**Along-wind Static Equivalent Wind Loads and Responses of Tall Buildings. Part II: Effects of Mode Shapes**

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**Abstract**

The reconsideration of the along-wind responses of tall buildings in part I is continued in the present paper. According to the literature to date, the effects of the mode shape on the along-wind responses of tall buildings are taken as to be negligible. In this paper, it is pointed out that only the effect on the first mode displacement response has been included in the previous literature. Closed form expressions for the mode shape correction are provided. The factor for the first mode displacement response is in good agreement with that in the literature, and it can be neglected; but the factors for some other wind-induced responses, such as the base shear force, cannot be neglected. At last, a new correction procedure, which provides directly the static equivalent wind load, is developed.

**Keywords:** Mode shape correction; Wind-induced responses in along-wind; Tall building

1. **Introduction**

The assumption of linear mode shape is adopted by Dr. Davenport (1967)[1] in his earliest paper introducing the “gust loading factor” method. Vickery (1970)[2] gives a mode shape correction formula for the “gust loading factor”, and concludes that “even for large deviations of the actual mode shape, the errors are roughly 1 to 3 percent, making the effect of moderate deviations from a straight line mode shape insignificant”. This conclusion is referred to by almost all the latterly developed codes and high frequency base balance[3] and other test techniques. As far as the current codes is concerned, the assumption of linear mode shape is accepted by almost all the codes of the major countries and associations, for example, the Canadian code NRCC-1995[4], the Australian code AS1170.2-89[5], the American code ANSI-1993[7] and the RLB-AIJ1993[6], etc. The mode shape recommended by the Chinese loading code, GBJ9-87[8], is also very close to a straight line. For actual building with non-linear mode shape, the formulae set out in these codes are directly used and no correction taking account of the effect of the mode shape is provided. In the process of revising for the new version RLB-AIJ1993, the effect of mode shape on the along-wind response is taken as a problem to be resolved. The study shows that “the effect of the mode shape on \(G_f\) … is within 4% for \(\beta = 0.5 - 2\)”[9], and the effect of mode shape is then neglected.

All these papers [2,9] are limited to the effect of mode shape on the “gust loading factor” or the “gust effect factor”, \(G_f\), in RLB-AIJ1993. However, as pointed out by the authors in part I[10] that the “gust loading factor” is essentially the first mode displacement “gust effect factor”. That is to say, only the effect on the first mode

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displacement response has been incorporated in the precious literature, and the effects on the other responses are ignored. Referring to the force balance technique, the effects of non-ideal mode shape on the wind-induced responses are discussed by Boggs et al. (1989) and other researchers. Because different distributions of applied wind forces from those generally used in the codes are adopted, and with some misleading analysis existing in these studies, the results of correction in those literature are not effective for along-wind problem.

In this paper, closed form corrections applicable to arbitrary wind-induced responses are provided. Similar with the treatment in part I, the correction factors are separately expressed for background and resonant responses. Then, they are discussed and compared with the results in the previous literature. Because the corrections are dependent on the response, a new correction procedure, which provides directly the equivalent wind loads, is developed. The new correction procedure is especially convenient for code use.

2. Equivalent wind loads and responses of tall buildings

The mean wind force has been generally recognized by the engineers and is not addressed in this paper. Considering the same building characteristics and using the same additional assumptions as those in the part I, the expressions for the background and resonant equivalent wind loads and responses components of tall buildings with arbitrary mode shapes and linear mode shapes are separately provided in this section. The detailed derivation is similar with and can be referred to those in part I.

2.1 Buildings with arbitrary mode shapes

The expressions for buildings with arbitrary mode shapes have set out in part I, and they are listed as follows. The background equivalent wind load regarding to certain response

$$
\hat{P}_B(z) = g_B Q(z) \sigma_p(z),
$$

(1)

$$
Q(z) = \frac{\int_{-H}^{0} \int_{-H}^{0} \int_{0}^{B} \int_{0}^{B} p(z_1, x_1) p(z_1, x_2) \dot{y}(z_1) dx_1 dx_2 dz_1 dz_2 df}{\left( \int_{-H}^{0} \int_{-H}^{0} \int_{0}^{B} \int_{0}^{B} p(z_1, x_1) p(z_2, x_2) \dot{y}(z_1) \dot{y}(z_2) dx_1 dx_2 dz_1 dz_2 df \right)^{1/2}}
$$

(2)

$$
\sigma_p(z) = p U_h C_d B \left( \int_{0}^{\infty} S_u(f) \left| J_H(f) \right|^2 df \right)^{1/2} (z / H)^{1/3},
$$

(3)

where $g_B$ is the background peak factor; $p(x, z)$ is the fluctuating wind pressure on the surface of the building at the point $(x, z)$; $\sigma_p(z)$ is the RMS fluctuating externally applied wind force evaluated at height $z$; $S_u(f)$ is the spectrum of the fluctuating wind velocity; $Q(z)$ is the load-response-correlation factor. The effect of the peak background equivalent wind loads on the tall building

$$
\hat{r}_B = \frac{g_B p U_h C_d B H_i}{(1 + \alpha + \beta_0)} \left( \int_{0}^{\infty} S_u(f) \left| J_H(f) \right|^2 \left| J_z(\alpha, \beta_0, f) \right|^2 df \right)^{1/2}.
$$

(4)

The peak resonant equivalent wind load
\[ \hat{P}_R(z) = \frac{g_r \rho U H C_d B(2\beta + 1)}{(1 + \alpha + \beta)} \left| J_H(f_1) \right| J_Z(\alpha, \beta, f_1) \left[ \frac{\pi f_1}{4\zeta_1} S_u(f_1) \left( \frac{z}{H} \right)^\beta \right], \]  

in which, \( f_1 \) and \( \zeta_1 \) are separately the natural frequency and the critical damping ratio of the first mode; \( g_r \) is the resonant peak factor. The effect of the resonant equivalent wind load on the building

\[ \hat{R} = \int_0^H \hat{P}_R(z) i(z) dz \]

\[ = \frac{g_r \rho U H C_d BH(2\beta + 1) i_c}{(1 + \beta + \beta_0)(1 + \alpha + \beta)} \left| J_H(f_1) \right| J_Z(\alpha, \beta, f_1) \left[ \frac{\pi f_1}{4\zeta_1} S_u(f_1) \right]. \]  

### 2.2. Buildings with linear mode shapes

As pointed out in part I, the actual response dependent background equivalent wind load can be well approximated with the \( 2\alpha \) exponent wind load equivalent with respect to the first mode displacement response; while the resonant component should be represented by the inertial wind load.

The \( 2\alpha \) exponent background equivalent wind load for building with linear mode is (refer to Eq.23 of part I)

\[ \hat{P}_b(z) = \frac{g_B \rho U H C_d B(2\alpha + 2)}{(2 + \alpha)} \left( \int_0^\infty S_u(f) \left| J_Z(\alpha, 1, f) \right|^2 \right) d\alpha \left( \frac{z}{H} \right)^{2\alpha}; \]

Its load effect on the building is

\[ \hat{R}_b = \int_0^H \hat{P}_b(z) i(z) dz \]

\[ = \frac{g_B \rho U H C_d BH(2\alpha + 2) i_c}{(1 + 2\alpha + \beta_0)(2 + \alpha)} \left( \int_0^\infty S_u(f) \left| J_Z(\alpha, 1, f) \right|^2 \right) d\alpha; \]

where the superscript \( \hat{\prime} \) means the expression is for building with linear mode shape.

Substituting the linear mode shape, the peak resonant equivalent wind load of building with linear mode shape can be obtained (refer to Eq.16 of part I)

\[ \hat{P}_R(z) = \frac{3g_B \rho U H C_d B}{(2 + \alpha)} \left| J_H(f_1) \right| J_Z(\alpha, 1, f_1) \left[ \frac{\pi f_1}{4\zeta_1} S_u(f_1) \left( \frac{z}{H} \right) \right]; \]

\[ = P_R \left( \frac{z}{H} \right) \]

Its load effect on the building is

\[ \hat{R}_R = \int_0^H \hat{P}_R(z) i(z) dz \]

\[ = \frac{3g_B \rho U H C_d BH i_c}{(2 + \beta_0)(2 + \alpha)} \left| J_H(f_1) \right| J_Z(\alpha, 1, f_1) \left[ \frac{\pi f_1}{4\zeta_1} S_u(f_1) \right]. \]

### 3. Correction factors for wind induced responses and parametric study

Because the distributions of equivalent wind loads for buildings with linear mode shapes are different from those of the actual ones, the correction factors in this section are defined for the load effects rather than the wind loads.

#### 3.1 Correction factors

\[ \hat{P}_R(z) = \frac{g_B \rho U H C_d B(2\beta + 1)}{(1 + \alpha + \beta)} \left| J_H(f_1) \right| J_Z(\alpha, \beta, f_1) \left[ \frac{\pi f_1}{4\zeta_1} S_u(f_1) \left( \frac{z}{H} \right)^\beta \right], \]  

\[ \hat{R} = \int_0^H \hat{P}_R(z) i(z) dz \]

\[ = \frac{g_B \rho U H C_d BH(2\beta + 1) i_c}{(1 + \beta + \beta_0)(1 + \alpha + \beta)} \left| J_H(f_1) \right| J_Z(\alpha, \beta, f_1) \left[ \frac{\pi f_1}{4\zeta_1} S_u(f_1) \right]. \]
Define background correction factor $C_B$ as the ratio of the actual background response to that by the linear mode shape method; and resonant correction factor $C_R$ as the ratio of the resonant response. Using the Eqs.4,8 and Eqs.6, 10, the correction factors can be obtained

\[
C_B = \frac{\hat{r}_B / \hat{r}_B^*}{(1 + 2\alpha + \beta \alpha_0) (2 + \alpha)} \left( \int_0^\infty S_u(f) J_H(f) df \right) \left( \int_0^\infty J_Z(\alpha, \beta, f) df \right)^{1/2}
\]

\[
C_R = \frac{\hat{r}_R / \hat{r}_R^*}{3(1 + \beta + \beta_0) (1 + \alpha + \beta)} \left( \int_0^\infty S_u(f) J_H(f) df \right) \left( \int_0^\infty J_Z(\alpha, 1, f) df \right)^{1/2}
\]

3.2 Parametric study of the correction factors

The background correction factor $C_B$ is dependent on parameters $\alpha$ and $\beta_0$, but is independent of $\beta$ or the mode shape. The sensitivity of $C_B$ to its parameters $\alpha$, $\beta_0$ is illustrated in Figure 1. Fig.1(a) demonstrates the sensitivity of $C_B$ to $\beta_0$ or the type of the response. $C_B$ is relatively sensitive to $\alpha$. When $\beta_0 = 1$ (the base bending moment response), $C_B = 1$, regardless of the value of $\alpha$. As $\beta_0 = \beta$, the factor can be then interpreted as that for the first mode displacement response, and is illustrated in Fig.1(b). The first mode displacement factor $C_B$ varies within 0.97~1.03 as $\beta$ varies from 0.5 to 2.0.

![Figure 1. Correction factor for background response](image)

The resonant correction factor $C_R$ is dependent on all the parameters $\alpha, \beta, \beta_0$. The sensitivity of $C_R$ to its parameters is illustrated in Fig.2. When $\beta = 1$ (linear mode shape), as expected, $C_R = 1$, regardless the values of $\alpha, \beta_0$. Fig.2(a) shows the sensitivity of $C_R$ to $\beta_0$ and $\beta$. It can be seen that the sensitivity of $C_R$ to $\beta$ (the mode shape) is evidently different for different value of $\beta_0$ or the type of response. For the base shear force response ($\beta_0 = 0$), $C_R$ is most sensitive to $\beta$. In Fig.2(b), $\beta_0 = 0$, $C_R$ is then the
correction factor for the resonant base shear force response. When $\alpha = 0.35$, $C_R$ decreases from 1.10 to 0.84 as $\beta$ increases from 0.5 to 2.0. Unlike the factor for the background responses, it is relatively insensitive to $\alpha$. As $\alpha$ varies from 0.1 to 0.35, the variation of $C_R$ is within 5% for all values of $\beta$. In Fig.2(c), $\beta_0 = 1$, $C_R$ is the correction factor for the resonant base bending moment response. The factor is generally lower than 1 but the deviation is within 5%. In Fig.2(d), $\beta_0 = \beta$, $C_R$ is the correction factor for the first mode displacement response. The factor varies within 0.99~1.03 as $\beta$ from 0.5 to 2.0.

![Graphs showing correction factors for different responses](image)

**Figure 2. Correction factor for resonant response**

### 3.3 Discussion

When $\beta_0 = \beta$ the correction factors in Eqs.11 and 12 are then both the factor for the first mode displacement response. As indicated in the companion paper [10], this correction factor is essentially the factor for the “gust loading factor” in the precious literature. The above analysis shows that the factor is within 5% when $\beta = 0.5~2.0$, which is consistent with the results in the previous literature. However, as have noted in the above analysis, the sensitivity of the resonant correction factor to $\beta$ or the mode shape is different for different type of response. The correction factor is more sensitive to $\beta$ for
some type response, for example, the base shear force response, than for the first mode
displacement response. When $\alpha = 0.35$, $\beta_0 = 0$, the background factor for the base shear
force $C_B = 1.03$, the resonant factor varies from 1.10 to 0.84 as $\beta$ from 0.5 to 2.0. The
latter can hardly be neglected even in the engineering viewpoint, while it has never been
emphasized in the previous literature.

4. New correction procedure for resonant equivalent wind load

The correction factors $C_B, C_R$ in section 4 are all dependent on $\beta_0$ or the response,
which means that the corrections for different responses are different. It is not convenient
to provide the correction factor for certain response for the code use. In this section, a
new correction procedure, which aims at the equivalent wind load, is developed.

Because the background equivalent wind load and response are both independent of $\beta$
or the mode shape, as well as the $2\alpha$ exponent equivalent wind load basing on the linear
mode shape is a fairly good approximation of the actual background equivalent wind
loads, the background equivalent wind load is not addressed in the new correction
procedure.

Define the new resonant correction factor as the correction factor for the resonant
equivalent wind load evaluated at the top height of the building. Using Eqs.5 and 9, and
letting $Z = H$, the factor can be derived as

$$
C_R \text{new} = \frac{\hat{P}_r(H) / \hat{P}_r(H)}{(2 + \alpha)(1 + 2\beta)\left\|J_Z(\alpha, \beta, f_1)\right\|} 
\frac{3(1 + \alpha + \beta)\left\|J_Z(\alpha,1, f_1)\right\|}. 
$$

Correspondingly, the new procedure for computing the actual resonant equivalent wind
load is

$$
\hat{P}(z) = C_{R-\text{new}} * P_1 * (z / H)^\beta, 
$$
in which, $P_1$ is the resonant equivalent wind load coefficient basing on the linear mode
shape (refer to Eqs.5 and 9).

Note that, the new correction factor above can also be interpreted as that for the first
mode resonant displacement response and acceleration evaluated at the top height of the
building except that for the equivalent wind load.

The sensitivity of the new correction factors $C_{R-\text{new}}$ to its parameters is illustrated in
Figure 3. It can be seen that the factor is insensitive to $\alpha$, and can be well approximated
with a linear function regarding parameter $\beta$ as

$$
C_{R-\text{new}} = 0.3 * \beta + 0.7. 
$$
The deviation of the approximation formula is within 5% for $\beta = 0.5 \sim 2.0$. 
Figure 3. New correction factor for resonant equivalent wind load

It is worthwhile to point out that the new correction formula in Eq.15 is very close to the correction factor for the acceleration at the top height of the building in across-wind set out in AS1170.2-89,\cite{5} which is \(0.24 \beta + 0.76\); and to the factor for the equivalent wind load or the first mode displacement response at the top of the building in across-wind provided by RLB-AIJ1993,\cite{6} which is \(0.27 \beta + 0.73\).

5. Conclusions

Only the effect on the “gust loading factor” or the first mode displacement response has been incorporated in the previous literature. In this paper, closed form expressions for the mode shape correction factors are derived. The factor for the first mode displacement response is in good agreement with that in the literature, and it can be neglected; but for some other wind-induced responses, such as the base shear force, the factor can not be neglected.

Because the correction factors are dependent on the responses, which is not convenient for code use, a new correction procedure for the equivalent wind loads is developed. The new procedure provides directly the actual resonant equivalent wind load and a good approximation of the background equivalent wind loads, the responses of tall buildings to gusty wind can be obtained with simply static analysis and no further correction is needed. The new correction procedure is especially convenient for code use.

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Reference:

[7]. ASCE, Minimum Design Loading for Buildings and Other Structures, ANSI/ASCE7-93, 1994