APPENDIX

A.1 Evaluation of Response Integral

In order to evaluate the response statistics of systems subject to random excitations with rational power spectra, the integrals are of the following form,

$$
I_n \equiv \int_{-\infty}^{\infty} \frac{\Xi_n(\omega) d\omega}{\Lambda_n(-i\omega)\Lambda_n(i\omega)}
$$
(A. 1)

where

 $\Xi_n(\omega) = \chi_{n-1}\omega^{2n-2} + \chi_{n-2}\omega^{2n-4} + ... + \chi_0$ and $\Lambda_n(i\omega) = \lambda_n(i\omega)^n + \lambda_{n-1}(i\omega)^{n-1} + \dots + \lambda_0$

This integral can be written in a matrix form as (Roberts and Spanos, 1990),

$$
I_{n} = \frac{\pi}{\lambda_{m}} \begin{bmatrix} \chi_{m-1} & \chi_{m-2} & \cdots & \cdots & \cdots & \chi_{0} \\ -\lambda_{m} & \lambda_{m-2} & -\lambda_{m-4} & \lambda_{m-6} & \cdots & \cdots \\ 0 & -\lambda_{m-1} & \lambda_{m-3} & -\lambda_{m-5} & \cdots & \cdots \\ \cdots & 0 & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \cdots & -\lambda_{2} & \lambda_{0} \end{bmatrix}
$$
\n
$$
I_{n} = \frac{\pi}{\lambda_{n}} \begin{bmatrix} \lambda_{m-1} & -\lambda_{m-3} & \lambda_{m-5} & -\lambda_{m-7} & \cdots & \cdots \\ -\lambda_{m} & \lambda_{m-2} & -\lambda_{m-4} & \lambda_{m-6} & \cdots & \cdots \\ 0 & -\lambda_{m-1} & \lambda_{m-3} & -\lambda_{m-5} & \cdots & \cdots \\ \cdots & 0 & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \cdots & -\lambda_{2} & \lambda_{0} \end{bmatrix}
$$
\n(A. 2)

where \vert denotes determinant of the matrix.

A.2 Building and Excitation Parameters (Example 4 in Chapter 5)

The building stiffness matrix is given by,

$$
K = \frac{4.5}{0.0254} \begin{bmatrix} 2000 & -1000 & 0 & 0 & 0 \ -1000 & 4800 & -1400 & 0 & 0 \ 0 & -1400 & 6000 & -1600 & 0 \ 0 & 0 & -1600 & 6600 & -1700 \ 0 & 0 & 0 & -1700 & 7400 \end{bmatrix}
$$
kN/m

and the excitation parameters in Eq. 5.30 are given as:

$$
a = 4.5 \begin{bmatrix} 675.45 \\ 700.45 \\ 615.15 \\ 555.25 \\ 475.05 \end{bmatrix} \text{kN}; \quad b = 4.5 \begin{bmatrix} 0.3 \\ 375 \\ 284.5 \\ 175.3 \\ 15.1 \end{bmatrix} \text{kN}; \quad c = 4.5 \begin{bmatrix} 735.5 \\ 655.15 \\ 564.45 \\ 90.15 \\ 18.6 \end{bmatrix} \text{kN}; \quad d = 4.5 \begin{bmatrix} 180.5 \\ 35.5 \\ 425.0 \\ 280.0 \\ 650.05 \end{bmatrix} \text{kN}
$$

A.3 Relation between $\mathbf{C}_{\mathbf{v}}$ **and ξ**

Most valve suppliers provide a different measure of flow characteristic than the headloss coefficient (ξ) used thoroughout this dissertation. The commonly used measure is the valve conductance which is defined as the mass flow of liquid through the valve, given by,

$$
Q = C_V \sqrt{\rho(\Delta p)} \tag{A. 3}
$$

where *Q* is the mass flow (Kg/s); C_V is the valve conductance (m²); ρ is the specific density of the liquid (Kg/m³); Δp is the pressure drop across the valve (Pa). The valve conductance is usually supplied in British rather than S.I. units. The parameter \tilde{C}_V in gall/min/ $(psi)^{1/2}$ can be related to C_V (in S.I. units) by the conversion factor,

$$
C_V = 2.3837 \times 10^{-5} \tilde{C}_V
$$
 (A. 4)

A 1.5 inch ball valve has been used for the experimental study described in chapter 7. The valve manufacturer provided the valve conductance values as a function of the valve opening angle (Fig. A.1 (a)). The headloss across a valve/orifice can be written as,

$$
\Delta p = \frac{\rho \xi V^2}{2} \tag{A.5}
$$

Equation A.5 can be rewritten as follows:

$$
\Delta p = \frac{Q^2}{\rho C_V^2} \tag{A.6}
$$

The flow through the pipe of diameter *D* is given by:

$$
Q = \rho A V = \frac{\pi \rho D^2}{4} V \tag{A.7}
$$

Comparing Eqs. A.3 and A.7, we obtain:

$$
\xi = \frac{\pi^2 D^4}{8 C_V^2} \tag{A. 8}
$$

Equation A.8 has been plotted for the 1.5 inch ball valve as a function of the angle of valve opening.

Figure A.1 (a) Variation of Valve Conductance (b) Variation of headloss coefficient with the angle of valve opening