1 Concept Review and Preliminaries

Text, Chapter 2.

1.1 Warm-Up

1. Eviews documentation and help are available on my course website.

2. Powerpoint makes professors even more boring than otherwise and that it makes students stupid

3. Try to take some notes during class. This is especially useful if you are a mechanical learner. I'll make available some brief typed notes after class. Review your hand-written notes and compare with the notes that I circulate to see if you are understanding the material. **Do not just take notes** then forget about them.

1.2 What is Financial Econometrics?

1. **Financial econometrics** is the **structured** study of the behavior of financial **asset returns** (asset prices).

2. A **financal asset** is a **durable store of wealth** that may (or may not) pay an income stream (dividends for equity, interest for bonds, rent for real estate). Gold and Bitcoin are assets that don't pay an income stream.

3. A **return** is the rate of change in the value of the asset over a specific time period (More on this below). **Asset returns** or **asset prices** embody people's **beliefs** about the asset, how worldly shocks affect the asset, and how the particular asset interacts with other assets. What we do in financial econometrics is to try to interpret and make sense of these beliefs.

4. **The structural** part here means we do things in the context of a **model** (theory). Theory provides framework for organized thinking. World is complicated. Need theory to identify systematic mechanism driving some phenomenon.

   Financial returns are **time-series** data. Time-series econometrics is a bit different from the standard econometrics you learned in Econ 30331. Hence, we need to spend a little time on the econometric analysis of time-series data.

   We employ the **scientific method**, whereby we come up with a theory (i.e., make some assumptions from which we get conclusions or predictions) and **test** the predictions. If the theory is **rejected**, then it's probably no good. If it is **unrejected**, then use the theory to think about how the world works. (The **tests** come in the form of **t-tests**.)

1.3 Returns

1. **Rate** of return $= r$

2. **Gross return** $= 1 + r$

3. The **log** approximation: $\ln(1 + r) \simeq r$ for small r.

   This comes from the first-order Taylor appximation of an arbitrary, continusously differentiable function around the point x_0,

   $$f(x) = f(x_0) + f'(x_0)(x - x_0) + R$$

   where R is the approximation error. Now let $x = 1 + r$, and $f() = ln()$ be the log function. Expand around $x_0 = 1$ (i.e., $r_0 = 0$. then $\ln(1 + r) \simeq r$ for 'small r'.

4. **Bond** one period **gross holding period** return. C is the coupon. Let P_t^b be the bond price at t

   $$1 + r_{t+1} = \frac{P_{t+1}^b + C}{P_t^b} = \frac{P_{t+1}^b}{P_t^b} + \underbrace{\frac{C}{P_t^b}}_{\text{coupon rate}}$$

   Subtract 1 to get the rate of return.

   $$r_{t+1} = \frac{P_{t+1}^b + C}{P_t^b} - 1$$

5. **Equity** one period holding period return. D_t is the dividend paid in the interval between t and $t + 1$. Let P_t be the stock price at t.

   $$1 + r_{t+1} = \frac{P_{t+1}^e + D_t}{P_t^e} = \frac{P_{t+1}^e}{P_t^e} + \underbrace{\frac{D_t}{P_t^e}}_{\text{dividend yield}}$$

   Subtract 1 to get the rate of return,

   $$r_{t+1} = \frac{P_{t+1}^e + D_t}{P_t^e} - 1$$

6. **Two-period gross returns**

   $$1 + r_{t,t+2} = (1 + r_{t+1})(1 + r_{t+2})$$

Subtract 1 to get the two-period rate return.

$$r_{t,t+2} = (1 + r_{t+1})\,(1 + r_{t+2}) - 1$$

If returns are 'small', we can use the log approximation

$$r_{t,t+2} \simeq r_{t+1} + r_{t+2}$$

7. **Multiperiod** ($k-$period) holding periods,

$$(1 + r_{t,t+k}) = \prod_{j=1}^{k} (1 + r_{t+j})$$

Over long periods of time, returns tend to be pretty large, so you don't want to use the log approximation.

8. **Equally-weighted portfolio returns**. For concreteness, consider a portfolio with 2 assets whose rates of return are $r_{t,1}$ and $r_{t,2}$. Let r_t^p be the portfolio return. The equally-weighted return (EWR) is

$$(1 + r_t^p) = (1 + r_{t,1}) + (1 + r_{t,2})$$

9. **Value-weighted portfolio returns**. Let $N_{t,1}$ be the number of shares of asset 1 in the portfolio and $N_{t,2}$ be the number of shares of asset 2. The value of the portfolio is

$$W_t^p = P_{t,1}N_{t,1} + P_{t,2}N_{t,2}$$

The value-weighted portfolio return is

$$1 + r_t^p = \omega_{t,1}(1 + r_{t,1}) + \omega_{t,2}(1 + r_{t,2})$$

where

$$\omega_{t,j} = \frac{P_{t,j}N_{t,j}}{W_t^p}$$

Notice the weights sum to 1.

10. For a portfolio of n assets,

$$\left(1 + r_{t+1}^p\right) = \sum_{j=1}^{n} \omega_{t,j}\,(1 + r_{t,j})$$

$$1 = \sum_{j=1}^{n} \omega_{t,j}$$

The return on the Standard and Poors 500 has $n = 500$, $\omega_{t,j}$ is the market value of firm j divided by the total S&P market value.

11. **Real and nominal** returns. The above has implicitly been talking about nominal returns. To generate real returns, we need to deflate by a price index, such as the CPI. However, the CPI is available only monthly and we often have returns data at a more frequent horizon. Most of the time, we going to work with nominal returns.

12. **Rule of 70**. If something grows at 1 percent per year, it will double in 70 years. Hence, for it to double in 35 years, it needs to grow 2 percent per year, double in 17.5 years, 4 percent, 8 3/4 years, 8 percent, and so on.

    Question: What is the average stock market return? How long does it take to double it's value?

    How we get rule of 70. Start with 1. Find the value of n^* that makes this true:

$$\underbrace{1}_{\text{Starting}} (1+r)^{n^*} = 2 \tag{1}$$

    Solve for n^*,

$$n^* = \frac{\ln(2)}{\ln(1+r)} \simeq \frac{0.693}{r} = \frac{69.3}{r(100)} \simeq \frac{70}{r(100)} \tag{2}$$

13. Let's take a look at some returns.

    Open the Eviews work file: **f-f_research_data_factors_daily.wf1**

    - Note how different assets have different **return characterisitics**. Here, mean and **standard deviation**, often referred to as **volatility**.
    - Note the absense of trend. Time-series **stationarity**.
    - Excess return
    - How characteristics change as holding period horizon changes.

1.4 Time-Series

1.4.1 What do we mean by a 'model'?

1. Mostly, we mean mathematical or statistical models. Here's a really simple one: Each day, we take a draw from the normal distribution. Observations are serially independent–independent and identically distributed (i.i.d.). We call these **shocks**.

$$y_t \sim N(0, \sigma^2)$$