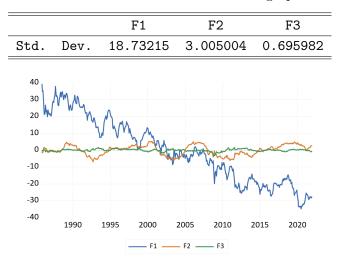
## Solution to Midterm 2 Financial Econometrics, Econ 40357 University of Notre Dame Prof. Mark Due 1:45 p.m. Thursday 8 December

100 points total. The rules are the same as last time: Test is open book, open note, open internet, but **you may not communicate with other people** in any way. Any such communication will be considered cheating. **Do not cheat!** Submit via Canvas, a pdf of your own work by 1:45 p.m. Thursday 8 December. No late submissions (no exceptions).

For questions that ask for a numerical answer, give that answer first, preferably in bold type. Below your answer, if you wish, state "Explanation:" and give a brief explanation for what you did. This might help get partial credit if you get the numerical answer wrong.

- 1. (5 points) Louie says, if you do the write-up entirely with a **word processor**, if you are **clear**, organized, and have your **name** written at top of first page, you get 5 points.
- Louie wants you to do an analysis of the term structure of interest rates. For this problem, consult the Eviews workfile Midterm2.wf1. The sheet LW\_monthly contains monthly yields on U.S. Treasury debt for time-to-maturity from 1 month (m1) to 360 months (m360). The time span is 1985M11 to 2021M12.
  - (a) (7 points) Compute the first three principal components (PCs) of this data set. In doing so, normalize the loadings. Report the standard deviation of the three PCs along with a graph of them. Put all three PCs on the same graph.



(b) (7 points) Regress m1 on the first PC. Next regress m360 on the first PC. Report the  $R^2$  values from these regressions (should you include a constant these regressions?).

m1 m360

0.779382 0.975342

And YES (always include constant)

(c) (7 points) Explain why the first PC explains such a large proportion of the variation in these yields?

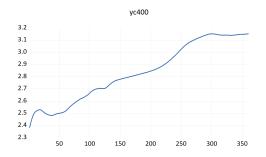
First PC is 'level'. It is approximately the cross-sectional average of yields.

(d) (7 points) Regress (m1-m360) on the second PC. Report the  $R^2$  from this regression. Explain why the second PC explains such a large proportion of the variation in this yield and interpret what the second PC represents.

R-square: 0.896556

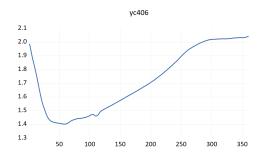
Second PC is 'slope'. m1-m360 is the short-long yield differential--the inverse of the yield curve slope.

- (e) For this problem, you need to transpose the panel data set of yields. Place the transposed data in a new page called CX with 360 observations (hint: pagecreate(page=CX) u 1 360). Either write out the data set, use Excel to transpose it, then read it into Eviews, or consult problem set 4 on Fama-MacBeth, where we also had to transpose the data set of returns. You should now have 434 series with 360 observations each. Also, for your reference, the U.S. experienced a recession from 2020Q1–2020Q2.
  - i. (7 points) Plot the yield curve for 2019M02 (observation 400). What is the bond market predicting about the economy in 2019M02? Explain your answer.



Generally upward sloping. High future interest rates predict high future consumption growth rates. The explanation comes from the investor's Euler equations for investing in bonds of varying maturities. Under log utility, we showed  $r_{2,t}-r_{1,t} = (1/2) E_t (\Delta c_{t+2} - \Delta c_{t+1})$ , where  $r_{2,t}$  is the yield on the long-term bond,  $r_{1t}$  is the yield on the short term bond. If you think you will be rich in the future,  $(\Delta c_{t+2} > \Delta c_{t+1})$ , borrow now to repay at t+2 when you're going to be rich because it'll be easy to repay then.

ii. (7 points) Plot the yield curve for 2019M08 (observation 406). What is the bond market predicting about the economy in 2019M08? Explain your answer.



Predicting a recession coming in the next 50 to 100 months. From the Euler equation reasoning in part i, the market is predicting future (consumption) growth to slow  $(\Delta c_{t+2} < \Delta c_{t+1})$ .

- 3. Louie wants you to examine the impact of monetary policy on consumption inequality between old and young households. For this problem, consult the sheet entitled **MoPo** on the Eviews workfile. **old\_to\_young** is log consumption of old people minus log consumption of young people (old is where the head of household is 65 and older, young is where head of household is between 25 and 35). **gk** is a monetary policy shock measured as the change in current month federal funds futures within a 60-minute window around FOMC announcements. An increase in gk means a tightening of monetary policy.
  - (a) (7 points) If we want to study how monetary policy affects the consumption of the old relative to the young with a vector autoregression, which variable should be estimated in equation 1 and which in equation 2? Explain.

The shock goes first. Orthogonalization will make the second variable affect the first with a lag.

(b) (7 points) Let the error terms from the first and second equations be  $\epsilon_{1t}$  and  $\epsilon_{2t}$ . Suppose that

$$V_t = \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{\Sigma})$$
$$\mathbf{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$\mathbf{\Sigma} = \begin{pmatrix} 0.01202 & 0.00176 \\ 0.00176 & 0.00373 \end{pmatrix}$$

To do the impulse response analysis, why do we need to orthogonalize the error terms? So that we can unambigiously attribute the shock to the variable in question. (c) (7 points) Using the information provided, orthogonalize into

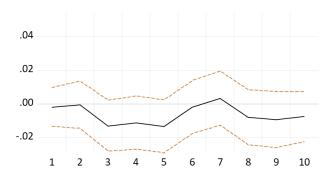
$$z_{1t} = a_1 \epsilon_{1t} + a_2 \epsilon_{2t}$$
$$z_{2t} = b_1 \epsilon_{1t} + b_2 \epsilon_{2t}$$

where  $z_{1t}$  and  $z_{2t}$  are independent standard normal variables. Report numerical values for the coefficients  $a_1, a_2, b_1, b_2$ .

First, we need to compute the correlation 
$$\rho = \frac{0.00176}{\sqrt{0.00373}\sqrt{0.01202}} = 0.262\,85$$
  
 $a_1 = \frac{1}{\sqrt{\sigma_{11}}} = \frac{1}{\sqrt{0.01202}} = 9.121\,1$   
 $a_2 = 0$   
 $b_1 = -\frac{\rho\sqrt{\sigma_{22}}}{\sqrt{\sigma_{11}\sigma_{22}(1-\rho^2)}} = \frac{0.262\,85\sqrt{0.00373}}{\sqrt{(1-0.262\,85^2)}\sqrt{(0.01202)}\sqrt{(0.00373)}} = -2.4849$   
 $b_2 = \frac{\sqrt{\sigma_{11}}}{\sqrt{\sigma_{11}\sigma_{22}(1-\rho^2)}} = \frac{\sqrt{0.01202}}{\sqrt{(1-0.262\,85^2)}\sqrt{(0.01202)}\sqrt{(0.00373)}} = 16.97$ 

(d) (7 points) Run an 8th order vector autoregression. Report the  $R^2$  from the first equation and from the second equation.

(e) (7 points) Report a plot of the impulse response of old\_to\_young to a shock in gk.



Response of OLD\_TO\_YOUNG to GK Innovation

(f) (7 points) How does consumption of the old respond to a monetary tightening relative to the young?

Declines. Old consumption more sensitive to higher interest rates than young.

(g) (5 points extra credit) Suggest an economic mechanism to explain the impulse response pattern you found.

Old income and wealth from interest bearing assets.

4. Louie applies a two-factor model to two particular portfolios. Their excess returns are  $r_{1,t}^e$  and  $r_{2,t}^e$ . Let  $r_{mt}^e$  be the market excess return and  $r_{\text{hml},t}$  be the return on the high minus low book to

market portfolio. Louie obtains the following estimates: For portfolio 1:

$$r_{1,t}^e = -0.0446 + 1.261 r_{mt}^e + 0.497 r_{\text{hml},t} + \epsilon_{s,t}$$
$$R^2 = 0.791$$

and for portfolio 2

$$\begin{split} r^e_{2,t} &= \underbrace{0.0093}_{(0.5426)} + \underbrace{0.9570}_{(291.29)} \underbrace{-0.0203}_{(-4.134)} \\ R^2 &= 0.987 \end{split}$$

T-ratios in parentheses. Furthermore,

$$\bar{r}_1^e = 0.989$$
  
 $\bar{r}_2^e = 0.644$   
 $\bar{r}_m^e = 0.670$   
 $\bar{r}_{\rm hml} = 0.359$ 

(a) (5 points) If Louie is correct in thinking that  $r_{mt}^e$  and  $r_{hml,t}$  are the correct factors, what are the values of  $a_{1,\lambda_1,\beta_1,\lambda_2}, \delta_1, a_{2,\beta_2,}, \delta_2$ , in the relations

$$\bar{r}_1^e = a_1 + \lambda_1 \beta_1 + \lambda_2 \delta_1$$
$$\bar{r}_2^e = a_2 + \lambda_1 \beta_2 + \lambda_2 \delta_2$$

$a_{1,}$	$\lambda_{1,}$	$\beta_{1,}$	$\lambda_2,$	$\delta_1,$	$a_{2,}$	$\beta_{2,},$	$\delta_2$
0	0.670	1.261	0.359	0.497	0	0.9570	-0.0203

(b) (6 points) Does the model work for these portfolios? Explain. Include in your answer why portfolio 1 earns a higher return over time.

Yes, constants in regressions are insignificant.

Portfolio 1 has larger betas ( $eta_1, \delta_1$ ). Larger exposure to risk factors.

Louie wishes you a Merry Christmas and a terrific winter break (Prof. Mark does also). Who the heck is Louie?

https://www3.nd.edu/~nmark/FinancialEconometrics/DisapprovingLouie.jpg