10 Time-Varying Volatility

Text: Chapter 9

10.1 Motivation

Financial returns are not normally distributed. They exhibit

- 1. Leptokurtosis (fat tails relative to the normal)
- 2. Volatility clusters
- 3. The **unconditional** distribution of short-horizon returns aren't normal. But their **condi-tional** distributions could be normal.

Why we might care

1. Suppose we want to estimate the value of market risk. (Sharpe ratios).

$$\text{Sharpe} = \frac{E\left(r_p - r_f\right)}{\sigma_p}$$

where $E(r_p - r_f)$ is portfolio excess return and σ_p is portfolio volatility. Sharpe ratio is the average portfolio return per unit of volatility (a risk concept).

- 2. Volatility is a key parameter for pricing financial derivatives. All modern option pricing techniques rely on a volatility parameter for price evaluation.
- 3. Volatility is used for risk management assessment and in general portfolio management. Financial institutions want to know the current value of the volatility of the managed assets.
- 4. They also want to predict their future values. Volatility forecasting is important for institutions involved in options trading and portfolio management.
- 5. Volatility changes over time, which makes these pricing examples conditional on the current environment (high, low volatility). We want to model how volatility changes and what it depends on.

Dependent Variable: ERRTSQ Method: Least Squares Sample (adjusted): 7/02/1926 9/30/2019 Included observations: 24578 after adjustments



Figure 5: Market excess return

Let r_{mt}^e be the market excess return. Suppose we have only one observation. How would you form the sample variance? The sample standard deviation?



Figure 6: Square root of squared daily market excess returns

Does staring at this picture make you want to regress it on lags of itself?

Dependent Variable: ERRTSQ Sample (adjusted): 7/13/1926 9/30/2019 Included observations: 24571 after adjustments

10.2 The ARCH/GARCH class of models

- Popular way to model is with ARCH (autoregressive conditional heteroskedasticity) and GARCH (generalized ARCH).
- ARCH was invented by Robert Engle. The Nobel committee gave him the economics prize in part for this.
- GARCH was invented by Tim Bollerslev, who was Engle's student at UCSD.
- There's also,
 - EGARCH (exponential GARCH)
 - IGARCH (integrated GARCH)
 - STARCH (smooth-transition ARCH)
 - TARCH (threshold ARCH)
 - FIGARCH (fractionally integrated GARCH)
 - SWARCH (switching ARCH).
- Return on some asset

$$r_t = a + bx_t + \epsilon_t$$
$$\epsilon_t \sim N\left(0, \sigma_t^2\right)$$

Notice t subscript on variance. σ_t^2 is the conditional variance of ϵ_t . Conditional on past observations of ϵ_t

$$\sigma_t^2 = E\left[\left(\epsilon_t - E_t\left(\epsilon_t\right)\right)^2 | \epsilon_{t-1}, \epsilon_{t-2}, \ldots\right] = \operatorname{Var}\left(\epsilon_t | \epsilon_{t-1}, \epsilon_{t-2}, \ldots\right)$$

This says the conditional variance changes over time. It is time-varying. It **moves around** over time. ARCH is a **parametric model** of the conditional variance.

- Intuition: remember how we want to think of conditional expectation as regression?
- Estimation done by maximum likelihood

Figure 7: Robert Engle Nobel Laureat and Heck of a Nice Guy



Nobel Prize citation: "for methods of analyzing economic time series with time- varying volatility (ARCH)"



Figure 8: Robert Engle Does Ice Dancing!

10.3 ARCH class of models

ARCH(1)

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2$$

ARCH(2)

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2$$

ARCH(q)

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \cdots + \alpha_q \epsilon_{t-q}^2$$

Test for ARCH effects

• Run the main regression

$$r_t = \hat{a} + \hat{\beta}x_t + \hat{u}_t$$

save the residuals \hat{u}_t

• Regress the squared residuals \hat{u}_t^2 on q lags of itself (to test for ARCH(q)).

$$\hat{u}_t^2 = b_0 + b_1 \hat{u}_{t-1}^2 + \dots + b_q \hat{u}_{t-q}^2 + v_t$$

where v_t is the error term. You can do an F-test on the coefficients.

• You can also do a Lagrange multiplier (LM) test. Get the R^2 from this regression.

$$TR^2 \sim \chi_q^2$$

• What does the F-test and LM test test?

$$H_0: (b_1 = 0) \cap (b_2 = 0) \cap \cdots (b_q = 0)$$

The alternative is H_A : NOT H_0 .

• Test for and Estimate ARCH model in EViews, use f-f_research_data_factors_daily.wf1

Equation Estimation	\times	E
Specification Options		P
Mean equation Dependent followed by regressors & ARMA terms OR explicit equation: mkt_rf_c_mkt_rf(-1) ARCH-M: None		
Variance and distribution specification		
Model: GARCH/TARCH Variance regressors:		
Order: ARCH: 1 Threshold order: 0		
GARCH: 0 Error distribution:		L
Restrictions: None \checkmark Normal (Gaussian) \checkmark		
Estimation settings		
Method: ARCH - Autoregressive Conditional Heteroskedasticity \lor		L
Sample: 7/01/1926 8/30/2019		
OK Cancel		

Dependent Variable: MKT_RF Method: ML ARCH - Normal distribution (BFGS / Marquardt steps) Date: 11/18/19 Time: 15:12 Sample (adjusted): 7/02/1926 8/30/2019 Included observations: 24558 after adjustments Convergence achieved after 11 iterations

Coefficient covariance computed using outer product of gradients Presample variance: backcast (parameter = 0.7) GARCH = C(3) + C(4)*RESID(-1)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
С	0.028996	0.004540	6.386850	0.0000
MKT_RF(-1)	0.246425	0.002153	114.4349	0.0000
	Variance I	Equation		
С	0.656324	0.003491	188.0134	0.0000
RESID(-1) ²	0.473991	0.008016	59.13110	0.0000
R-squared	-0.028109	Mean dependent var		0.029196
Adjusted R-squared	-0.028151	S.D. dependent var		1.062371
S.E. of regression	1.077221	Akaike info	criterion	2.768679
Sum squared resid	28494.91	Schwarz cri	terion	2.770000
Log likelihood	-33992.61	Hannan-Qu	inn criter.	2.769107
Durbin-Watson stat	2.323311			

Equation: EQ02 Workfile: VOLATILITY::Untitled View Proc Object Print Name Freeze Estimate Forecast Stats Resids View Proc Object || Print | Name | Freeze || Estimate | Forecast | Stats | Resids || Dependent Variable: IMKT_RF Method: ML ARCH - Normal distribution (BFGS / Marquardt steps) Date: 11/18/19 Time: 15:13 Sample (adjusted): 7/02/1926 8/30/2019 Included observations: 24558 after adjustments Confectent covariance computed using outer product of gradients Presample variance: backcast (parameter = 0.7) GARCH = C(3) + C(4)*RESID(-14)*2 + C(6)*RESID(-2)*2 + C(6)*RESID(-3)*2 + C(7)*RESID(-4)*2 + C(8)*RESID(-5)*2 + C(9)*RESID(-6)*2 + C(10)*RESID(-7)*2 + C(11)*RESID(-8)*2 + C(12)*RESID(-9)*2 Coefficient Std. Error z-Statistic Variable Prob. 0.053366 0.127875 0.004332 0.006636 12.31774 19.27105 0.0000 0.0000 C MKT_RF(-1) Variance Equation

С	0.160815	0.002808	57.26586	0.0000
RESID(-1) ²	0.122085	0.004743	25.74071	0.0000
RESID(-2) ²	0.113064	0.004983	22.69077	0.0000
RESID(-3) ²	0.110880	0.006016	18.43183	0.0000
RESID(-4)^2	0.120884	0.005092	23.74082	0.0000
RESID(-5)^2	0.094493	0.005543	17.04883	0.0000
RESID(-6)^2	0.095819	0.005632	17.01452	0.0000
RESID(-7)^2	0.057432	0.004902	11.71529	0.0000
RESID(-8) ²	0.091738	0.004974	18.44226	0.0000
RESID(-9)^2	0.078895	0.004899	16.10544	0.0000
R-squared	-0.000094	Mean deper	ndent var	0.029196
Adjusted Disquared	0.000125	C D donon	hont vor	1 060071

Oy! Too many parameters!

10.4 The GARCH class of models

GARCH(1,1)

$$r_{t} = a + \beta x_{t} + \epsilon_{t}$$
$$\epsilon_{t} \sim N\left(0, \sigma_{t}^{2}\right)$$
$$\sigma_{t}^{2} = \alpha_{0} + \alpha_{1}\epsilon_{t-1}^{2} + \beta\sigma_{t-1}^{2}$$

The (1,1) refers to number of lags of ϵ^2 and σ^2 , and where

$$0 \le \beta \le 1$$

GARCH(1,1) is constrained infinite ordered ARCH. Observe,

$$\sigma_{t-1}^{2} = \alpha_{0} + \alpha_{1}\epsilon_{t-2}^{2} + \beta\sigma_{t-2}^{2}$$

$$\sigma_{t-2}^{2} = \alpha_{0} + \alpha_{1}\epsilon_{t-3}^{2} + \beta\sigma_{t-3}^{2}$$

substitute this into previous

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta \underbrace{\left(\alpha_0 + \alpha_1 \epsilon_{t-2}^2 + \beta \sigma_{t-2}^2 \right)}_{\sigma_{t-1}^2} \\ = \alpha_0 \left(1 + \beta \right) + \alpha_1 \epsilon_{t-1}^2 + \alpha_1 \beta \epsilon_{t-2}^2 + \beta^2 \sigma_{t-2}^2 \\ = \alpha_0 \left(1 + \beta \right) + \alpha_1 \left(\epsilon_{t-1}^2 + \beta \epsilon_{t-2}^2 \right) + \beta^2 \left(\alpha_0 + \alpha_1 \epsilon_{t-3}^2 + \beta \sigma_{t-3}^2 \right) \\ = \alpha_0 \left(1 + \beta + \beta^2 \right) + \alpha_1 \left(\epsilon_{t-1}^2 + \beta \epsilon_{t-2}^2 + \beta^2 \epsilon_{t-3}^2 \right) + \beta^3 \sigma_{t-3}^2$$

Keep going. $\beta^k = 0$ as $k \to \infty$.

$$\sigma_t^2 = \frac{\alpha_0}{1-\beta} + \frac{\alpha_1}{\beta} \sum_{j=1}^{\infty} \beta^j \epsilon_{t-j}^2$$

GARCH(2,1)

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \beta \sigma_{t-1}^2$$

GARCH(1,2)

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-1}^2$$

Usually, GARCH(1,1) does the job.

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Dependent Variable: M Method: ML ARCH - N Date: 11/18/19 Time: Sample (adjusted): 7// Included observations Convergence achieved Coefficient covariance Presample variance: b GARCH = C(3) + C(4)	/IKT_RF lormal distribut 15:25 02/1926 8/30/2 : 24558 after a d after 26 itera computed usi vackcast (paral *RESID(-1)^2 -	tion (BFGS / M 019 djustments tions ng outer prod meter = 0.7) + C(5)*GARCH	/larquardt sto uct of gradie H(-1)	eps) nts
Variable	Coefficient	Std. Error	z-Statistic	Prob.
С	0.047571	0.004436	10.72388	0.0000
MK1_RF(-1)	0.126446	0.006698	18.87856	0.0000
	Variance I	Equation		
С	0.012641	0.000497	25.43512	0.0000
RESID(-1) ²	0.103414	0.002076	49.82175	0.0000
GARCH(-1)	0.886041	0.002373	373.4385	0.0000
R-squared	0.000338	Mean deper	ndent var	0.029196
Adjusted R-squared	0.000298	S.D. dependent var		1.062371
S.E. of regression	1.062213	Akaike info criterion		2.428462
Sum equared resid	27706.47	Schwarz criterion		2.430113
Sulli squaleu lesiu		Hannan Oui	inn criter	2 428997
Log likelihood	-29814.08	nannan-Qu	initi critter.	2.120001

Recover the GARCH series, makes the conditional variance series eqno.makegarch cvar where cvar is the series name series csd = $cvar^{0.5}$

10.5 ARCH-M, GARCH-M (in the mean)

You might be interested in whether higher volatility associated with higher or lower returns? Here is a GARCH-M example.

$$r_t^e = a + b\sigma_t + \epsilon_t$$
$$\epsilon_t \sim N\left(0, \sigma_t^2\right)$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

- Use volatility as 'regressor' to preserve units.
- b > 0, high volatility, r^e expected to be large. b < 0, high volatility, r^e expected to be small.
- Estimation is by maximum likelihood.
- To implement, choose the option in EViews

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