11 The Beta-Risk Model

Here, we start talking about risk **factors**. Material is also covered in **Brooks** pp. 586-588.

11.1 The Market Model and the CAPM

- Finance people like to talk about (common) **factors**
- Factor is systematic component driving the cross-section (all securities) over time.
- Factor may be observed or latent (unobservered)
- Returns driven by common and idiosyncratic factors
- Investors are only compensated for bearing systematic risk (i.e., that part driven by common factors)
- CAPM is a single-factor model. Factor is the market return.
- Later, we talk about multi-factor models.
- Finance people like to embed factor models within the **beta-risk** framework.

11.2 The Beta-Risk Model

- **Question is**: Over long periods of time, **why** do some assets pay high returns and why do others pay low returns?

  e.g., Big versus small firms. Do small firms pay more or less? If more, what's the risk in small firms that make people afraid of them?

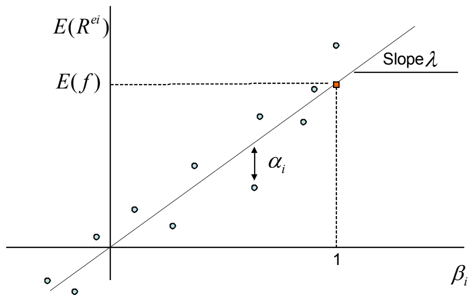
- **Answer is** those assets with greater **exposure** to the **risk factor**. Measure exposure with **beta**. The **big question** here, is **what is (are) risk factor(s)**?
- **Asset pricing theory**: Let $r^e_{t,i}$ be asset $i's$ excess return. Let $\bar{r}^e_i = E\left(r^e_{t,i}\right)$ is the asset's mean excess return. In finance, all asset pricing models take the form

  $$\bar{r}^e_i = \beta_{1i}\lambda_1 + \beta_{2i}\lambda_2 + \cdots + \beta_{ki}\lambda_k$$

  where

  $$r^e_{t,i} = \alpha_i + \beta_{1i}f_{1t} + \beta_{2i}f_{2t} + \cdots + \beta_{ki}f_{kt} + \epsilon_{t,i}$$

  The f_{jt} are called (common) risk factors. The λ_j are called risk prices. The β_{ji} are called betas. They measure the risk exposure of asset i to factor j.



Picture shows relation between risk and return. Risk is **covariance**. Excess returns vary proportionally to β_i. α_i is the deviation (Jensen's alpha). β is the asset's exposure to the risk factor, f. It says, the risk-premium (expected excess return) varies in proportion to the asset's exposure to risk factor. λ is that factor of proportionality.

- Lets start simple, with a single factor model.

$$\bar{r}_i^e = \beta_i \lambda \tag{32}$$

where

$$r_{t,i}^e = \alpha_i + \beta_i f_t + \epsilon_{t,i} \tag{33}$$

  - f_t is the common risk factor. In the CAPM, factor $f_t = r_{t,m}^e$ is the **market excess return.** We assume that the excess return is generated by (33). (32) is the prediction of the model. It says an asset's average return is proportional to its beta. The factor of proportionality is the price of risk, λ. Higher beta stock pay higher returns on average, because they are risker in the sense that their returns covary more with the risk factor.
  - What happened to the α_i in (32)? α_i is Jensen's alpha. It is the risk-adjusted performance measure. If the theory (about f_t being the only risk factor) is correct, $\alpha_i = 0$. If α_i is not zero then the average return on the security or portfolio is higher (or lower) than that predicted by the theory.
  - If the assets are professionally managed portfolios, then $\alpha_i > 0$ tells us the portfolio manager has special talent.
  - What is λ? Take expectations of (33) assuming the theory is true, $\bar{r}_i^e = \beta_i \bar{f}$, then

$$\bar{f} = E\left(f_t\right) = \lambda \tag{34a}$$

(32)-(34a) form the crux of the beta-risk model.

All finance models take beta-risk form (short version)

- The marginal Investor's Euler equation. $x_{t+1,j}$ is payoff from asset j that costs $p_{t,j}$.

$$p_{t,j}u'(c_t) = E_t\left[\beta u'(c_{t+1})\, x_{t+1,j}\right] \tag{35}$$

  If asset is stock, $x_{t+1,j} = p_{t+1,j} + d_t$. If asset is coupon bond, replace d_t with coupon. If asset is discount bond, $x_{t+1,j} = 1$.

- Express in return form,

$$1 = E_t\left[\frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{x_{t+1,j}}{p_{t,j}}\right]$$

- Change notation: $m_{t+1} = \beta u'(c_{t+1})/u'(c_t)$ is the **stochastic discount factor**. $(1+r_{t+1,j}) = x_{t+1,j}/p_{t,j}$ is the gross return.

- Rewrite the Euler equation one more time

$$1 = E_t(m_{t+1}(1 + r_{t+1,j})) \tag{36}$$

- Holds for all traded assets $j = 1, ..., N$. Also holds for the **risk free** asset whose return is $1 + r_t^f$.

$$1 = E_t(m_{t+1}(1 + r_t^r)) \tag{37}$$

- Subtract (37) from (36) to get

$$0 = E_t(m_{t+1}r^e_{t+1,j})$$

  Take unconditional expectations of both sides,

$$0 = E(m_{t+1}r^e_{t+1,j})$$

  Now the timing $t+1$, t doesn't matter.[1]

- Finance bros aren't fans of consumption data. It is poorly measured, and we can't really observe the consumption of the marginal investor. So we assume that the SDF of the

[1] Think of a predictive regression, $y_{t+1} = \alpha + \beta x_t + \epsilon_{t+1}$. Take conditional expectation,

$$E_t(y_{t+1}) = \alpha + \beta x_t$$

. Now take the unconditional expectation of the conditional expectation,

$$E(E_t(y_{t+1})) = \alpha + \beta E(x_t) = E(y_{t+1})$$

marginal investor has a **one-factor** representation for the SDF. $\Leftarrow$ this is key

$$m_t = 1 - b(f_t - \mu_f) \tag{38}$$

What is factor f_t? Could be consumption growth, could be asset returns.

- Substitute (38) into Euler equation to get the beta-risk representation

$$\begin{aligned} 0 &= E(r_t^e(1 - b(f_t - \mu_f))) \\ &= E(r_t^e) - bCov(r_t^e, f_t) \\ &= E(r_t^e) - \underbrace{bVar(f_t)}_{\lambda}\underbrace{\frac{Cov(r_t^e, f_t)}{Var(f_t)}}_{\beta} \end{aligned} \tag{39}$$

Hence,

$$\bar{r}_t^e = \lambda\beta$$

11.3 Estimate and Test the CAPM with the Time-Series Method

- The time-series method works when **factor** is an **excess return**.
- Preliminary analysis
  - Estimate and test if price of risk $E(f_t) = \lambda$ is statistically significant: Run the regression

    $$f_t = \lambda + \epsilon_t$$

    of the factor (excess return) on constant. The factor needs to be some sort of excess return.
  - Constant is estimate of λ. Do Newey-West on the constant, test if it is greater than 0.

- Run the time-series regression

$$r_{t,i}^e = \alpha_i + \beta_i f_t + \epsilon_{t,i}$$

for each asset $i = 1, ..., n$, using Newey-West. Do individual t-tests on the $\alpha_{i.}$ If the factor explains everything about why this asset pays an excess return, $\alpha_i = 0$. If not, then the model is lacking. Impose the restriction that mean returns are proportional to betas,

$$\bar{r}_i^e = \beta_i\lambda = \alpha_i + \beta_i\bar{f}$$

Then the intercept should be

$$\alpha_i = \beta_i(\lambda - E(f_t))$$

- Plot the $\bar{r}_i^e$ against the β_i. Do they line up? Does the regression line of $\bar{r}_i^e$ on β_i go through the origin?

- A cheap and not entirely correct joint test: If all the α_i are zero, then the sum of the α_i is zero. If the α_i estimates are independent, then

$$t_1^2 + t_2^2 + \cdots t_n^2 \sim \chi_n^2$$

  where t_i^2 is the squared value of the Newey-West t-ratio on α_i.

  This test is not entirely right because it ignores possible correlation across the α_i

- Details for doing the correct joint test on the $\alpha' s$. Let

$$\underline{\hat{\alpha}} = \begin{pmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \\ \vdots \\ \hat{\alpha}_N \end{pmatrix}$$

$$\Sigma_\alpha = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\ \vdots & & & \\ \sigma_{N1} & \cdots & & \sigma_{NN} \end{pmatrix}$$

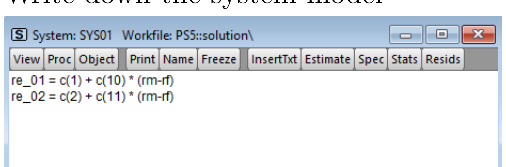
$$\sigma_{ii} = Var(\alpha_i), \quad \sigma_{ij} = Cov(\alpha_i, \alpha_j)$$

  Then test statistic is,

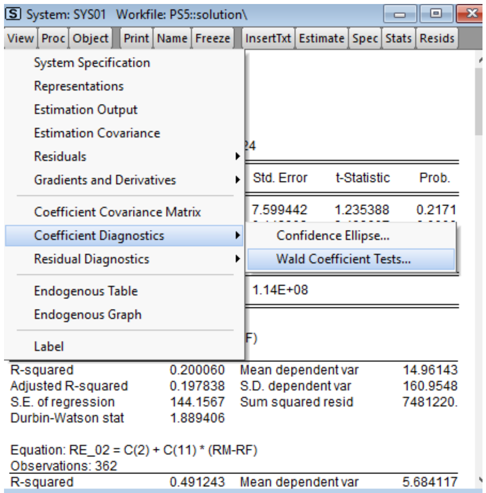
$$(\underline{\hat{\alpha}}' \Sigma_\alpha^{-1} \underline{\hat{\alpha}}) \sim \chi_N^2$$

- How to do this in Eviews? Estimate as system, ask for the joint test. (betarisk_dow.wf1)

  - Object → New Object → System
  - Write down the system model



  - Estimate by Ordinary Least Squares → View → Coefficient Diagnostics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System Specification
Representations
Estimation Output
Estimation Covariance
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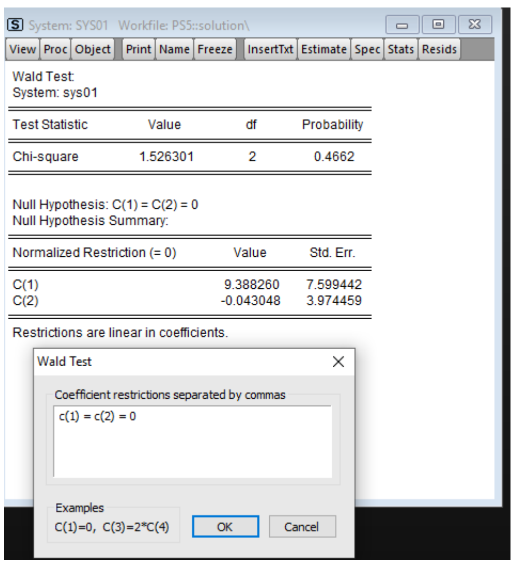
Coefficient Covariance Matrix
Coefficient Diagnostics
Residual Diagnostics
Endogenous Table
Endogenous Graph
Label
Confidence Ellipse...
Wald Coefficient Tests...
Std. Error t-Statistic Prob.
7.599442 1.235388 0.2171
1.14E+08
R-squared 0.200060 Mean dependent var 14.96143
Adjusted R-squared 0.197838 S.D. dependent var 160.9548
S.E. of regression 144.1567 Sum squared resid 7481220.
Durbin-Watson stat 1.889406
Equation: RE_02 = C(2) + C(11) * (RM-RF)
Observations: 362
R-squared 0.491243 Mean dependent var 5.684117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View Proc Object Print Name Freeze InsertTxt Estimate Spec Stats Resids
Wald Test:
System: sys01
Test Statistic Value df Probability
Chi-square 1.526301 2 0.4662
Null Hypothesis: C(1) = C(2) = 0
Null Hypothesis Summary:
Normalized Restriction (= 0) Value Std. Err.
C(1) 9.388260 7.599442
C(2) -0.043048 3.974459
Restrictions are linear in coefficients.
Wald Test
Coefficient restrictions separated by commas
c(1) = c(2) = 0
Examples
C(1)=0, C(3)=2*C(4)
OK
Cancel