Explanation about Question 6 on Midterm 1
Financial Econometrics, Econ 40357
University of Notre Dame
Prof. Mark

(10 points) Consult Sheet01 from the accompanying Eviews workfile. The last observation for $r01$ is 05/10/2023. Using all the data, estimate an ARMA(2,2) model for $r01$. Use your results to forecast $r01$ for 05/11/2023 and 05/12/2023. **Report the numerical value of your forecasts.**

`Here,` $t = 05/10/2023. r_{t-1} = -0.00277, r_t = -0.02097, \epsilon_{t-1} = -0.003539628, \epsilon_t = -0.021084637, \rho_1 = 0.126226,$
$\rho_2 = 0.706492, \theta_1 = -0.114136, \theta_2 = -0.724068, c = 0.000440$

$$
\begin{aligned}
r_t &= c + \rho_1 r_{t-1} + \rho_2 r_{t-2} + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \epsilon_t \\
E_t\left(r_{t+1}\right) &= c + \rho_1 r_t + \rho_2 r_{t-1} + \theta_1 \epsilon_t + \theta_2 \epsilon_{t-1} \\
E_t\left(r_{t+2}\right) &= E_t\left(c + \rho_1 r_{t+1} + \rho_2 r_t + \theta_1 \epsilon_{t+1} + \theta_2 \epsilon_t\right) \\
&= c + \rho_1 E_t\left(r_{t+1}\right) + \rho_2 r_t + \theta_2 \epsilon_t
\end{aligned}
$$

$$
\begin{aligned}
E_t\left(r_{t+1}\right) &= 8.0551 \times 10^{-4} \\
&= 0.000440 + 0.126226\,(-0.02097) + 0.706492\,(-0.00277) \\
&+ (-0.114136)\,(-0.021084637) + (-0.724068)\,(-0.003539628)
\end{aligned}
$$

Here's what Eviews does

$$
\begin{aligned}
E_t\left(r_{t+1}\right) &= 0.000440 + 0.126226\,(-0.02097 - 0.000440) + 0.706492\,(-0.00277 - 0.000440) \\
&+ (-0.114136)\,(-0.021084637) + (-0.724068)\,(-0.003539628) \\
&= 4.3911 \times 10^{-4}
\end{aligned}
$$

ARMA(2,2) representation

$$
\begin{aligned}
r_t &= \mu + z_t \\
z_t &= \rho_1 z_{t-1} + \rho_2 z_{t-2} + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2}
\end{aligned}
$$

This is equivalent to the way we present it. To cut down on notation, let's work through an ARMA(1,1) instead. We've been looking at an ARMA(1,1) as

$$
r_t = \mu\left(1 - \rho_1\right) + \rho_1 r_{t-1} + \theta_1 \epsilon_{t-1} + \epsilon_t \tag{1}
$$

Eviews wants to look at it as

$$
\begin{aligned}
r_t &= \mu + z_t \qquad (2) \\
z_t &= \rho_1 z_{t-1} + \epsilon_t + \theta_1 \epsilon_{t-1}
\end{aligned}
$$

Multiply (2) by ρ_1,

$$
\rho_1 r_{t-1} = \rho_1 \mu + \rho z_{t-1} \tag{3}
$$

Subtract (3) from (2)

$$
\begin{aligned}
r_t - \rho_1 r_{t-1} &= \mu\left(1 - \rho_1\right) + z_t - \rho_1 z_{t-1} \\
&= \mu\left(1 - \rho_1\right) + \epsilon_t + \theta_1 \epsilon_{t-1}
\end{aligned}
$$

and we get back (1). So now, when Eviews computes a forecast

$$\begin{aligned} E_t r_{t+1} &= \mu + E_t z_{t+1} \\ &= \mu + \rho_1 z_t + \theta_1 \epsilon_t \\ &= \mu + \rho_1 (r_t - \mu) + \theta_1 \epsilon_t \end{aligned}$$

where $z_t = r_t - \mu$.

Going back to the ARMA(2,2), I thought the constant Eviews estimates is

$$c = \mu (1 - \rho_1 - \rho_2)$$

but it is actually μ, according to the representation they used. Their manual describes the estimation and forecast procedure the same way we talked about it in class. I had to get this information from one of their engineers.