1.4 Time-Series

1.4.1 What do we mean by a 'model'?

1. We are referrring to the **structural** part of how we study financial returns.

2. Usually, we mean **mathematical or statistical** models, as opposed to an economic model.

3. Here's a really simple one: Each day, we take a draw from the same normal distribution. Observations are serially independent–independent and identically distributed (i.i.d.). We call these **shocks**.

4. The basic **building block** of **stochastic** modeling. Stochastic means random. Observations are **unforecastable.** Handed down from God.

   $$y_t \sim N(0, \sigma^2)$$

   Create a workfile called Demo_01, with interger date size 100. Generate a series of standard normal draws and plot.

   ```
   wfcreate(wf=Demo_01) u 1 100

   series y = nrnd

   show y
   ```

5. Series with dependence over time.

   ```
   series x = y + y(-1) + y(-2) +y(-3)
   ```

6. **Moments:** A bit of review.

   - First moment is the mean. $\mu = E(y_t)$. Note the date t. Refering to distribution that y_t is drawn from. Ditto down below.
   - Second moment: $E(y_t^2)$
   - Third moment: $E(y_t^3)$
   - Second central moment is variance: $\sigma^2 = E(y_t - \mu)^2$
   - First-order autocovariance: $\gamma_1 = E(y_t - \mu)(y_{t-1} - \mu)$ measures strength of serial dependence.

     Second-order autocovariance: $\gamma_2 = E(y_t - \mu)(y_{t-2} - \mu)$

     Notice, we've assumed $E(y_t) = E(y_{t-1})$. We've assumed the series is **strictly stationary**.

7. Properties of the **normal** distribution (The **benchmark** or **default** distribution).

- Standard normal. For $-\infty \leq y \leq \infty <$

$$f(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2} \tag{3}$$

- (General) Normal

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{y-\mu}{\sigma}\right)^2} \tag{4}$$

- **Distribution** is **symmetric** around μ (mean, location) Dispersion regulated by σ (**scale**). How is σ a measure of scale? If y is household income in dollars, then $100y$ is household income in cents.

$$\sqrt{\text{Var}(100y)} = 100\sqrt{\text{Var}(y)} = 100\text{sd}(y) \tag{5}$$

- In finance, **standard deviation** $\Leftrightarrow$ **volatility**
- **Tail probabilities** converge to 0 at a well defined rate. Loosely speaking normal tail probabilities converge to 0 quickly (even though it's possible to have realizations that are arbitraily large or small).
- Conclusion: Assessments of normality involve checking for distributional **symmetry** and appropriate **tail thickness**. How do we do that? Through examination of **sample moments**.

8. **Higher-ordered** moments:

   (a) The k-th theoretical moment of a distribution (the k-th theoretical moment of a random variable y) is

$$E\left(y^k\right)$$

   The k-th central moment is

$$E(y-\mu)^k$$

   where μ is the first moment, $\mu = E(y)$.

   Sample moments are the sample counterparts. Let $\{y_t\}_{t=1}^T$ be a sequence of time-series observations (e.g., returns).

   Third moment for symmetry/asymmetry. **Skewness** measure

$$\frac{E(y_t-\mu)^3}{\sigma^3}; \qquad sk_T = \frac{\frac{1}{T-1}\sum_{t=1}^T (y_t-\hat{\mu})^3}{\hat{\sigma}^3}$$

   For **normal** distribution, skewness measure is **0**. It is **0** for all symmetric distributions.

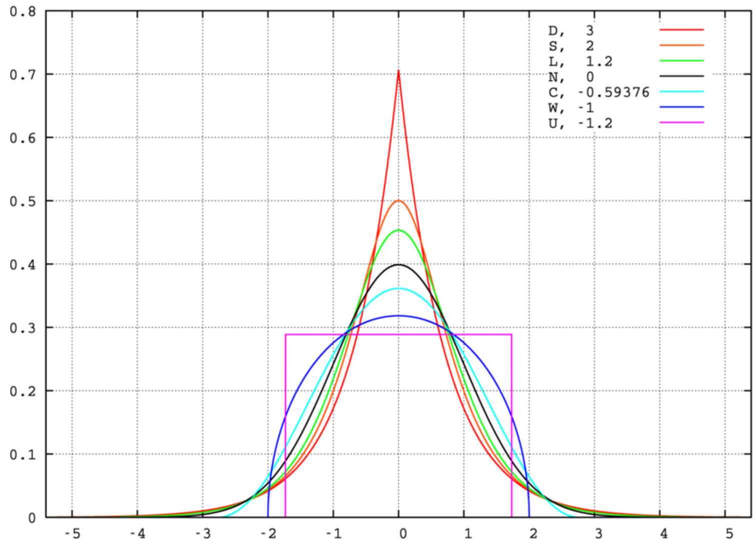
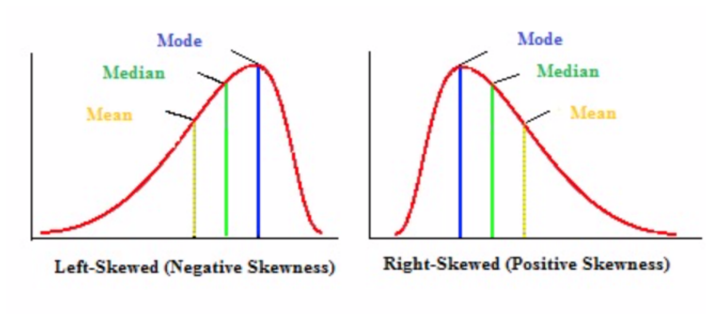


Figure 2: Distributions with differing kurtosis

Figure 1: Skewed Left Skewed Right



Fourth moment measures tail thickness. The theoretical measure is **kurtosis**

$$\frac{E\left(Y_t-\mu\right)^4}{\sigma^4}; \qquad kurt_T=\frac{\frac{1}{T-1}\sum_{t=1}^{T}\left(y_t-\hat{\mu}\right)^4}{\hat{\sigma}^4}$$

For **normal** distribution, kurtosis is **3**.

Distribution has **excess kurtosis** if the measure **exceeds 3**. These are **fat-tailed** distributions and **peaked**. There is a higher probability of extreme events than predicted by the normal. Called **leptokurtotic.**

(b) In applications, pay attention to whether the software computes **kurtosis** or **excess kurtosis**. Excess kurtosis subtracts 3 from the kurtosis measure. Eviews reports **raw**

kurtosis.

9. **Strict stationarity** means each observation drawn from the same distribution. **Weak stationarity** means the mean, variance, and autocovariance is invariant over time.

   At this moment, think of stationarity as **no trend**.

   Here's how to generate a nonstationary series.

   ```
   series z = 0.35*@trend + @cumsum(y)
   ```

   What is the meaning of $E(y_t)$? Pick a t. This is the mean at time t. If stationary, is also the mean at $t+k$. But y_t is a single data point. A single observation. So what does the mean mean?

   ```
   for !k = 1 to 10
   series u{!k} = nrnd
   next
   group aa
   for !k = 1 to 10
   aa.add u{!k}
   next
   show aa
   ```

10. **Ergodicity**–is the technical condition that allows us to estimate the cross-sectional mean $\mu = E(y_t)$ with the time-series mean,

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} y_t$$