Financial Econometrics Econ 40357
The Fama and French 3-Factor Framework

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Brooks, pp. 590

Creating Portfolios

- There are thousands of stocks. You want to test a hypothesis on factor exposure on these stocks, but you don't want to run 10,000 individual regressions against the factor.
  - The factor loadings on individual stocks may not be stable over time. If so, you'll end up with incorrect estimates of factor premia.
- Instead, create portfolios from the universe of stocks, and allow the stocks shift from one portfolio to another based on changing factor exposures. These are **dynamic portfolios**.
  - Don't just randomly select into portfolios. If you do that, you get portfolios that look like the market. (Why is that?) Then the only factor that will explain these portfolios is the market. Not that interesting.
  - Strategy is to sort by characteristic that you think causes a stock's return to correlate with the factor. Create 25 portfolios, such that each portfolio has **increasing** exposure to factor. Try to have at least 10 stocks in each portfolio

Review of the Framework

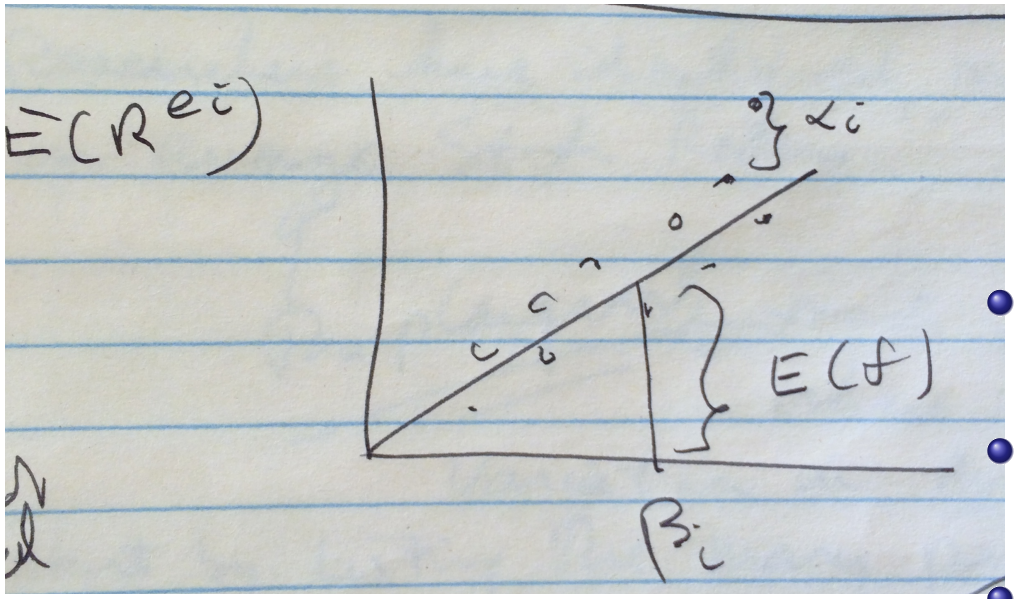


Figure is what we exepct to see if model is true.

- α_i is the part of expected return not explained by beta. If model is correct, $\alpha_i = 0$
- The slope of the line in the figure is the factor risk premium. It is the mean of the factor.
- Two **key questions** addressed by Fama and French are, **what are we allowed to use as factors? How does the model work?**

$$E\left(R^{ei}\right) = \alpha_i + \beta_i E\left(f\right)$$

$$R_t^{ei} = \alpha_i + \beta_i f_t + \epsilon_{it}$$

Fama and French's 3-Factor Strategy

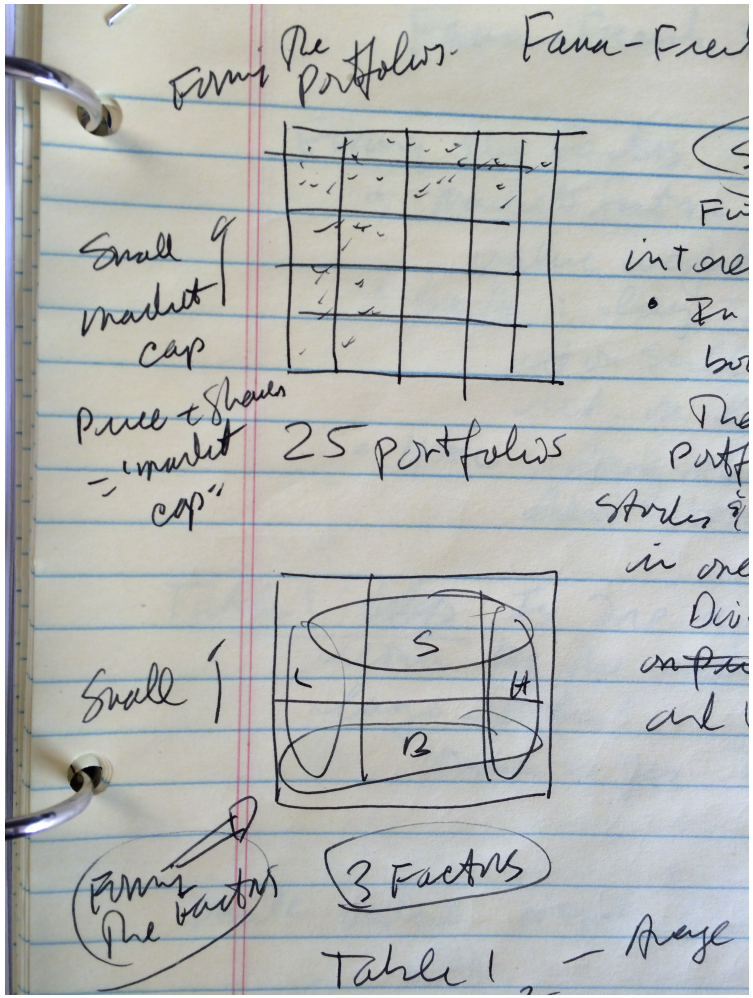
- Factors are
  1. Market excess return r_m
  2. Excess return on high minus low book-to-market value firms
  3. Excess return on small minus big market capitalization firms.
- Test assets (portfolios) are formed by a double sort of returns into
  1. 5 grids of low to high book-to-market,
  2. 5 grids of large-to-small firms.

  That is, they took the intersection of firms into each of the 25 sets.
- FF looked for securities where there is substantial variation in average returns. In their paper, the start by listing the many of these discovered patterns in **average** stock returns that they want to explain.
- They say that **anomalies** are captured by their sensitivities to three factors:
  - R^m: the market return,
  - *SML*: **small minus big (in capitalization)** portfolio and
  - *HML*:**high minus low book to market portfolios.**

$$E\left(R^i\right) - R^f = b_i\left(E\left(R^m\right) - R^f\right) + s_i\left(E\left(SML\right)\right) + h_i\left(E\left(HML\right)\right)$$

For $i = 1, ..., 25$.

Forming the portfolios



They want to find portfolios with **interesting** average excess returns.

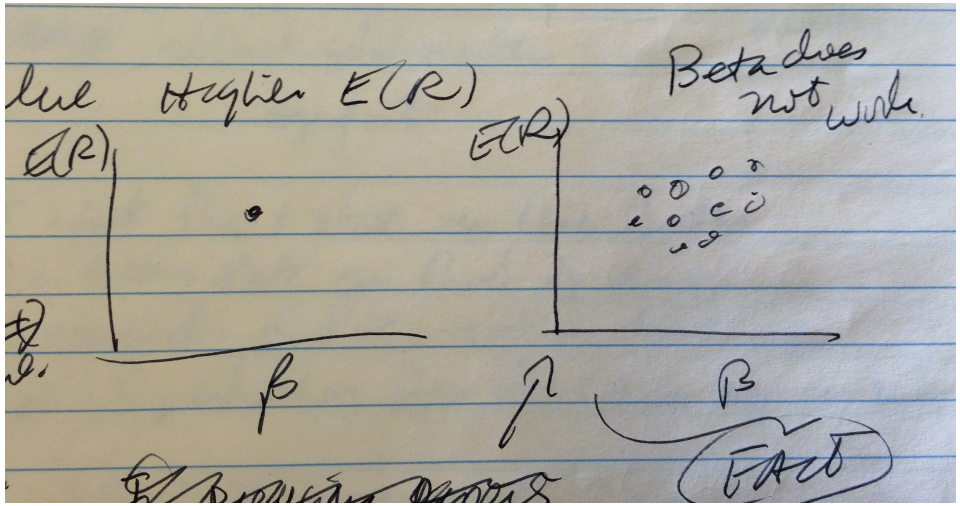
In June, look at the size and book to market of every stock. In the following January, form portfolios. Put the smallest stocks and all the highest value stocks on one portfolio. Then do the same for the next smallest and next highest value.

In all, divide into 25 bins based on size (price times outstanding shares) and value (book to market). These are called value stocks because high book to market signals that the market price is low.

Look at the characteristics of the stocks in their Table 1 In the table, we can ask “Is there a **difference** in return between the value, growth, small and large stocks? The answer is **yes**, and they are **huge**.

	Book-to-Market Equity (BE/ME) Quintiles									
Size	Low	2	3	4	High	Low	2	3	4	High
	Panel A: Summary Statistics									
	Means					Standard Deviations				
Small	0.31	0.70	0.82	0.95	1.08	7.67	6.74	6.14	5.85	6.14
2	0.48	0.71	0.91	0.93	1.09	7.13	6.25	5.71	5.23	5.94
3	0.44	0.68	0.75	0.86	1.05	6.52	5.53	5.11	4.79	5.48
4	0.51	0.39	0.64	0.80	1.04	5.86	5.28	4.97	4.81	5.67
Big	0.37	0.39	0.36	0.58	0.71	4.84	4.61	4.28	4.18	4.89

They form three factors. The market factor is all stocks together, value weighted. SMB is the long minus short portfolio return on small stocks subtract the return on large stocks. HML is the long-short excess return on high minus low book to market portfolios.



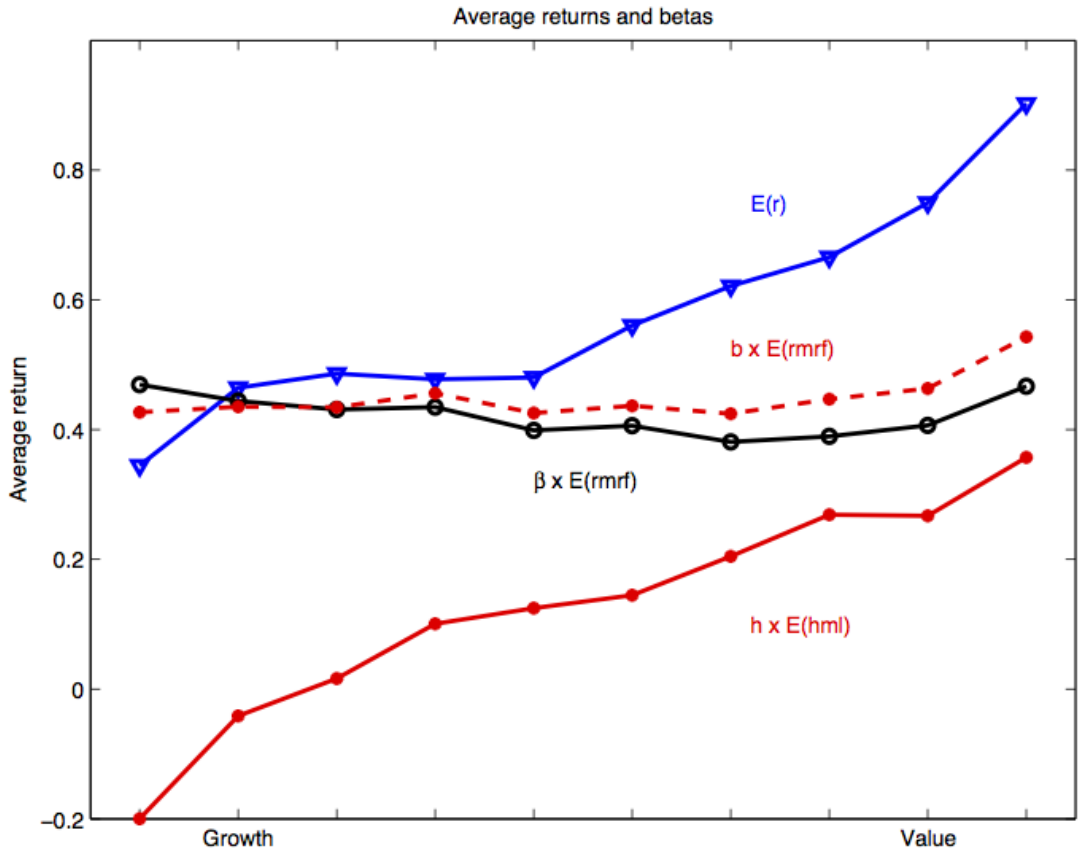
- **Left** shows how things would look if all expected returns are the same. This would be a very boring world. On the
- **Right** shows the facts. Market beta does not work. Expected returns vary enormously (that's what the table showed). In earlier papers, Fama and French show those higher excess returns did not correspond to higher betas.
- So, the question is **can the three beta model** explain excess returns?

What the model is and isn't about

- The model is **not about** the t-statistics on the betas. Significant t-stats on betas tell you that they are well measured. But it's not a test of the model whether these things are big or not.
- The model is about **average returns versus betas**. Not about the betas themselves. The model is also not about the R^2 in the time-series regression.

  High time-series R^2 tells you covariance of all 25 portfolios is driven by the 3 factors. Yes, there is a strong factor structure and it is a good model for explaining covariance. But it doesn't tell you that it is a good model for explaining means. When you run the time-series regressions, the purpose is to produce the **data** (of the betas).

This plot shows model **success**. Market beta doesn't show any variation. HML factor beta is increasing with value, which is increasing in average return.



Average returns and betas for Fama - French 10 B/M sorted portfolios. Monthly data 1963-2010.

Panel B: Regressions: $R_i - R_f = a_i + b_i(R_M - R_f) + s_i SMB + h_i HML + e_i$

	a					t(a)				
Small	−0.45	−0.16	−0.05	0.04	0.02	−4.19	−2.04	−0.82	0.69	0.29
2	−0.07	−0.04	0.09	0.07	0.03	−0.80	−0.59	1.33	1.13	0.51
3	−0.08	0.04	−0.00	0.06	0.07	−1.07	0.47	−0.06	0.88	0.89
4	0.14	−0.19	−0.06	0.02	0.06	1.74	−2.43	−0.73	0.27	0.59
Big	0.20	−0.04	−0.10	−0.08	−0.14	3.14	−0.52	−1.23	−1.07	−1.17
	b					t(b)				
Small	1.03	1.01	0.94	0.89	0.94	39.10	50.89	59.93	58.47	57.71
2	1.10	1.04	0.99	0.97	1.08	52.94	61.14	58.17	62.97	65.58
3	1.10	1.02	0.98	0.97	1.07	57.08	55.49	53.11	55.96	52.37
4	1.07	1.07	1.05	1.03	1.18	54.77	54.48	51.79	45.76	46.27
Big	0.96	1.02	0.98	0.99	1.07	60.25	57.77	47.03	53.25	37.18
	s					t(s)				

	s					t(s)				
Small	1.47	1.27	1.18	1.17	1.23	39.01	44.48	52.26	53.82	52.65
2	1.01	0.97	0.88	0.73	0.90	34.10	39.94	36.19	32.92	38.17
3	0.75	0.63	0.59	0.47	0.64	27.09	24.13	22.37	18.97	22.01
4	0.36	0.30	0.29	0.22	0.41	12.87	10.64	10.17	6.82	11.26
Big	−0.16	−0.13	−0.25	−0.16	−0.03	−6.97	−5.12	−8.45	−6.21	−0.77

	h					t(h)				
Small	−0.27	0.10	0.25	0.37	0.63	−6.28	3.03	9.74	15.16	23.62
2	−0.49	0.00	0.26	0.46	0.69	−14.66	0.34	9.21	18.14	25.59
3	−0.39	0.03	0.32	0.49	0.68	−12.56	0.89	10.73	17.45	20.43
4	−0.44	0.03	0.31	0.54	0.72	−13.98	0.97	9.45	14.70	17.34
Big	−0.47	0.00	0.20	0.56	0.82	−18.23	0.18	6.04	18.71	17.57

	R^2					s(e)				
Small	0.93	0.95	0.96	0.96	0.96	1.97	1.49	1.18	1.13	1.22
2	0.95	0.96	0.95	0.95	0.96	1.55	1.27	1.28	1.16	1.23
3	0.95	0.94	0.93	0.93	0.92	1.44	1.37	1.38	1.30	1.52
4	0.94	0.92	0.91	0.88	0.89	1.46	1.47	1.51	1.69	1.91
Big	0.94	0.92	0.87	0.89	0.81	1.19	1.32	1.55	1.39	2.15

Table III

Three-Factor Time-Series Regressions for Monthly Excess Returns (in Percent) on the LSV Equal-Weight Deciles: 7/63–12/93, 366 Months

$$R_i - R_f = a_i + b_i(R_M - R_f) + s_i\text{SMB} + h_i\text{HML} + e_i$$

The formation of the BE/ME, E/P, C/P, and five-year-sales-rank (5-Yr SR) deciles is described in Table II. The explanatory returns, $R_M - R_f$, SMB, and HML are described in Table I. $t(\ )$ is a regression coefficient divided by its standard error. The regression R^2s are adjusted for degrees of freedom. GRS is the F-statistic of Gibbons, Ross, and Shanken (1989), testing the hypothesis that the regression intercepts for a set of ten portfolios are all 0.0. $p(GRS)$ is the p-value of GRS, that is, the probability of a GRS value as large or larger than the observed value if the zero-intercepts hypothesis is true.

C/P	**Low**									**High**		
a	0.02	−0.08	−0.07	−0.00	−0.04	0.00	0.00	0.05	0.06	0.01		
b	1.04	1.06	1.08	1.06	1.05	1.04	0.99	1.00	0.98	1.14		
s	0.45	0.50	0.54	0.51	0.55	0.50	0.53	0.48	0.57	0.92		
h	−0.39	−0.18	0.07	0.11	0.23	0.31	0.36	0.50	0.67	0.79		
$t(a)$	0.22	−1.14	−1.00	−0.04	−0.51	0.00	0.06	0.72	0.92	0.14	0.49	0.898
$t(b)$	51.45	61.16	62.49	64.15	59.04	61.28	60.02	63.36	58.92	46.49		
$t(s)$	15.56	20.32	22.11	21.57	21.49	20.72	22.19	21.17	24.13	26.18		
$t(h)$	−12.03	−6.52	2.56	4.28	7.85	11.40	13.52	19.46	24.88	19.74		
R^2	0.93	0.95	0.95	0.95	0.94	0.94	0.94	0.94	0.94	0.92		

Test of the model

- Are α's jointly zero? Fama-FrenchF reject this hypothesis. Model is statistically rejected. But doesn't mean the model is not useful. Fama-French took finance away from pure hypothesis testing towards getting into the data and seeing how the model works and doesn't work.
- **What Fama and French do with the three factor model**. We might ask, Isn't it tautological, that the 25 portfolios sorted by size and book-to-market, are explained by the factors that are also size and book-to-market? Cochrane argues the answer is no, because size and book-to-market factors can explain many other puzzles. That is, explanation of returns that cannot be explained by market beta. Here are a couple of examples.
- **Google versus Sears.** Stocks that have had sales disasters give higher returns. They are not explained by market beta. But the Fama-French 3 factor model does explain these kinds of returns. It says the slow sales stocks are not about size or book-to-market. It is a different puzzle. Slow sales stocks are not value stocks, but they **behave like value stocks.**
- **Achieves Data Reduction.** The model needs only 3 factors that explains all returns. Use as controls over known puzzles or anomalies. Now, the theory of finance only needs to explain the factor premiums, not the set of 25 portfolio returns.