Timeline going forward

- 3 more classes of materials: Local Projections, Principal Components, Value at Risk.
- 29 November: Papers due. 3 group presentations. Shoot for 20 minutes per presentation.
- 1 December: Another set of 3 presentations
- 6 December: 2 sets of presentations and review for test.
- 8 December: IN class test, problem set 5 due.
- Live a happy life.

More on VARs:
Orthogonalization: Let ϵ_{1t} and ϵ_{2t} both have variance 1.

$$\begin{aligned}
y_t &= \epsilon_t + A\epsilon_{t-1} + ... \\
&= \Lambda z_t + A\Lambda z_{t-1} + ... \\
\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} &= \underbrace{\begin{pmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix}}_{\Lambda} \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} \\
&+ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix} \begin{pmatrix} z_{1t-1} \\ z_{2t-1} \end{pmatrix} + ... \\
\begin{pmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix} \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} &= \begin{pmatrix} z_{1t} \\ \rho z_{1t} + z_{2t}\sqrt{-\rho^2+1} \end{pmatrix} \\
\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix} &= \begin{pmatrix} a_{11} + \rho a_{12} & a_{12}\sqrt{-\rho^2+1} \\ a_{21} + \rho a_{22} & a_{22}\sqrt{-\rho^2+1} \end{pmatrix}
\end{aligned}$$

After orthogonalization, the shock to y_{2t} (z_{2t}) has a delayed effect on y_{1t},while the shock to y_{1t} (z_{1t}) has an instantaneous effect on y_{1t} and y_{2t}.

1 Local Projections

A different way to estimate impulse responses. And local projection is just a fancy name for REGRESSION! Let's revisit our 2 variable system.

$$\begin{aligned}
y_{1,t+1} &= a_{11}y_{1,t} + a_{12}y_{2,t} + \epsilon_{1,t+1} \\
y_{2,t+1} &= a_{21}y_{1,t} + a_{22}y_{2,t} + \epsilon_{2,t+1}
\end{aligned}$$

Let's take the first equation and run the sequence of regressions.

$$\begin{aligned}
y_{1,t+1} &= a\left(1\right) y_{1,t} + b\left(1\right) y_{2,t} + \epsilon_{1,t+1} \\
y_{1,t+2} &= a\left(2\right) y_{1,t} + b\left(2\right) y_{2,t} + \epsilon_{1,t+2} \\
y_{1,t+3} &= a\left(3\right) y_{1,t} + b\left(3\right) y_{2,t} + \epsilon_{1,t+3} \\
&\vdots \\
y_{1,t+k} &= a\left(k\right) y_{1,t} + b\left(k\right) y_{2,t} + \epsilon_{1,t+k}
\end{aligned}$$

Some really smart guys have figured out that the sequence $b\left(1\right), b(2), ..., b\left(k\right)$ are asymptotically equivalent to the impulse response of y_{2t} shocks on y_{1t} in the VAR. And the sequence $a(1)a\left(2\right), ..., a\left(k\right)$ are the impulse responses of y_{1t} shocks on y_{1t}.

1.1 What's the point?

In finite samples, the LP is more robust. When we estimate VAR, it is estimating short-run correlations from which we infer long-horizon impulse responses. The LP estimates the long-horizon correlation directly.

2 Principal Components

Text on pages 175-179.

Statistical factor analysis. We don't necessarily identify the factors with economic variables. δ_i is the factor loading. In a single factor model,

$$r_{i,t} = \alpha_i + \delta_i f_t + \epsilon_{i,t}$$

In a two-factor model,

$$r_{i,t} = \alpha_i + \delta_{1i} f_{1t} + \delta_{2i} f_{2t} + \epsilon_{i,t}$$

Principal components is a way to estimate the factors and the loadings from the data on returns $r_{i,t}.i = 1, .., N, t = 1, ..., T$. This is very popular technique in studying the term structure of interest rates.

How this works. We start with the single factor model. Write in matrix form. Ignore the constant α_i

$$\underbrace{\begin{pmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,N} \\ r_{2,1} & r_{2,2} & & r_{2,N} \\ \vdots & \vdots & & \vdots \\ r_{T,1} & r_{T,2} & & r_{T,N} \end{pmatrix}}_{r} = \begin{pmatrix} f_1\delta_1 & f_1\delta_2 & \cdots & f_1\delta_N \\ f_2\delta_1 & f_2\delta_2 & & f_2\delta_N \\ \vdots & & & \\ f_T\delta_1 & f_T\delta_2 & & f_T\delta_N \end{pmatrix} + (\text{Residual Matrix})$$

$$= \underbrace{\begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_T \end{pmatrix}}_{f} \underbrace{\begin{pmatrix} \delta_1 & \delta_2 & \cdots & \delta_N \end{pmatrix}}_{\delta'} + (\text{Residual Matrix})$$

Assume $N = 2, T = 3$.

$$(r - f\delta')' \underbrace{(r - f\delta')}_{\tilde{r}} = \tilde{r}'\tilde{r}$$

$$\tilde{r} = \begin{pmatrix} \tilde{r}_{1,1} & \tilde{r}_{1,2} \\ \tilde{r}_{2,1} & \tilde{r}_{2,2} \\ \tilde{r}_{3,1} & \tilde{r}_{3,2} \end{pmatrix}$$

$$\tilde{r}' = \begin{pmatrix} \tilde{r}_{1,1} & \tilde{r}_{2,1} & \tilde{r}_{3,1} \\ \tilde{r}_{1,2} & \tilde{r}_{2,2} & \tilde{r}_{3,2} \end{pmatrix}$$

Trace of a matrix is the sum of the diagonal elements.

$$\tilde{r}'\tilde{r} = \begin{pmatrix} \tilde{r}_{1,1} & \tilde{r}_{2,1} & \tilde{r}_{3,1} \\ \tilde{r}_{1,2} & \tilde{r}_{2,2} & \tilde{r}_{3,2} \end{pmatrix} \begin{pmatrix} \tilde{r}_{1,1} & \tilde{r}_{1,2} \\ \tilde{r}_{2,1} & \tilde{r}_{2,2} \\ \tilde{r}_{3,1} & \tilde{r}_{3,2} \end{pmatrix} = \begin{pmatrix} \tilde{r}_{1,1}^2 + \tilde{r}_{2,1}^2 + \tilde{r}_{3,1}^2 & \tilde{r}_{1,1}\tilde{r}_{1,2} + \tilde{r}_{2,1}\tilde{r}_{2,2} + \tilde{r}_{3,1}\tilde{r}_{3,2} \\ \tilde{r}_{1,1}\tilde{r}_{1,2} + \tilde{r}_{2,1}\tilde{r}_{2,2} + \tilde{r}_{3,1}\tilde{r}_{3,2} & \tilde{r}_{1,2}^2 + \tilde{r}_{2,2}^2 + r_{3,2}^2 \end{pmatrix}$$

$$\begin{aligned} \operatorname{Tr}(\tilde{r}'\tilde{r}) &= \left(\tilde{r}_{1,1}^2 + \tilde{r}_{2,1}^2 + \tilde{r}_{3,1}^2\right) + \left(\tilde{r}_{1,2}^2 + \tilde{r}_{2,2}^2 + r_{3,2}^2\right) \\ &= \left[(r_{11} - f_1\delta_1)^2 + (r_{2,1} - f_2\delta_1)^2 + (r_{3,1} - f_3\delta_1)^2\right] + \left[(r_{1,2} - f_1\delta_2)^2 + (r_{2,2} - f_2\delta_2)^2 + (r_{3,2} - f_3\delta_2)^2\right] \end{aligned}$$

the sum of squared errors of every observation $r_{i,t}$. To be continued : In general, this last line is

$$\sum_{t=1}^{T} (r_{t,1} - f_t\delta_1)^2 + \sum_{t=1}^{T} (r_{t,2} - f_t\delta_2)^2 + \cdots \sum_{t=1}^{T} (r_{t,N} - f_t\delta_N)^2$$

and PC chooses f_t and δ_i, $t = 1, ..., T$ $i = 1, ..., N$ to minimize this thing. In regression, $y_t = \alpha + \beta x_t + \epsilon_t$, we choose α and β to minimize the sum of squared errors, $\sum_{t=1}^{T} \epsilon_t^2$. We call f_t the first principal component (first factor) and δ_i the first factor loading.

In matrix form, we have

$$\begin{aligned} r &= f\delta' + r^o \\ &= fc\,(\delta'/c) + r^o \end{aligned}$$

the second equation says the pc and loadings are not unique. To take care of this, we normalize either f or δ. If we normalize f, we set things so that $Var(f_t) = 1$. Then, Var($r_{t,i}$)=Var($f_t\delta_i + r^o_{t,i}$)=δ_i^2+var$\left(r^o_{t,i}\right)$,so δ_i^2/Var($r_{t,i}$) gives us proportion of variance explained by f_t.

From above, we called $\tilde{r}_{t,i} = r_{t,i} - f_t\delta_i$. This is the first pc. Let's rename f_t to be $f_{1,t}$ and rename δ_i to be δ_{1i}. Rename

$$\tilde{r}_{t,i} = r_{t,i} - f_{1t}\delta_{1i}$$

We know f_{1t} and δ_{1i}. Now let

$$r^*_{t,i} = \tilde{r}_{t,i} - f_{2t}\delta_{2i}$$

Instead of choosing f_{1t} and δ_{1i} to minimize Tr$(\tilde{r}'\tilde{r})$, we choose f_{2t} and δ_{2i} to minimize Tr$(r^{*\prime}r^*)$. f_{2t} is the second principal component and δ_{2i} is the loading on the second pc. Can keep going through the Nth pc. In practice, we are interested in only the first few PCs. The idea is to understand the dynamics of a large number of time series (assets) in terms of a small number of variables (factors, or PCs).

How to compute PCs in Eviews.

Construct a group, click on Proc, select Make Principal Components. Eviews 'scores' is what I called factors or pc. Give names to the factors and select normalization.

People really like PC to study the term structure of interest rates. The terms structure is summarized in the yield curve. The yield curve is the relation between the yield to maturity and time to maturity on different government bonds. The economics behind the yield curve: Let's start with the Euler equation

$$P_{1t}u'(C_t) = \beta E_t u'(C_{t+1})$$

where $P_{1t} = \frac{1}{1+r_{1t}}$.Let $u(C_t) = \ln(C_t)$,then $u'(C_t) = 1/C_t$.Let $\beta = 1/(1+\delta)$ where δ is the pure rate of time preference. Let's expunge uncertainty. Rewrite as

$$\begin{aligned}
\frac{1}{1+r_{1t}}\frac{1}{C_t} &= \frac{1}{1+\delta}\frac{1}{C_{t+1}} \\
1 &= \frac{1+r_{1t}}{1+\delta}\frac{C_t}{C_{t+1}} \\
\ln(1) &= \ln(1+r_{1t}) - \ln(1+\delta) + \ln(C_t) - \ln(C_{t+1}) \\
0 &= r_{1t} - \delta - \Delta c_{t+1} \\
r_{1t} &= \Delta c_{t+1} + \delta
\end{aligned}$$

where $c_t = \ln (C_t)$ .Pricing a 2-period bond.

$$\begin{aligned}
\left(\frac{1}{1 + r_{2t}} \right)^2 u' (C_t) &= \beta^2 u' (C_{t+2}) \\
1 &= \frac{(1 + r_{2t})^2}{(1 + \delta)^2} \frac{C_t}{C_{t+2}} \\
0 &= 2r_{2t} - 2\delta - (c_{t+2} - c_t) \\
r_{2t} &= \frac{1}{2} (c_{t+2} - c_t) + \delta \\
r_{2t} - r_{1t} &= \frac{1}{2} (c_{t+2} - c_{t+1} + c_{t+1} - c_t) - (c_{t+1} - c_t) \\
&= \frac{1}{2} (\Delta c_{t+2} - \Delta c_{t+1})
\end{aligned}$$