\overline{R}\mathbf{u}(k) \quad (10)$$

which is subjected to the following linear inequality constraints:

$$\mathbf{u}(k)\geqslant\mathbf{u}_{\min}(k), \quad \mathbf{u}(k)\leqslant\mathbf{u}_{\max}(k), \quad \Psi(k)\geqslant\Psi_{\min}(k),$$
$$\Psi(k)\leqslant\Psi_{\max}(k) \quad (11)$$

The preceding problem is solved utilizing a quadratic programming algorithm. Introducing $\mathbf{v}(k)=\mathbf{u}(k)-\mathbf{u}_{\min}(k)$, the optimization problem can be written as a standard quadratic programming problem as

$$J_q=\max\left\{a^T(k)\mathbf{v}(k)-\frac{1}{2}\mathbf{v}^T(k)B\mathbf{v}(k)\right\} \quad (12)$$

which is subjected to the generalized inequality constraints

$$A\mathbf{v}(k)\leqslant b(k) \quad (13)$$

where

Table 1. Evaluation Criteria for Across-Wind Excitations

Root mean square response ($\Delta K=0\%$)				Peak response ($\Delta K=0\%$)			
Evaluation criteria	LQG	MPC[1]	MPC[2]	Evaluation criteria	LQG	MPC[1]	MPC[2]
J_1	0.369	0.363	0.346	J_7	0.381	0.381	0.349
J_2	0.417	0.410	0.391	J_8	0.432	0.438	0.428
J_3	0.578	0.572	0.563	J_9	0.717	0.716	0.712
J_4	0.580	0.574	0.565	J_{10}	0.725	0.725	0.720
J_5	2.271	2.260	2.410	J_{11}	2.300	2.282	2.400
J_6	11.99	11.96	14.52	J_{12}	71.87	79.59	88.37
σ_u (kN)	34.07	32.23	36.95	$\max\lvert u(t)\rvert$	118.24	118.1	118.0
σ_{xm} (cm)	23.03	22.90	24.44	$\max\lvert x_m\rvert$	74.29	73.37	77.52

Note: LQG=linear quadratic Gaussian, and MPC=model predictive control.

$$a(k)=H^T\overline{Q}[\Psi_r(k)-Y_z\hat{x}(k|k-1)-Y_e\hat{e}(k|k)]-B^T\mathbf{u}_{\min}(k) \tag{14}$$

$$B=H^T\overline{Q}H+\overline{R} \tag{15}$$

$$A=\begin{bmatrix}-I\\ I\\ -H\\ H\end{bmatrix}$$

$$b(k)=\begin{bmatrix}0\\ \mathbf{u}_{\max}(k)-\mathbf{u}_{\min}(k)\\ -\Psi_{\min}(k)+H\mathbf{u}_{\min}(k)+Y_z\hat{x}(k|k-1)+Y_e\hat{e}(k|k)\\ \Psi_{\max}(k)-H\mathbf{u}_{\min}(k)-Y_z\hat{x}(k|k-1)-Y_e\hat{e}(k|k)\end{bmatrix} \tag{16}$$

This problem can be solved as a standard quadratic programming problem using *MATLAB* (The MathWorks, Inc., Natick, Mass., 1998). The optimal solution is obtained in the constrained space. In order to accomplish this, the quadratic problem involving an active set strategy is utilized. A feasible solution is first obtained by solving a linear programming problem and then it is used as an initial point for the iterative solution involved in the quadratic programming problem. Then an iterative sequence of feasible points that converge to the desired solution are generated. The optimal point obtained in this manner is the optimal predictive control force in the constrained space which maximizes the objective function.

Results and Discussion

In this study, accelerometers are placed on the top two floors and on the ATMD. An acceleration feedback based MPC scheme is used here (Mei et al. 2002). The Kalman-Bucy filter is used to estimate the state of the system from the measurement. The feedback gain of the observer is obtained from

$$\Gamma_e=PC_y^T(C_yPC_y^T+V)^{-1} \tag{17}$$

where P matrix=solution of the Riccati equation which is unique, symmetric, and positive definite

$$P=\Phi[P-PC_y^T[C_yPC_y^T+R_v]^{-1}C_yP]\Phi^T+\Gamma_yQ_w\Gamma_y^T \tag{18}$$

and $E[WW^T]=Q_w$, $E[vv^T]=R_v$, $Q_w=Q_w^T$, $Q_w>0$, $R_v=R_v^T$, and $R_v>0$. W and v are assumed to be independent. The estimated state of the system is then used in the controller design as shown in Fig. 3.

In the following examples, the MPC based controllers are designed for the 76-story building with designed stiffness, which is referred to as a nominal building. Furthermore, to show the robustness of the controller, the uncertainty of building stiffness is considered. The controller obtained for the nominal building is applied to buildings with ±15% variations in stiffness matrix. The peak and RMS response quantities and evaluation criteria for these three buildings are presented and compared to the LQG control design.

Nominal Building

First the nominal building with designed stiffness is studied using the MPC scheme without consideration of the hard constraints. The limits on the control force and displacement of ATMD are satisfied by adjusting weighting matrices Q and R. Here the weighting matrix Q is same as the one used in Yang et al. (2000), the weight R on control force is chosen as 55, and the prediction

Table 2. Peak Response Using Passive Tuned Mass Damper (TMD), Linear Quadratic Gaussian (LQG), and Model Predictive Control (MPC) Schemes

Floor number	No control		Passive (TMD)		LQG control $u_{\max}=118.2$ kN		MPC[1] $u_{\max}=118.1$ kN		MPC[2] $u_{\max}=118.0$ kN	
	x_{pio} (cm)	$\ddot{x}_{pio}$ (cm/s^2)	x_{pi} (cm)	$\ddot{x}_{pi}$ (cm/s^2)	x_{pi} (cm)	$\ddot{x}_{pi}$ (cm/s^2)	x_{pi} (cm)	$\ddot{x}_{pi}$ (cm/s^2)	x_{pi} (cm)	$\ddot{x}_{pi}$ (cm/s^2)
1	0.053	0.22	0.044	0.21	0.041	0.23	0.041	0.24	0.040	0.25
30	6.84	7.14	5.60	4.68	5.14	3.38	5.14	3.92	5.11	3.77
50	16.59	14.96	13.34	9.28	12.22	6.73	12.21	7.09	12.14	6.77
55	19.41	17.48	15.54	10.74	14.22	8.05	14.21	8.15	14.12	8.09
60	22.34	19.95	17.80	12.69	16.27	8.93	16.26	8.86	16.16	8.92
65	25.35	22.58	20.10	14.72	18.36	10.06	18.35	10.13	18.24	10.14
70	28.41	26.04	22.43	16.77	20.48	10.67	20.46	10.79	20.33	10.55
75	31.59	30.33	24.84	19.79	22.67	11.56	22.64	11.55	22.50	10.59
76	32.30	31.17	25.38	20.52	23.15	15.89	23.13	9.26	22.99	16.36
md			42.60	46.18	74.27	72.64	73.70	78.81	77.52	80.31

Note: md=mass damper.

Table 3. Root Mean Square Response Using Passive Tuned Mass Damper (TMD), Linear Quadratic Gaussian (LQG), and Model Predictive Control (MPC) Schemes

Floor number	No control		Passive (TMD)		LQG control $\sigma_u = 37.99$ KN		MPC[1] $\sigma_u = 32.23$ KN		MPC[2] $\sigma_u = 36.95$ KN	
	σ_{xi} (cm)	$\sigma_{\ddot{x}i}$ (cm/s^2)	σ_{xi} (cm)	$\sigma_{\ddot{x}i}$ (cm/s^2)	σ_{xi} (cm)	$\sigma_{\ddot{x}i}$ (cm/s^2)	σ_{xi} (cm)	$\sigma_{\ddot{x}i}$ (cm/s^2)	σ_{xi} (cm)	$\sigma_{\ddot{x}i}$ (cm/s^2)
1	0.017	0.06	0.012	0.06	0.010	0.06	0.010	0.06	0.010	0.06
30	2.15	2.02	1.48	1.23	1.26	0.89	1.24	0.92	1.23	0.89
50	5.22	4.78	3.57	2.80	3.04	2.03	3.01	2.00	2.96	1.92
55	6.11	5.59	4.17	3.26	3.55	2.41	3.51	2.36	3.46	2.28
60	7.02	6.42	4.79	3.72	4.08	2.81	4.03	2.75	3.97	2.66
65	7.97	7.31	5.43	4.25	4.62	3.16	4.57	3.10	4.50	2.99
70	8.92	8.18	6.08	4.76	5.17	3.38	5.11	3.30	5.03	3.16
75	9.92	9.14	6.75	5.38	5.74	3.34	5.67	3.31	5.58	2.97
76	10.14	9.35	6.90	5.48	5.86	4.70	5.80	2.68	5.71	4.70
md			12.757	13.86	23.03	22.40	22.90	24.60	24.43	24.48

Note: md=mass damper.

horizon $p=20$ and control horizon $\lambda=1$. Then MPC considering constraints on the control force and ATMD displacement is applied to this nominal building. Table 1 gives performance criteria under different control schemes including LQG, MPC[1] (with no constraints), and MPC[2] (with constraints). Table 2 shows the peak values of the displacement and acceleration at different floors under different control schemes. Table 3 lists the RMS values of the displacement and acceleration response at different floors under different control strategies.

It has been noted by Rodellar et al. (1987), and Mei and Kareem (1998) that the MPC[1] (with no constraints) scheme has equivalent control effectiveness as the LQG control design. As shown in the performance criteria in Table 1, performance of the MPC scheme is better than the TMD and is similar to LQG. Under MPC[1] scheme the peak control force is 118.1 kN while it is 118.2 kN under LQG. The RMS value of the control force is 32.23 kN under MPC[1] and 34.07 kN under LQG. Most of the criteria are a little smaller under MPC[1] except that J_8 (related to average peak displacement reduction) and J_{12} (peak value of control power) are smaller under LQG. The controlled top floor acceleration 9.26 cm/s^2 is smaller than that of LQG, which is 15.89 cm/s^2 as listed in Table 2. Similar results are obtained for the RMS values in Table 3. For example, the RMS values of the 76th floor acceleration under MPC[1] is 43% smaller than that of the LQG control.

Following the unconstrained case, the controlled response is evaluated using constrained MPC. The weight R on control force is chosen as 50 so that the maximum control force is 128 kN if not constrained. The range of the control force is chosen as [−118 kN,+118 kN] in this example. The constraint on the output is the limit on the ATMD displacement, which requires the maximum displacement to be 95 cm. The maximum control force reaches the constraint (118 kN) and an optimal solution within the boundary is obtained from the constrained MPC scheme. The results for MPC[2] (with constraints) are shown in Tables 1, 2, and 3. Under MPC[2] scheme the criteria J_1 to J_4, and J_7 to J_{10} are smaller, which means better response reduction, while J_5, J_6, J_{11}, and J_{12} are larger, which implies larger AMD stroke and more control power. This leads to more response reduction than the MPC[1] and LQG schemes while the peak control force remains 118 kN as prescribed.

Buildings with ±15% of Original Stiffness

To show the robustness of the controller, the uncertainty of building stiffness is taken into consideration. In addition to the ''nominal building,'' two additional buildings are taken into account. One case is with a +15% higher stiffness of the building and the other with a −15% lower stiffness, which are referred to as the +15% building and the −15% building, respectively, in the benchmark problem. The stiffness matrices for the two buildings are obtained by multiplying each element of the stiffness matrix of the nominal building by 1.15 and 0.85, respectively. The controller designed previously for the nominal building is applied to the ±15% buildings. The performance criteria of the ±15% buildings are presented in Table 4. The peak and the RMS values of displacement and acceleration of the two buildings are listed in Tables 5 and 6.

As noted from these tables, MPC[1] and MPC[2] designed for the nominal building result in reducing the response of the ±15%

Table 4. Evaluation Criteria for Across-Wind Excitations

Root mean square response: Evaluation criteria	$\Delta K=15\%$ MPC[1]	$\Delta K=15\%$ MPC[2]	$\Delta K=-15\%$ MPC[1]	$\Delta K=-15\%$ MPC[2]	Peak response: Evaluation criteria	$\Delta K=15\%$ MPC[1]	$\Delta K=15\%$ MPC[2]	$\Delta K=-15\%$ MPC[1]	$\Delta K=-15\%$ MPC[2]
J_1	0.345	0.335	0.390	0.376	J_7	0.386	0.381	0.461	0.451
J_2	0.389	0.379	0.439	0.425	J_8	0.432	0.430	0.537	0.529
J_3	0.477	0.472	0.710	0.702	J_9	0.607	0.611	0.780	0.751
J_4	0.479	0.474	0.711	0.704	J_{10}	0.614	0.618	0.788	0.760
J_5	1.883	1.992	2.548	2.748	J_{11}	1.840	1.931	2.702	2.750
J_6	9.732	11.440	14.34	17.25	J_{12}	61.69	70.43	99.04	96.54
σ_u (kN)	30.23	33.86	37.50	43.16	$\max\|u(t)\|$	112.5	118.0	135.5	118.0
σ_{xm} (cm)	19.08	20.19	25.83	27.86	$\max\|x_m\|$	59.44	62.35	87.26	88.83

Note: MPC=model predictive control.

Table 5. Results of Model Predictive Control for +15% Building

Floor number	MPC[1] $u_{max}=112.5$ kN		MPC[2] $u_{max}=118.0$ kN		MPC[1] $\sigma_u=30.23$ kN		MPC[2] $\sigma_u=33.86$ kN	
	x_{pi} cm	$\ddot{x}_{pi}$ cm/s^2	x_{pi} cm	$\ddot{x}_{pi}$ cm/s^2	σ_{xi} cm	$\sigma_{\ddot{x}i}$ cm/s^2	σ_{xi} cm	$\sigma_{\ddot{x}i}$ cm/s^2
1	0.034	0.24	0.034	0.24	0.01	0.06	0.01	0.06
30	4.34	3.77	4.37	3.71	1.04	0.92	1.03	0.91
50	10.33	6.54	10.40	6.62	2.51	1.91	2.49	1.86
55	12.03	7.83	12.11	7.79	2.93	2.25	2.91	2.20
60	13.77	8.96	13.87	8.90	3.37	2.61	3.34	2.56
65	15.54	10.01	15.66	10.20	3.81	2.95	3.78	2.89
70	17.33	11.13	17.46	10.75	4.26	3.15	4.22	3.07
75	19.19	11.71	19.32	11.55	4.73	3.08	4.68	2.93
76	19.60	17.49	19.74	17.17	4.83	4.54	4.79	4.54
md	59.44	68.58	62.35	72.48	19.08	20.87	20.19	22.10

Note: md=mass damper.

buildings. As observed from the results of Tables 5 and 6, like the LQG case, the acceleration response quantities are robust for the MPC schemes. In comparison with the nominal structure, the displacement of the 75th floor, stroke, active control force, and control power for the −15% building under MPC[1] increase by about 24.2, 12.8, 16.4, and 19.9%, respectively. Under MPC[2], they increase by 24.7, 14.0, 16.8, and 18.8%, respectively. For the +15% building, The displacement, stroke, active control force, and control power in comparison with the nominal building are reduced by 16.6, 16.7, 6.2, and 18.6% by MPC[1], respectively. Using MPC[2], the reductions are 16.1, 17.4, 8.4, and 21.2%, respectively. For the MPC[2] scheme, the maximum absolute control force is always limited to be less than 118 kN for both the +15% and the −15% buildings. With a lager control power, the response reduction is better than those of MPC[1] and LQG. The RMS value of the ATMD displacement and the peak value of ATMD displacement both remain within the prescribed limits.

Figs. 4 and 5 compare the changes in the displacement of the 75th floor, actuator stroke, control force, and control power under LQG, MPC[1], and MPC[2] schemes when the structural stiffness has variations of ±15%. Compared to the LQG scheme, for the ±15% buildings, the displacement of 75th floor under MPC schemes is a little more sensitive to the stiffness uncertainty than that under the LQG scheme. However, the required actuator capacity (stroke, control force, and control power) under MPC schemes is much less sensitive to the stiffness uncertainty than those under the LQG scheme (Yang et al. 2000). These trends demonstrate that the MPC schemes are more robust to the uncertainty in the structural stiffness.

To sum up, from the numerical examples, MPC exhibits effectiveness similar to the LQG method. The ±15% changes in the stiffness of the building does notably affect the controller performance. MPC based schemes show more robustness in the event of uncertainty in the structural model. The MPC scheme can also address control under constraints more effectively. Simulations show that for the ATMD, MPC with constraints can restrict the control force within the prescribed limits and generate optimal control force at each time step. The damper displacement is also limited within the required range.

The most appealing feature of MPC for practical applications concerns its ability to explicitly account for constraints in the controller design. When constraints are not present, MPC is simply a linear quadratic regulator problem which has computational effort that varies linearly with the predicted horizon p. When constraints are included, different algorithms lead to different computational effort. In this paper, an active set algorithm was used which has a computational cost that varies as a cube of the prediction horizon. Improved algorithms can be explored, such as interior-point methods, which could help to reduce the computational effort to a level that is just above the one linearly related to the prediction horizon (Rao et al. 1998). Also with advances in high speed processors, the computational effort involving MPC would scale down rapidly and the benefits of MPC would become more apparent. Above all, the MPC scheme can accommodate practical civil engineering problems and provides a more effective means of handling physical constraints. Additional studies in MPC would increase its familiarity to the structural control com-

Table 6. Results of Model Predictive Control (MPC) for −15% Building

Floor number	MPC[1] $u_{max}=135$ kN		MPC[2] $u_{max}=118.0$ kN		MPC[1] $\sigma_u=37.50$ kN		MPC[2] $\sigma_u=43.16$ kN	
	x_{pi} cm	$\ddot{x}_{pi}$ cm/s^2	x_{pi} cm	$\ddot{x}_{pi}$ cm/s^2	σ_{xi} cm	$\sigma_{\ddot{x}i}$ cm/s^2	σ_{xi} cm	$\sigma_{\ddot{x}i}$ cm/s^2
1	0.044	0.228	0.043	0.23	0.012	0.058	0.012	0.059
30	5.59	3.77	5.39	3.75	1.54	0.99	1.53	0.96
50	13.26	7.98	12.79	7.89	3.72	2.15	3.68	2.08
55	15.44	9.98	14.88	9.89	4.35	2.54	4.30	2.48
60	17.67	11.20	17.03	11.11	5.00	2.97	4.95	2.90
65	19.96	12.72	19.23	12.46	5.67	3.34	5.60	3.26
70	22.27	13.98	21.45	13.68	6.34	3.56	6.27	3.44
75	24.66	13.88	23.75	13.52	7.04	3.42	6.96	3.21
76	25.20	20.10	24.27	19.84	7.20	5.15	7.12	5.16
md	87.25	82.63	88.83	82.96	25.83	23.83	27.86	25.41

Note: md=mass damper.

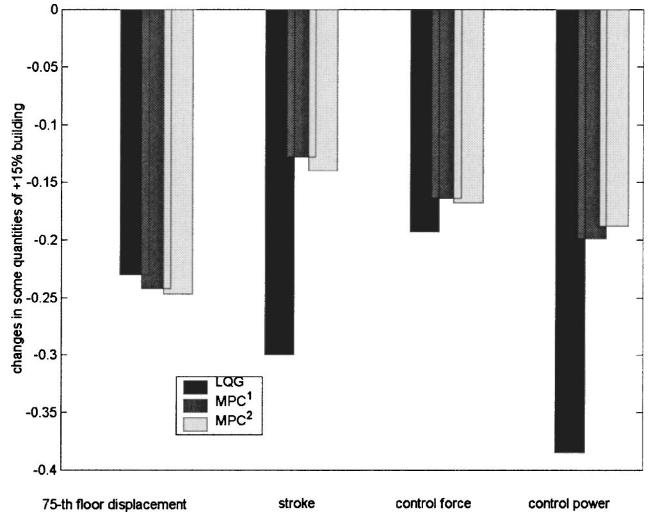

Fig. 4. Comparison of sensitivities of different control schemes to the +15% change in stiffness

munity and help to expedite its applications to structural engineering.

Concluding Remarks

In this paper, the model predictive control scheme was employed to reduce structural response of the benchmark problem under wind excitation with the input/output inequality constraints imposed on the structure and the control device. At each time step, MPC reduced to an optimal problem subjected to prescribed constraints on the input and output. This resulted in a quadratic programming problem with inequality constraints. Numerical studies of the nominal building demonstrated the effectiveness of the MPC scheme with or without the consideration of constraints. Two buildings with ±15% stiffness uncertainty highlighted the robustness of the MPC based schemes. The constraints for the control actuator were satisfied in both buildings with uncertain characteristics. The results demonstrated the effectiveness and robustness of the MPC based schemes for the benchmark study and the potential for applications to civil engineering structures.

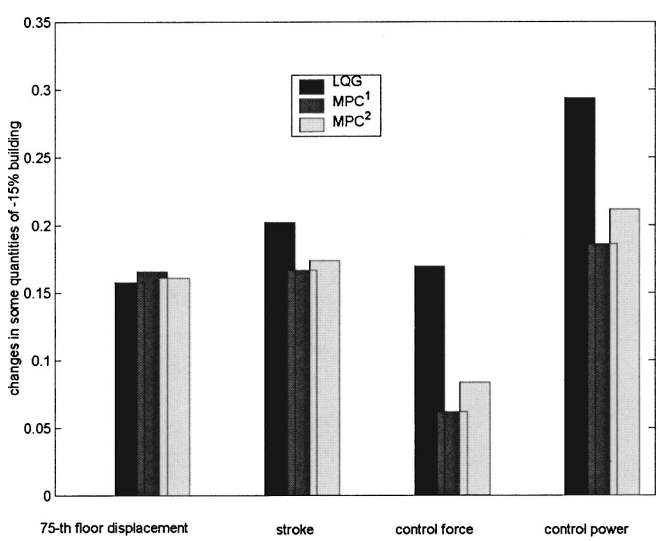

Fig. 5. Comparison of sensitivities of different control schemes to the −15% change in stiffness

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