DICE simplified*

July 19, 2019

Masako Ikefuji
School of Social and International Studies, University of Tsukuba

Roger J. A. Laeven
Amsterdam School of Economics,
University of Amsterdam, CentER, and EURANDOM

Jan R. Magnus
Department of Econometrics & Operations Research,
Vrije Universiteit Amsterdam and Tinbergen Institute

Chris Muris
Department of Economics, University of Bristol

*We are grateful to Peter Boswijk, Rick van der Ploeg, and Hiroaki Sakamoto for comments and suggestions. This research was funded in part by the Netherlands Organization for Scientific Research (NWO) under grant Vidi-2009 (Laeven).

Corresponding author:
Chris Muris
Department of Economics
University of Bristol
E-mail: chris.muris@bristol.ac.uk

Abstract: We analyze Nordhaus' DICE model and show that the temperature and CO_2 equations are needlessly complicated and can be simplified without loss of essence. In addition, we argue that the damage function can be altered in such a way that it lends itself to experiments involving extreme risk. We conclude that, within the philosophy of the DICE model, significant simplifications can be made which make the model more transparent, more robust, and easier to apply.

JEL Classification: C51; Q54

Keywords: DICE model; Climate change; Integrated assessment model; Optimal policy; Damage function

E-mail addresses:
ikefuki.masako.gn@u.tsukuba.ac.jp (Ikefuji)
R.J.A.Laeven@uva.nl (Laeven)
jan@janmagnus.nl (Magnus)
chris.muris@bristol.ac.uk (Muris)

Entia non sunt multiplicanda praeter necessitatem
William of Ockham, 1285(?)–1347

1 Introduction

This paper provides a critical assessment of the 2016 DICE (Dynamic Integrated model of Climate and the Economy) model originally developed by Nordhaus (1992), but since then continuously updated and altered (Nordhaus, 2018b). Integrated assessment models (IAMs) are being used extensively for the analysis of climate change policy and DICE has played an important part in projecting greenhouse gas emissions and temperature under various social and economic scenarios.

The assumptions and functional forms used in DICE have been under considerable scrutiny and criticism. Among others, Pindyck (2013, 2017) and Gerlagh and Liski (2018a, 2018b) argue that IAMs should be made simpler and more transparent, and an editorial in *Nature Climate Change* (Editorial, 2015) also emphasizes this view. DICE is complex, because it employs an equation for radiative forcing, two equations (a two-box model) for the climate system, and a three-reservoir model for the carbon cycle. Some of the other IAMs (for example, the FUND and PAGE models) are simpler, as they employ a single-equation climate model (Calel and Stainforth, 2017).

DICE includes a complex geophysical system, perhaps too complex. The purpose of this paper is to show that the temperature and CO_2 equations are needlessly complicated and can be much simplified. The reason why this simplification is possible lies in the fact that the two dynamic equations describing temperature in the DICE model have one approximate unit eigenvalue, and that the three equations describing CO_2 have one exact and one approximate unit eigenvalue. We derive the exact counterparts after differencing out the auxiliary variables and we provide simplifications.

In response to criticism that IAMs give the impression of being 'black boxes' (Editorial, 2015), some studies propose a simpler IAM in which an analytical formula for the optimal carbon price can be derived; see Golosov *et al.* (2014), Traeger (2015), Rezai and Van der Ploeg (2016), Van den Bijgaart *et al.* (2016), Dietz and Venmans (2018), and Van der Ploeg (2018). Among studies employing closed-form IAMs, the specifications of the climate system and the carbon cycle vary. Golosov *et al.* (2014) specify a two-and-a-half-box carbon system consisting of a permanent component (about 20% of carbon) and a transient component. The climate system is omitted. Under assumptions such as logarithmic utility, Cobb-Douglas technology, constant saving rate, and full depreciation of capital, they derive the optimal-tax for-

mula analytically. Rezai and Van der Ploeg (2016) employ a similar two-box carbon system. They allow for a lag between temperature and atmospheric carbon in addition to more general functional forms than those assumed by Golosov *et al.* (2014), and they derive a simple rule for the optimal carbon price.

The omission of the climate system in Golosov *et al.* (2014) is justified by recent findings in climate science that the climate response to a CO_2 emission is nearly instantaneous and remains almost constant over time; see Roe and Bauman (2013) and Ricke and Caldeira (2014). Based on the same findings, Dietz and Venmans (2018) assume that the global mean temperature is linearly proportional to cumulative CO_2 emissions, in contrast to the large thermal inertia of the climate system assumed in DICE and Lemoine and Rudik (2017). These differences may lead to different optimal transition paths of temperature. Thus, the optimal carbon price follows Hotelling's rule in Dietz and Venmans (2018), whereas it grows more slowly in Lemoine and Rudik (2017).

Our purpose is to stay as close as possible to Nordhaus' DICE model, but to simplify it using Ockham's razor quoted above ('more things should not be used than are necessary'), also in the spirit of Einstein's dictum: 'as simple as possible but not simpler'. A complex model is not necessarily a better model than a simple model. Leaving things out is arguably more difficult and more important than putting things in. Many statisticians believe that a more complex model will reduce the bias and increase the variance, but this is only half true. A more complex model does indeed increase the variance, but it does not necessarily reduce the bias (De Luca *et al.*, 2018). Hence, simplicity matters given the large uncertainties in the exogenous variables (such as population and technical knowledge) and the parameters.

The purpose of this paper is therefore not to propose a completely different climate system nor to examine climate sensitivity in the DICE model. Our purpose is more modest. We show that the temperature and CO_2 equations in the DICE model are needlessly complicated and that they can be much simplified. We also argue that the specification of the damage function can be altered in such a way that it lends itself to experiments involving extreme risk. Finally, we briefly discuss the assumption in DICE that the abatement fraction for CO_2 is allowed to become larger than one, which implies that emissions can become negative.

In Section 2 we present the Nordhaus DICE 2016R model. In Sections 3 and 4 we discuss and simplify the DICE equations for temperature and CO_2 concentration. In Section 5 we provide an alternative to the DICE damage function. Section 6 presents the SICE (simplified DICE) model and concludes.

2 Nordhaus' DICE 2016R model

The following equations are the equations from the beta version of DICE-2016R (Nordhaus, 2017, 2018a), a version with the identification DICE-2016R-091916ap.gms. A number of equations are redundant and have been deleted. A new variable ω_t has been introduced, some equations have been combined, and the equations have been reordered; see Ikefuji *et al.* (2019) for the details. Still, this is *precisely* the same model as Nordhaus' 2016R model.

Everybody works. In period t, the labor force L_t together with the capital stock K_t generate GDP Y_t through a Cobb-Douglas production function

$$Y_t = A_t K_t^{\gamma} L_t^{1-\gamma} \qquad (0 < \gamma < 1), \tag{1}$$

where A_t represents technological efficiency and γ is the elasticity of capital. Capital is accumulated through

$$K_{t+1} = (1-\delta)K_t + I_t \qquad (0 < \delta < 1), \tag{2}$$

where I_t denotes investment and δ is the depreciation rate of capital.

Carbon dioxide (CO_2) emissions consist of industrial emissions and non-industrial ('land-use') emissions. We denote the latter type by E_t^0 and consider it to be exogenous to our model. Total CO_2 emissions E_t are then given by

$$E_t = \sigma_t(1-\mu_t)Y_t + E_t^0, \tag{3}$$

where σ_t denotes the emissions-to-output ratio for CO_2 and μ_t is the abatement fraction for CO_2. The associated CO_2 concentration increase M_t in the atmosphere (GtC from 1750) accumulates through

$$M_{t+1} = (1-b_0)M_t + b_1 X_{1,t} + E_t, \tag{4a}$$
$$X_{1,t+1} = b_0 M_t + (1-b_1-b_3)X_{1,t} + b_2 X_{2,t}, \tag{4b}$$
$$X_{2,t+1} = b_3 X_{1,t} + (1-b_2)X_{2,t}, \tag{4c}$$

where $X_{1,t}$ and $X_{2,t}$ are auxiliary variables representing CO_2 concentration increases in shallow and lower oceans, respectively, also measured in GtC from 1750.

Temperature increase H_t (degrees Celsius from 1900) develops according to

$$H_{t+1} = (1-a_0)H_t + a_1 \log(M_{t+1}) + a_2 Z_t + F_{t+1}, \tag{5a}$$
$$Z_{t+1} = (1-a_3)Z_t + a_3 H_t, \tag{5b}$$

where Z_t is an auxiliary variable representing the temperature increase of the lower oceans, also measured in degrees Celsius from 1900, and F_{t+1} is exogenous radiative forcing.

In each period t, the fraction of GDP not spent on abatement or 'damage' is either consumed (C_t) or invested (I_t) along the budget constraint

$$\left(1 - \omega_t - \xi H_t^2\right) Y_t = C_t + I_t. \tag{6}$$

A fraction ω_t of Y_t is spent on abatement, and we specify the abatement cost fraction as

$$\omega_t = \psi_t \mu_t^\theta \qquad (\theta > 1). \tag{7}$$

When μ_t increases then so does ω_t, and a larger fraction of GDP will be spent on abatement.

Damage is represented by a fraction ξH_t^2 of Y_t and it depends only on temperature. The optimal temperature is $H_t = 0$, the temperature in 1900. Deviations from the optimal temperature cause damage. For very high and very low temperatures the fraction becomes large, but (given the value of ξ) it will still be a fraction between zero and one, unless in truly catastrophic cases.

As in Nordhaus (2017, 2018a) one period is five years. Period 1 refers to the time interval 2015–2019, period 2 to 2020–2024, and so on. Stock variables are measured at the beginning of the period; for example, K_1 denotes capital in the year 2015. We choose the exogenous variables such that $L_t > 0$, $A_t > 0$, $E_t^0 > 0$, $\sigma_t > 0$, and $0 < \psi_t < 1$. The policy variables must satisfy

$$C_t \geq 0, \quad I_t \geq 0, \quad \mu_t \geq 0. \tag{8}$$

Nordhaus (2018a, 2018b) allows 'negative-emission technologies' by setting an upper bound on μ_t of 1.2 (rather than 1.0) from period 30 onwards (year 2160), which implies that emissions can become negative by (3), and in fact this upper bound is reached in the DICE output from period 46 onwards (year 2240). The idea of negative emissions is controversial. Anderson and Peters (2016) state that negative-emission technologies are unjust and a high-stake gamble, while a recent editorial in *Nature* (Editorial, 2018) discusses the enormous effort required to carry out such technologies — an effort which would lead to a deterioration of the environment.

Given a utility function U we define welfare in period t as

$$W_t = L_t \, U(C_t / L_t). \tag{9}$$

The policy maker has a finite horizon and maximizes total discounted welfare

$$W = \sum_{t=1}^{T} \frac{W_t}{(1+\rho)^t} \qquad (0 < \rho < 1), \tag{10}$$

where ρ denotes the discount rate and $T = 100$ (500 years). Letting x denote per capita consumption, the utility function $U(x)$ is assumed to be defined and strictly concave for all $x > 0$. There are many such functions, but a popular choice is

$$U(x) = \frac{x^{1-\alpha} - 1}{1 - \alpha} \qquad (\alpha > 0), \tag{11}$$

where α denotes the elasticity of marginal utility of consumption. This is the so-called *power* function. Many authors, including Nordhaus, select this function. In earlier versions of the DICE model, Nordhaus (2008) chooses $\alpha = 2$ in which case $U(x) = 1 - 1/x$. Also popular is $\alpha = 1$; see Kelly and Kolstad (1999) and Stern (2007). In the 2016 version of the DICE model $\alpha = 1.45$.

3 Temperature

The DICE model thus consists of the seven equations (1)–(7). Four of these, equations (1)–(3) and (7), are not controversial. In the next three sections we shall discuss the CO_2 equation (4), the temperature equation (5), and the budget constraint (6).

We start with the temperature equations in (5), which we now write in matrix form as

$$x_{t+1} = Ax_t + a_{t+1}, \tag{12}$$

where

$$x_t = \begin{pmatrix} H_t \\ Z_t \end{pmatrix}, \quad A = \begin{pmatrix} 1 - a_0 & a_2 \\ a_3 & 1 - a_3 \end{pmatrix}, \quad a_t = \begin{pmatrix} a_1 \log(M_t) + F_t \\ 0 \end{pmatrix}.$$

The matrix A has two eigenvalues given by

$$1 - \frac{1 - \eta_1}{2} \pm \frac{1}{2}\sqrt{(1 - \eta_1)^2 - 4\eta_2},$$

where $\eta_1 = 1 - a_0 - a_3 = 0.8468$ and $\eta_2 = (a_0 - a_2)a_3 = 0.0030$, so that the eigenvalues are 0.9771 and 0.8697, respectively. The largest eigenvalue is thus close to one and it would be equal to one if (and only if) $\eta_2 = 0$.

We can 'difference out' the auxiliary variable Z and this gives

$$\begin{aligned} H_{t+1} = {} & (1 + \eta_1)H_t - (\eta_1 + \eta_2)H_{t-1} \\ & + \eta_3 \log(M_{t+1}) - (\eta_3 - \eta_4)\log(M_t) + \eta_{0t}, \end{aligned} \tag{13}$$

where $\eta_3 = a_1 = 0.5338$, $\eta_4 = a_1 a_3 = 0.0133$, and $\eta_{0t} = F_{t+1} - (1 - a_3)F_t$. Equation (13) does not contain Z but, compared to (5a), it contains an

additional lag in both H and $\log(M)$. Note that (13) is not invariant to scaling in M.

Letting Δ be the (backward) difference operator defined by $\Delta x_{t+1} = x_{t+1} - x_t$, we can write (13) alternatively as

$$\Delta H_{t+1} = \eta_1 \Delta H_t + \eta_3 \Delta \log(M_{t+1}) + \eta^*_{0t},$$

where $\eta^*_{0t} = -\eta_2 H_{t-1} + \eta_4 \log(M_t) + \eta_{0t}$. This equation in first differences can also be written as

$$H_{t+1} = \eta_0 + \eta_1 H_t + \eta_3 \log(M_{t+1}) + \sum_{j=1}^{t} \eta^*_{0j}, \tag{14}$$

where $\eta_0 = -3.3291$ is an integration constant. Notice that H_{t+1} in (14) depends on H_t but that the effect of H_{t-1} is negligible, which is another way of saying that the largest eigenvalue of the matrix A in (12) is close to one. Both are caused by the fact that η_2 is small. We emphasize that (12), (13), and (14) are *equivalent* descriptions of the DICE temperature equations. No approximation has yet taken place.

Given the DICE parameter values, in particular the fact that η_2 and η_4 are small, the partial sums $\sum_j \eta^*_{0j}$ are well approximated by a linear trend with slope 0.025. This implies that if we run a regression on the equation

$$H_{t+1} = \eta^*_0 + \eta^*_1 H_t + \eta^*_2 \log(M_{t+1}) + \eta^*_3 t \tag{15}$$

we will get a good fit. If we leave out the linear trend, then the estimate of η^*_1 increases somewhat and the estimate of η^*_2 decreases somewhat.

Table 1: Simplified temperature equations

	constant	H_t	$\log(M_{t+1})$	*trend*/10	s	$\max \lvert Q_t \rvert$
(a)	−3.6772	0.8707	0.5826	0.0064	0.0032	0.43%
	(0.0511)	(0.0014)	(0.0076)	(0.0003)		
(b)	−2.8672	0.8954	0.4622	—	0.0068	1.70%
	(0.0561)	(0.0012)	(0.0082)			

More precisely, we obtain the results in Table 1, where we note that in all regressions, figures, and numerical experiments that follow, H_t (and similarly M_t and other variables) take the optimal values as obtained from the DICE gams routine, which optimizes welfare in (10).

Under (a) we report the estimated coefficients and standard errors from a regression of H_{t+1} on a constant, H_t, $\log(M_{t+1})$, and a time trend, as in (15).

The fit is very good. In particular, letting e denote the vector of residuals and $\hat{H}_t$ the predicted value of H_t from the regression, and defining the regression variance s^2 and the relative deviations Q_t as

$$s^2 = e'e/(n-k), \qquad Q_t = 100(\hat{H}_t - H_t)/H_t,$$

we find that $s = 0.0032$ and $\max_t |Q_t| = 0.43$ with $n = 99$ and $k = 4$. This shows that for a temperature increase of, say, 3 degrees Celsius the maximum error will be 0.013 degrees.

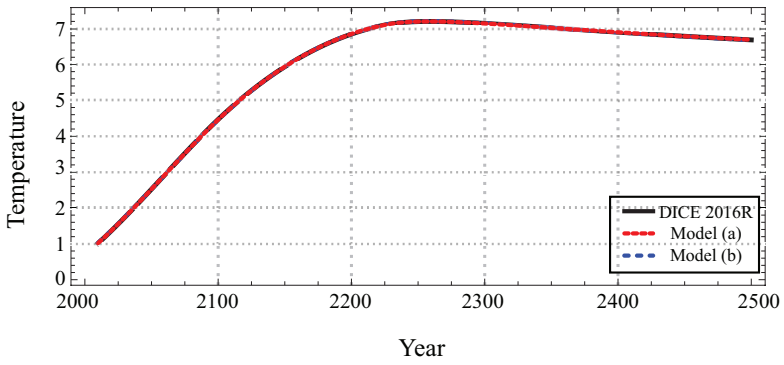

Figure 1: Temperature — time path for DICE 2016R and two simplified models

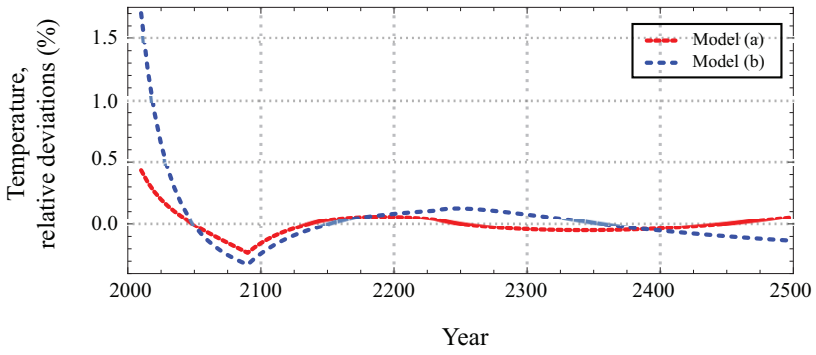

Figure 2: Temperature — Deviations (%) of two simplified models relative to DICE 2016R

If we leave out the linear trend we obtain (b) which is almost as good, except that the error in the first few periods is somewhat higher. This is illustrated in Figures 1 and 2. In Figure 1 the time paths of temperature

of DICE and the two models (a) and (b) are indistinguishable, reaching a maximum of 7.2 in 2270. The relative deviations Q_t are graphed in Figure 2. They are all below 0.5% except the first four periods in Model (b). Even though the estimated coefficient on the time trend is 'significant' it is not important, and the fit is essentially the same.

Summarizing, the simplified equation (15) provides a good approximation because (a) the coefficients in (13) correspond approximately to a first-order difference equation; and (b) the omitted variable is essentially constant. The second approximation (without trend) is almost as good as the approximation in (15), and suffices for practical applications.

4 CO_2 concentration

Next we consider the CO_2 equations in (4), which we also write in matrix form as

$$x_{t+1} = Ax_t + a_t, \tag{16}$$

where now

$$x_t = \begin{pmatrix} M_t \\ X_{1,t} \\ X_{2,t} \end{pmatrix}, \quad A = \begin{pmatrix} 1-b_0 & b_1 & 0 \\ b_0 & 1-b_1-b_3 & b_2 \\ 0 & b_3 & 1-b_2 \end{pmatrix}, \quad a_t = \begin{pmatrix} E_t \\ 0 \\ 0 \end{pmatrix}.$$

One of the three eigenvalues of A equals one, and the remaining eigenvalues are given by

$$1 - \frac{1-\phi_1}{2} \pm \frac{1}{2}\sqrt{(1-\phi_1)^2 - 4\phi_2},$$

where $\phi_1 = 1 - b_0 - b_1 - b_2 - b_3 = 0.675535$ and $\phi_2 = b_0b_2 + b_0b_3 + b_1b_2 = 0.001303$, so that the two remaining eigenvalues take the values 0.995933 and 0.679602, respectively. The largest eigenvalue is thus equal to one and the next eigenvalue is close to one; it would be equal to one if (and only if) $\phi_2 = 0$. This suggests that we should difference not once (as in the previous section) but twice, and this is precisely what we shall do.

As in the previous section we can 'difference out' the auxiliary variables X_1 and X_2, and this gives

$$M_{t+1} = (\phi_1 + 2)M_t - (1 + 2\phi_1 + \phi_2)M_{t-1} + (\phi_1 + \phi_2)M_{t-2} + E_t^*, \tag{17}$$

where

$$E_t^* = E_t - (1 + \lambda_1)E_{t-1} + (\lambda_1 + \lambda_2)E_{t-2}.$$

This equation does not contain X_1 and X_2 but it contains two additional lags in both M and E. We can write (17) alternatively as

$$\Delta M_{t+1} = (\phi_1 + 1)\Delta M_t - (\phi_1 + \phi_2)\Delta M_{t-1} + E_t^*,$$

where we notice that there is no remainder term in M_{t-2} because the largest eigenvalue of A equals one exactly given the DICE parameters. This leads to

$$M_{t+1} = \phi_0 + (\phi_1 + 1)M_t - (\phi_1 + \phi_2)M_{t-1} + E_t^{**}, \tag{18}$$

where $E_t^{**} = \sum_{j=1}^{t} E_j^*$ and $\phi_0 = 0.8761$ is an integration constant. This, in turn, can be written as

$$\Delta M_{t+1} = \phi_0 + \phi_1 \Delta M_t - \phi_2 M_{t-1} + E_t^{**},$$

so that

$$M_{t+1} = \phi_{00} + \phi_0 t + \phi_1 M_t - \phi_2 \sum_{j=1}^{t-1} M_j + \sum_{j=1}^{t} w_{tj} E_j, \tag{19}$$

where $\phi_{00} = 263.2837$ is an integration constant and

$$\begin{aligned}\sum_{j=1}^{t} w_{tj} E_j = \sum_{j=1}^{t} E_j^{**} = \sum_{j=1}^{t}\sum_{i=1}^{j} E_i^* = \sum_{j=1}^{t}(t-j+1)E_j^* \\ = E_t + (1-\lambda_1)\sum_{j=1}^{t-1} E_j + \lambda_2 \sum_{j=1}^{t-2}(t-j-1)E_j,\end{aligned}$$

so that

$$w_{tt} = 1, \qquad w_{tj} = 1 - \lambda_1 + (t-j-1)\lambda_2 \quad (j = 1, \ldots, t-1).$$

The DICE weights w_{tj} are thus slightly *increasing* rather than decreasing, which is a little awkward. Notice that equations (16)–(19) are *equivalent* descriptions of the DICE CO_2 equations. No approximation has yet taken place.

Since $\phi_2 = 0.0013$ and $\lambda_2 = 0.0003$ are close to zero, (19) will be well approximated by

$$M_{t+1} \approx \phi_{00} + \phi_0 t + \phi_1 M_t + E_t + \sum_{j=1}^{t-1}(1-\lambda_1)E_j.$$

In fact we will run regressions on the equation

$$M_{t+1} = \phi_0^* + \phi_1^* M_t + \phi_2^* E_t + \phi_3^* t \tag{20}$$

Table 2: Simplified CO_2 equations

	constant	M_t	E_t	*trend*	s	$\max \lvert Q_t \rvert$
(a)	16.27	0.9900	0.6166	0.0317	1.52	0.35%
	(1.08)	(0.0004)	(0.0067)	(0.0098)		
(b)	17.94	0.9902	0.6001	—	1.59	0.39%
	(0.99)	(0.0004)	(0.0046)			
(c)	—	0.9975	0.6549	—	3.33	0.92%
		(0.0002)	(0.0073)			
(d)	—	0.9942	1.0000	—	16.35	1.66%
		(0.0007)	(—)			

and simplifications thereof.

This leads to the results in Table 2. Under (a) we regress M_{t+1} on all four variables, under (b) we delete the trend, under (c) we also delete the constant term, and under (d) we restrict the coefficient of E_t to be one. The last model is the simplest and mirrors capital accumulation in (2). The fit is very good in all cases, as can be seen from the values of s and $\max |Q_t|$, and also from Figures 3 and 4.

In Figure 3 the time paths of CO_2 of DICE and the four models (a)–(d) are indistinguishable, reaching a maximum of 2707 in 2230. The relative deviations Q_t are graphed in Figure 4. In Models (a) and (b) the relative deviations are all below 0.4% in absolute value. In Model (c) the relative deviations are all below 0.6% in absolute value, except in the first three periods. In Model (d) the relative deviations are larger than 0.6% up to period 36 (year 2190) and smaller than 0.6% afterwards, with a maximum of 1.7% in period 12 (year 2070) where $M_t = 1402$ (the DICE output) and $\hat{M}_t = 1425$ (the predicted value of M_t from the regression).

The simplified equation (20) thus provides an excellent approximation to the DICE results because the coefficients in (17) correspond approximately to a second-order difference equation. Model (b) is possibly the preferred approximation although the simplest Model (d) will suffice for most practical applications.

5 Damage and abatement

The damage-abatement function is DICE specifies two fractions, ω_t (abatement) and ξH_t^2 (damage), of Y_t which reduce Y_t so that less money is available for investment and consumption along the budget constraint. In DICE this

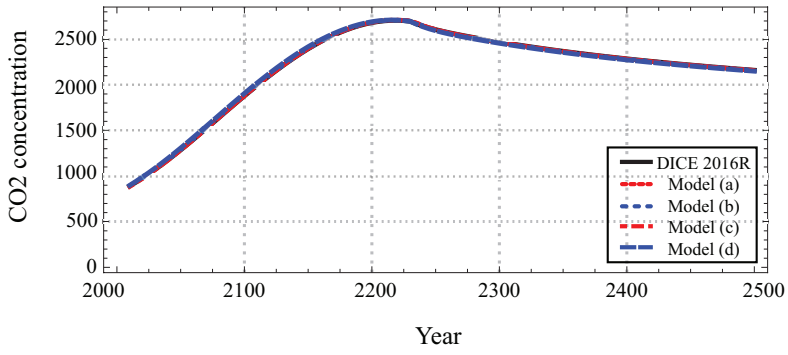

Figure 3: CO_2 concentration — time path for DICE 2016R and four simplified models

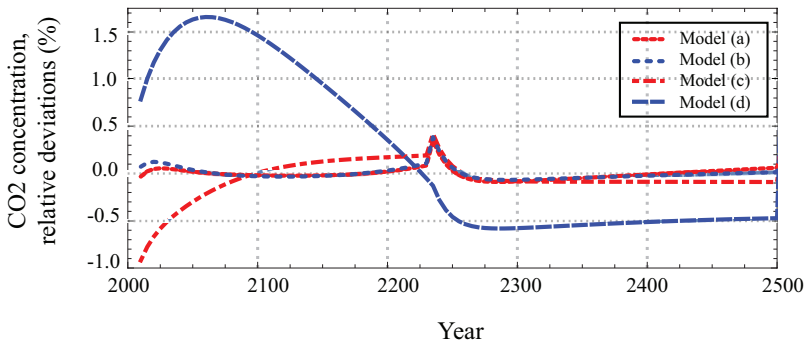

Figure 4: CO_2 concentration — Deviations (%) of four simplified models relative to DICE 2016R

fraction is specified as

$$1 - \omega_t - \xi H_t^2.$$

For very high and very low temperatures the fraction becomes large, but (given the value of ξ) it will still be a fraction between zero and one, unless in truly catastrophic cases when $H_t > 20.58$, that is, when the temperature in period t is more than 20 degrees Celsius higher than in 1900. Of course, other forms of the damage function are possible; see Stern (2007), Weitzman (2009), Ackerman *et al.* (2010), and Nordhaus (2013). Howard and Sterner (2017) emphasize the importance of the damage function in accurately estimating coefficient and standard error bias.

In Figure 5 the graph labeled DICE 2016R contains the time path of this

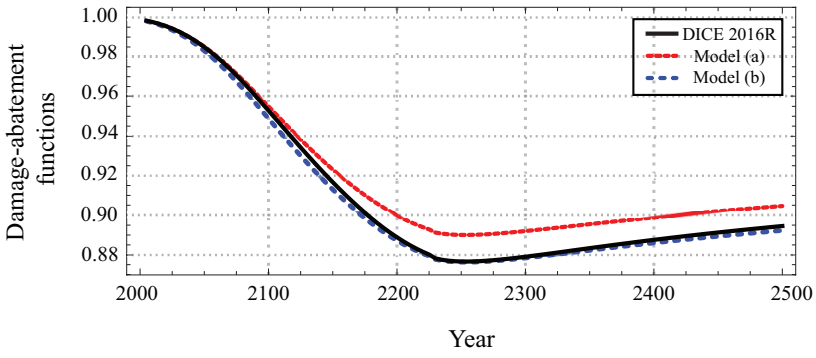

Figure 5: Damage-abatement functions

fraction. Models (a) and (b) use an alternative specification, namely

$$\frac{1-\omega_t}{1+\xi H_t^2},$$

based on the fact that the difference

$$D_t = 1-\omega_t-\xi H_t^2 - \frac{1-\omega_t}{1+\xi H_t^2} = \frac{-(\omega_t+\xi H_t^2)(\xi H_t^2)}{1+\xi H_t^2}$$

is small, about 1% in relative terms.

The alternative specification is of interest because we may wish to randomize ξ, as in Ikefuji *et al.* (2019). This is difficult under the DICE specification because $1-\omega_t-\xi H_t^2$ could become negative (under extreme circumstances), while the alternative specification is always positive provided $\xi > 0$.

In Model (a) we use the same value for ξ as in the DICE model, while in Model (b) we use an 'optimal' value $\xi^* = 0.00265$ which brings the lines closer together; in fact, DICE and Model (b) are indistinguishable in the figure. The value ξ^* is obtained by minimizing the sum of squares

$$\sum_{t=1}^{T}\left(1-\omega_t-\xi H_t^2 - \frac{1-\omega_t}{1+\xi^* H_t^2}\right)^2.$$

with respect to ξ^*. In summary, for a suitable choice of ξ we obtain an alternative for the DICE damage function which lends itself better to studying situations of uncertainty or catastrophe.

6 The SICE model

We summarize our proposed SICE (simplified DICE) model with the relevant parameters. The SICE model is the DICE model, but with the temperature equation replaced by the new (simplest) temperature equation

$$H_{t+1} = \eta_0^* + \eta_1^* H_t + \eta_2^* \log(M_{t+1}) \tag{21}$$

with

$$\eta_0^* = -2.8672, \quad \eta_1^* = 0.8954, \quad \eta_2^* = 0.4622,$$

and the CO_2 equation replaced by

$$M_{t+1} = \phi_1^* M_t + E_t, \qquad \phi_1^* = 0.9942. \tag{22}$$

For the damage equation we propose

$$\frac{1-\omega_t}{1+\xi^* H_t^2}, \quad \xi^* = 0.00265, \tag{23}$$

instead of the DICE specification

$$1 - \omega_t - \xi H_t^2, \quad \xi = 0.00236.$$

In addition, one may wish to set $\mu \leq 1.0$ instead of the upper bound $\mu \leq 1.2$ as used in DICE.

We conclude that, within the philosophy of Nordhaus' DICE model, significant simplifications can be made which make the model more transparent, more robust, and easier to apply.

References

Ackerman, F., Stanton, E. A., Bueno, R. (2010). Fat tails, exponents, extreme uncertainty: Simulating catastrophe in DICE. *Ecological Economics*, 69, 1657–1665.

Anderson, K., Peters, G. (2016). The trouble with negative emissions. *Science*, 354, 14 October, 182–183.

Calel, R., Stainforth, D.A. (2017). On the physics of three integrated assessment models. *Bulletin of the American Meteorological Society*, 98, 1199–1216.

De Luca, G., Magnus, J. R., Peracchi, F. (2018). Balanced variable addition in linear models. *Journal of Economic Surveys*, 32, 1183–1200.

Dietz, S., Venmans, F. (2018). Cumulative carbon emissions and economic policy: In search of general principles. Centre for Climate Change Economics and Policy, Working Paper 317.

Editorial (2015). IAM helpful or not? *Nature Climate Change*, 5, 81.

Editorial (2018). Why current negative-emissions strategies remain 'magical thinking'. *Nature*, 554, 21 February, doi: 10.1038/d41586-018-02184-x

Gerlagh, R., Liski, M. (2018a). Consistent climate policies. *Journal of the European Economic Association*, 16, 1–44.

Gerlagh, R., Liski, M. (2018b). Carbon prices for the next hundred years. *The Economic Journal*, 128, 728–757.

Golosov, M., Hassler, J., Krusell, P., Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82, 41–88

Howard, P. H., Sterner, T. (2017). Few and not so far between: A meta-analysis of climate damage estimates. *Environmental Resource Economics*, 68, 197–225.

Ikefuji, M., Laeven, R. J. A., Magnus, J. R., Muris, C. (2019). Expected utility and catastrophic risk in a stochastic economy-climate model. *Journal of Econometrics*, to appear.

Kelly, D. L., Kolstad, C. D. (1999). Bayesian learning, growth, and pollution. *Journal of Economic Dynamics & Control*, 23, 491–518.

Lemoine, D., Rudik, I. (2017). Steering the climate system: Using inertia to lower the cost of policy. *American Economic Review*, 107, 2947–2957.

Nordhaus, W. D. (1992). An optimal transition path for controlling greenhouse gases. *Science*, 258, 1315–1319.

Nordhaus, W. D. (2008). *A Question of Balance: Weighing the Options on Global Warming Policies*. Yale University Press, New Haven, CT.

Nordhaus W. D. (2013). *The Climate Casino: Risk, Uncertainty, and Economics for a Warming World*. Yale University Press, New Haven, CT.

Nordhaus, W. D. (2017). Revisiting the social cost of carbon. *Proceedings of the U. S. National Academy of Sciences*, 114, 1518–1523, February 14.

Nordhaus, W. D. (2018a). Projections and uncertainties about climate change in an era of minimal climate policies. *American Economic Journal: Economic Policy*, 10, 333–360.

Nordhaus, W. D. (2018b). Evolution of modeling of the economics of global warming: Changes in the DICE model, 1992–2017. *Climatic Change*, 148, 623–640.

Pindyck, R. S. (2013). Climate change policy: What do the models tell us? *Journal of Economic Literature*, 51, 860–872.

Pindyck. R. S. (2017). The use and misuse of models for climate policy. *Review of Environmental Economics and Policy*, 11, 100–114.

Rezai, A., Van der Ploeg, F. (2016). Intergenerational inequality aversion, growth, and the role of damages: Occam's rule for the global carbon tax. *Journal of the Association of Environmental and Resource Economists*, 3, 493–522.

Ricke, K. L., Caldeira, K. (2014). Maximum warming occurs about one decade after a carbon dioxide emission. *Environmental Research Letters*, 9, 124002.

Roe, G. H, Bauman, Y. (2013). Climate sensitivity: Should the climate tail wag the policy dog? *Climatic Change*, 117, 647–662.

Stern, N. (2007). *The Economics of Climate Change: The Stern Review*. Cambridge University Press, Cambridge, UK.

Traeger, C. P. (2015). Analytic integrated assessment and uncertainty. Mimeo.

Van den Bijgaart, I., Gerlagh, R., Liski, M. (2016). A simple formula for the social cost of carbon. *Journal of Environmental Economics and Management*, 77, 75–94.

Van der Ploeg, F. (2018). The safe carbon budget. *Climatic Change*, 147, 47–59.

Weitzman, M. L. (2009). On modeling and interpreting the economics of catastrophic climate change. *The Review of Economics and Statistics*, 91, 1–19.

Appendix

In this Appendix we present three tables which together contain all variable and parameter definitions required to compute the optimum in DICE (the Nordhaus model) and SICE (our simplified version of DICE): the variables employed in SICE and DICE and their relationship (Table 3), the initial values of the six state variables of DICE (Table 4), and the parameters employed in SICE and DICE and their relationship (Table 5).

Table 3: Variables in SICE and DICE

SICE	DICE
State variables	
H_t	TATM(t)
K_t	K(t)
M_t	MAT(t)
Policy variables	
I_t	$tstep \times$ I(t)
μ_t	MIU(t)
C_t	$tstep \times$ C(t)
Exogenous variables	
A_t	$tstep \times$ AL(t)
E_t^0	$tstep \times$ ETREE(t)/3.666
F_t	$-a_1 \log(588) + 0.1005 \times$ FORCOTH(t)
L_t	L(t)/1000
ψ_t	COST1(t)
σ_t	SIGMA(t)/3.666
Auxiliary variables	
E_t	$tstep \times$ E(t)/3.666
Y_t	$tstep \times$ YGROSS(t)
ω_t	defined in (7)
$X_{1,t}$	MU(t)
$X_{2,t}$	ML(t)
Z_t	TOCEAN(t)

Table 4: Initial values of state variables in SICE and DICE

H_1	K_1	M_1	$X_{1,1}$	$X_{2,1}$	Z_1
0.85	223	851	460	1740	0.0068

Table 5: Parameters in SICE and DICE

SICE	DICE	Value	Description
Basic parameters			
	a1	0	
ξ	*a2*	0.00236	
	a3	2	
	tstep	5	
	dk	0.1	
δ		0.40951	$1-\delta=(1-dk)^{tstep}$
	prstp	0.015	
ρ		0.077284	$1+\rho=(1+prstp)^{tstep}$
θ	*expcost2*	2.6	
γ	*gama*	0.3	
α	*elasmu*	1.45	
a_0		0.128189	$a_0=(c1)(fco22x)/(t2xco2)+(c1)(c3)$
a_1		0.533755	$a_1=(c1)(fco22x)/\log 2$
a_2	$c1\times c3$	0.008844	
a_3	*c4*	0.025	
b_0	*b12*	0.12	
b_1	*b21*	0.196	
b_2	*b32*	0.001465	
b_3	*b23*	0.007	
Additional parameters			
η_0		-3.329093	
η_1		0.846811	$\eta_1=1-a_0-a_3$
η_2		0.002984	$\eta_2=(a_0-a_2)a_3$
η_3		0.533755	$\eta_3=a_1$
η_4		0.013344	$\eta_4=a_1a_3$
ϕ_0		0.876134	
ϕ_{00}		263.283680	
ϕ_1		0.675535	$\phi_1=1-b_0-b_1-b_2-b_3$
ϕ_2		0.001303	$\phi_2=b_0b_2+b_0b_3+b_1b_2$
λ_1		0.795535	$\lambda_1=1-b_1-b_2-b_3$
λ_2		0.000287	$\lambda_2=b_1b_2$