Problem Set 3
Econ 40357 Financial Econometrics
University of Notre Dame
Professor Nelson Mark
Fall 2022

Due Thursday 6 October, electronic submission through Canvas. 100 points total on this problem set. Please read the problem set carefully, especially about **what you should report** as your answers. As before, submit **a single pdf** document. The first part of the document will have the group answers. The second part will be an appendix with individual Eviews output. The first page of the solutions needs to have your group number (or group name) with a list of your group members. In the appendix, list your name on the first page of your Eviews output.

1. This first question pertains to the log-likelihood function and information criteria

   (a) (5 points) What is the log-likelihood function?

   Take the joint pdf of the error terms, which are model specific. Substitute the model for the error terms, then take logarithms. This is the log likelihood function. It is a function of the data.

   (b) (5 points) What is maximum likelihood estimation?

   Choosing the parameters of the model to maximize the likelihood (or log likehood) function.

   (c) (5 points) What are the information criteria AIC, BIC, and HQIC, what is the role of the log-likelihood function?

   They are used to choose p and q in an ARMA(p,q) model. When one adds variables (additional lags of y_t or ϵ_t), and therefore parameters, the log-likelihood will increase so we can't choose the best model based on the highest log-likelihood from varying the model specification. These information crieteria attach a penalty to the log-likelihood for adding parameters (variables).

2. Use the Eviews workfile PS03.wf1, sheet entitled FF_3Factors. Mkt is the monthly market return–value-weight return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ (stated in percent). RF is the one-month Treasury bill rate (from Ibbotson Associates, also stated in percent).

   (a) (5 points) Report the first 4 autocorrelations for mkt and rf.

lag	MKT	RF
1	0.101	0.976
2	-0.018	0.962
3	-0.095	0.954
4	0.013	0.942

(b) (5 points) Use Hannan-Quinn information criteria in automatic ARIMA forecasting to choose the best ARMA model for mkt. Report the specification (that is, the p and q in ARMA(p,q). Then estimate that model. Report the coefficients, identifying which ones are autoregressive coefficients and which are moving average.

```
Automatic ARIMA Forecasting
Selected dependent variable:  MKT
Date:  10/01/22 Time:  17:02
Sample:  1926M07 2019M07
Included observations:  1117
Forecast length:  0
Model maximums:  (12,12)0(0,0)
Regressors:  C
```

```
Number of estimated ARMA models:  169
Number of non-converged estimations:  0
Selected ARMA model:  (2,2)(0,0)
HQ value:  6.17323759904
```

Dependent Variable: MKT
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 10/01/22 Time: 17:09
Sample: 1926M07 2019M07
Included observations: 1117
Convergence achieved after 44 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.936540	0.170881	5.480654	0.0000
AR(1)	−0.276112	0.082053	−3.365069	0.0008
AR(2)	−0.697589	0.048684	−14.32893	0.0000
MA(1)	0.393392	0.079683	4.936968	0.0000
MA(2)	0.746030	0.047735	15.62860	0.0000
SIGMASQ	27.50426	0.627918	43.80230	0.0000

R-squared	0.027830	Mean dependent var	0.936473
Adjusted R-squared	0.023455	S.D. dependent var	5.321370
S.E. of regression	5.258592	Akaike info criterion	6.163047
Sum squared resid	30722.25	Schwarz criterion	6.190004
Log likelihood	−3436.062	Hannan-Quinn criter.	6.173238
F-statistic	6.360943	Durbin-Watson stat	2.023253
Prob(F-statistic)	0.000008		

Inverted AR Roots	-.14-.82i	-.14+.82i
Inverted MA Roots	-.20-.84i	-.20+.84i

(c) (5 points) Repeat part b for rf

Automatic ARIMA Forecasting
Selected dependent variable: RF
Date: 10/01/22 Time: 17:11
Sample: 1926M07 2019M07
Included observations: 1117
Forecast length: 0
Model maximums: (12,12)0(0,0)
Regressors: C

Number of estimated ARMA models: 169
Number of non-converged estimations: 0
Selected ARMA model: (9,8)(0,0)
HQ value: -3.03966979646

Dependent Variable: RF
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 10/01/22 Time: 17:13
Sample: 1926M07 2019M07
Included observations: 1117
Convergence achieved after 128 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.265624	0.137309	1.934495	0.0533
AR(1)	1.369343	0.059289	23.09614	0.0000
AR(2)	−1.706490	0.064819	−26.32702	0.0000
AR(3)	2.722751	0.075226	36.19413	0.0000
AR(4)	−2.596321	0.103182	−25.16257	0.0000
AR(5)	2.555586	0.105301	24.26925	0.0000
AR(6)	−2.369292	0.087609	−27.04380	0.0000
AR(7)	1.499884	0.065679	22.83664	0.0000
AR(8)	−1.036626	0.059674	−17.37146	0.0000
AR(9)	0.542130	0.031388	17.27194	0.0000
MA(1)	−0.641658	0.059096	−10.85795	0.0000
MA(2)	1.379917	0.037440	36.85674	0.0000
MA(3)	−1.729819	0.072255	−23.94047	0.0000
MA(4)	1.441654	0.064190	22.45909	0.0000
MA(5)	−1.602910	0.077750	−20.61614	0.0000
MA(6)	1.231775	0.043625	28.23539	0.0000
MA(7)	−0.652042	0.047088	−13.84742	0.0000
MA(8)	0.697462	0.033412	20.87461	0.0000
SIGMASQ	0.002610	$5.44E-05$	47.98065	0.0000

R-squared	0.959041	Mean dependent var	0.273653
Adjusted R-squared	0.958370	S.D. dependent var	0.252558
S.E. of regression	0.051531	Akaike info criterion	−3.071939
Sum squared resid	2.915623	Schwarz criterion	−2.986576
Log likelihood	1734.678	Hannan-Quinn criter.	−3.039670
F-statistic	1428.306	Durbin-Watson stat	1.986713
Prob(F-statistic)	0.000000		

Inverted AR Roots	.99	.82+.32i	.82-.32i	.20-.88i
	.20+.88i	-.25-.91i	-.25+.91i	-.58-.81i
	-.58+.81i			
Inverted MA Roots	.88-.31i	.88+.31i	.26-.88i	.26+.88i
	-.24+.95i	-.24-.95i	-.58+.80i	-.58-.80i

3. In PS03.wf1, go to the second worksheet entitled 'Shiller'. gr is the gross monthly return on the S&P index, and rf is the monthly risk-free rate (stated as a pure number–not in percent). Thus, the monthly excess return on the market is $er = gr - 1 - rf$. dy is the monthly dividend yield on the index.

   (a) (7 points) Write a program to generate the future 12-month excess (rate of) return. Call it FER12 and regress it on the dividend yield. Report the coefficient estimates, t-ratios, and $R^{2.}$. See hint at the bottom of the problem set.

	Estimate	T-ratio
C	-0.029103	-1.800939
DY	3.122437	8.125233

   (b) (7 points) If we interpret the fitted part of the regression as an estimate of the risk premium, what is the implied annual risk premium if the dividend yield is 0.138?

$$-0.029103 + 3.122437\,(0.138) = 0.401\,79 \text{ or } 40.179\%$$

   :

(c) (7 points) Repeat part b for a dividend yield of 0.011.

$$-0.029103 + 3.122437\,(0.011) = 0.005243\,8 \text{ or } 0.5243\%$$

:

(d) (7 points) Write a program to generate the future 96-month excess (rate of) return. Call it FER96. Repeat parts a,b, and c with FER96. **Note:** We want the implied **annual** risk premium here.

	Estimate	T-ratio
C	-0.320548	-5.776272
DY	27.75556	21.67808

The first number is the exact annualized risk premium. The second is the approximate value

$$[1 + (-0.320548 + 27.75556\,(0.138))]^{\frac{1}{8}} - 1 = 0.207\,17 \text{ or } 20.0717\%$$
$$\frac{(-0.320548 + 27.75556\,(0.138))}{8} = 0.438\,71 \text{ or } 43.87\%$$

$$[1 + (-0.320548 + 27.75556\,(0.011))]^{\frac{1}{8}} - 1 = -0.001917\,4$$
$$\frac{(-0.320548 + 27.75556\,(0.011))}{8} = -0.001904\,6$$

4. Consider the following matrices,

$$A = \begin{pmatrix} 1 & 6 \\ -2 & 4 \end{pmatrix}, B = \begin{pmatrix} -3 & -8 \\ 6 & 4 \end{pmatrix}, C = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, D = \begin{pmatrix} 6 & -2 \\ 0 & -1 \\ 3 & 0 \end{pmatrix}$$

(a) (5 points) Which pairs of matrices can be multiplied together?
AB, AC, BC, CD, DC.

(b) (5 points) For those pairs that can be multiplied, perform the multiplications.

$$AB = \begin{pmatrix} 33 & 16 \\ 30 & 32 \end{pmatrix}, AC = \begin{pmatrix} 25 & 32 & 39 \\ 14 & 16 & 18 \end{pmatrix}, BC = \begin{pmatrix} -35 & -46 & -57 \\ 22 & 32 & 42 \end{pmatrix}$$

$$CD = \begin{pmatrix} 15 & -4 \\ 42 & -13 \end{pmatrix}, DC = \begin{pmatrix} -2 & 2 & 6 \\ -4 & -5 & -6 \\ 3 & 6 & 9 \end{pmatrix}$$

(c) (5 points) What is 2A?

$$\begin{pmatrix} 2 & 12 \\ -4 & 8 \end{pmatrix}$$

(d) (5 points) What is C' (transpose of C)?

$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

(e) (5 points) Calculate $A + B$

$$\begin{pmatrix} -2 & -2 \\ 4 & 8 \end{pmatrix}$$

(f) (5 points) Calculate $B - A$

$$\begin{pmatrix} -4 & -14 \\ 8 & 0 \end{pmatrix}$$

:

(g) (5 points) If $A = \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$, find A^{-1}. $\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}^{-1}$.

$$\begin{pmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{pmatrix}$$

5. (7 points) For being a good person, which I know you are!

Hint: This will get you started on the program.

```
series g = 1+er
series FER12 = g
for !j = 1 to 12
```

```
    series FER12 = FER12*g(+!j)
next
series FER12 = FER12 - 1
```