Endogenous Money and Exchange Rates

Nelson C. Mark

IMF Institute Seminar, 21 March 2008
1. **A set of exchange rate puzzles**

1. The PPP Puzzle:

- Why is the real exchange rate both highly persistent and volatile?
- Why is the nominal exchange rate so highly correlated with the real exchange rate?
- A successful model will explain these facts. Chari, Kehoe, McGrattan do with price stickiness lasting 4 quarters and relative risk aversion coefficient of 5.
- Flexible price models have difficulty explaining volatility (without high risk aversion). Sticky price models have difficulty explaining persistence (without unrealistically sticky prices).
2. Traditional monetary approaches

- The PPP theory: Higher inflation should lead to dollar weakening.

\[ s = p - p^* \rightarrow \Delta s = \pi - \pi^* \]
• Monetary models
  
  – Lucas JME 1982

\[
s = (m - m^*) - \mu (y - y^*)
\]


\[
s = (m - m^*) - \frac{1}{\epsilon} (c - c^*)
\]

– Original monetary model.

\[
\begin{align*}
m_t - p_t & = \alpha + \gamma y_t - \lambda i_t + v_t \\
m_t^* - p_t^* & = \alpha + \gamma y_t^* - \lambda i_t^* + v_t^* \\
q_t & = s_t + p_t^* - p_t \\
i_t - i_t^* & = E_t(s_{t+1}) - s_t
\end{align*}
\]
- Substitute into UIP for the difference equation

\[ s_t = \frac{1}{1 + \lambda} \left( \frac{(m_t - m_t^*) - \gamma (y_t - y_t^*) + q_t - (v_t - v_t^*)}{f_t} \right) + \frac{\lambda}{1 + \lambda} E_t (s_{t+1}) \]

\[ f_t = (m_t - m_t^*) - \gamma (y_t - y_t^*) + q_t \]

- Iterate forward,

\[ s_t = \frac{1}{1 + \lambda} E_t \sum_{j=0}^{\infty} \left( \frac{\lambda}{1 + \lambda} \right)^j (f_{t+j} + z_{t+j}) \]

- If \( f_t \) is AR(1) with coefficient \( \rho \),

\[ s_t = \frac{1}{1 + \lambda (1 - \rho)} [ (m_t - m_t^*) - \gamma (y_t - y_t^*) ] \]
- Low-brow econometric results.
3. Relatively new empirical evidence on news and the exchange rate.

- Real-time (5 minute sampling) exchange rate and announcement news (errors). Anderson et. al. AER 2003.
We model the 5-minute spot exchange rate, $R_t$, as a linear function of $I$ lagged values of itself, and $J$ lags of news on each of $K$ fundamentals:

\begin{equation}
R_t = \beta_0 + \sum_{i=1}^{I} \beta_i R_{t-i} \\
+ \sum_{k=1}^{K} \sum_{j=0}^{J} \beta_{kj} S_{k,t-j} + \epsilon_t,
\end{equation}

$t = 1, \ldots, T$. 
Their findings:

- Only unanticipated shocks to the fundamentals have a statistically significant effect on the exchange rate.

- Good real news good news for the dollar: Unexpectedly strong real indicator is followed by a strengthening of the dollar. Consistent with most theory.

- Bad news about inflation is good news for the dollar: Unexpectedly high inflation is followed by a strengthening of the dollar. This is contrary to most theory.
Inflation Differentials and Real Dollar–DM Rate

log real DM-dollar rate

DEU-USA inflation


1987.4 1986.4 1994.2

Inflation Differentials wrt Structural Break and Real Dollar–DM Rate
4. Simplified Taylor-rule model with uncovered interest parity. Taylor rule (without interest rate smoothing).

- Let $i_t$ be the target rate,
- $\bar{\pi}$ be targeted inflation,
- $x_t = y_t - y_t^p$ be the output gap,
- $\bar{\bar{i}}$ be the natural nominal rate,
- $s_t = p_t - p_t^*$ be an exchange rate target. Then the deviation from the targeted exchange rate is $q_t = s_t + p_t^* - p_t$. 

In Germany,

\[ i_t = \gamma_{\pi} \left( E_t \pi_{t+1} - \bar{\pi} \right) + \gamma_x x_t + \gamma_q q_t \]

In the U.S., no exchange rate feedback

\[ i_t^* = \gamma_{\pi} \left( E_t \pi_{t+1}^* - \bar{\pi}^* \right) + \gamma_x x_t^* \]

Important for \( \gamma_{\pi} > 1 \). This is called the Taylor Principle.

– Take Germany to be the home country. Ignore constant terms. Assume identical targets and coefficients across countries.
Exploit UIP

\[ E_t s_{t+1} - s_t = i_t - i_t^* \]

\[ E_t s_{t+1} - E_t (p_{t+1} - p_{t+1}^*) - s_t - (p_t - p_t^*) = i_t - i_t^* - E_t (\pi_{t+1} - \pi_{t+1}^*) \]

\[ E_t q_{t+1} - q_t = (\gamma_\pi - 1) E_t (\pi_{t+1} - \pi_{t+1}^*) + \gamma_x (x_t - x_t^*) + \gamma_q q_t \]

\[ q_t = \frac{(1 - \gamma_\pi)}{1 + \gamma_q} \sum_{j=0}^{\infty} \left( \frac{1}{1 + \gamma_q} \right)^j E_t (\pi_{t+1+j} - \pi_{t+1+j}^*) \]

\[ -\frac{\gamma_x}{1 + \gamma_q} \sum_{j=0}^{\infty} \left( \frac{1}{1 + \gamma_q} \right)^j E_t (x_{t+j} - x_{t+j}^*) \]
– Assume inflation differentials and output gap follow AR(1) processes

\[ q_t = \frac{(1 - \gamma_\pi) \rho_{\pi}}{1 + \gamma_q \rho_{\pi}} (\pi_t - \pi_t^*) - \frac{\gamma_x \rho_x}{1 + \gamma_q \rho_x} (x_t - x_t^*) \]

* Significantly different character of fundamentals.

* Good news for home output is good news for the exchange rate.

* Bad news about inflation means what about the exchange rate?
  Depends on \( \gamma_\pi \)
2 The existence of interest rate rules
2.1 Clarida, Gali, Gertler QJE: For the Fed.

\( r_t^* = r^* + \beta (E\{\pi_{t,k} | \Omega_t\} - \pi^*) + \gamma E\{x_{t,q} | \Omega_t\}, \)

\( r \) is the nominal rate (FFR)

\( rr_t^* = rr^* + (\beta - 1) (E\{\pi_{t,k} | \Omega_t\} - \pi^*) + \gamma E\{x_{t,q} | \Omega_t\}, \)

\( rr^* = r^* - \pi^* \)

\( r_t = \rho(L) r_{t-1} + (1 - \rho) r_t^* \)

\( r_t = (1 - \rho) [rr_t^* - (\beta - 1)\pi^* + \beta \pi_{t,k} + \gamma x_{t,q}] + \rho(L) r_{t-1} + \varepsilon_t, \)

where \( \varepsilon_t \equiv -(1 - \rho) [\beta (\pi_{t,k} - E\{\pi_{t,k} | \Omega_t\}) + \gamma (x_{t,q} - E\{x_{t,q} | \Omega_t\})] \). No-
<table>
<thead>
<tr>
<th></th>
<th>$\pi^*$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\rho$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-Volcker</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.24</td>
<td>0.83</td>
<td>0.27</td>
<td>0.68</td>
<td>0.834</td>
</tr>
<tr>
<td></td>
<td>(1.09)</td>
<td>(0.07)</td>
<td>(0.08)</td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td><strong>Volcker-Greenspan</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.58</td>
<td>2.15</td>
<td>0.93</td>
<td>0.79</td>
<td>0.316</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
<td>(0.40)</td>
<td>(0.42)</td>
<td>(0.04)</td>
<td></td>
</tr>
</tbody>
</table>

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation: output gap, the federal funds rate, the short-long spread, and commodity price inflation.
<table>
<thead>
<tr>
<th>TABLE III</th>
<th>ALTERNATIVE VARIABLES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\pi^*$</td>
</tr>
<tr>
<td>Detrended output</td>
<td></td>
</tr>
<tr>
<td>Pre-Volcker</td>
<td>4.17</td>
</tr>
<tr>
<td></td>
<td>(0.68)</td>
</tr>
<tr>
<td>Volcker-Greenspan</td>
<td>4.52</td>
</tr>
<tr>
<td></td>
<td>(0.58)</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td></td>
</tr>
<tr>
<td>Pre-Volcker</td>
<td>3.80</td>
</tr>
<tr>
<td></td>
<td>(0.87)</td>
</tr>
<tr>
<td>Volcker-Greenspan</td>
<td>4.42</td>
</tr>
<tr>
<td></td>
<td>(0.44)</td>
</tr>
<tr>
<td>CPI</td>
<td></td>
</tr>
<tr>
<td>Pre-Volcker</td>
<td>4.56</td>
</tr>
<tr>
<td></td>
<td>(0.53)</td>
</tr>
<tr>
<td>Volcker-Greenspan</td>
<td>3.47</td>
</tr>
<tr>
<td></td>
<td>(0.79)</td>
</tr>
</tbody>
</table>

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation, output gap, the federal funds rate, the short-long spread, and commodity price inflation.
2.2 Other central banks

Germany, Japan, US, UK, France, Italy. CGG 1998 EER. Data 1979-1993
<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\rho$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline</strong></td>
<td>1.31</td>
<td>0.25</td>
<td>0.91</td>
<td>3.14</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.04)</td>
<td>(0.01)</td>
<td>(0.28)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\rho$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline</strong></td>
<td>2.04</td>
<td>0.08</td>
<td>0.93</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.44)</td>
</tr>
</tbody>
</table>
Table 3
FED reaction functions

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\rho_1 + \rho_2$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>1.79</td>
<td>0.07</td>
<td>0.92</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(0.06)</td>
<td>(0.03)</td>
<td>(0.85)</td>
</tr>
<tr>
<td>post 82:10</td>
<td>1.83</td>
<td>0.56</td>
<td>0.97</td>
<td>-0.10</td>
</tr>
<tr>
<td></td>
<td>(0.45)</td>
<td>(0.16)</td>
<td>(0.03)</td>
<td>(1.54)</td>
</tr>
</tbody>
</table>

Table 4
Bank of England reaction functions

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\rho$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.98</td>
<td>0.19</td>
<td>0.92</td>
<td>5.76</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.04)</td>
<td>(0.01)</td>
<td>(0.69)</td>
</tr>
</tbody>
</table>
3 Taylor rules in New Keynesian (New Open Economy) Models

Benigno, JME

PPP puzzle: Flexible price models have difficulty explaining volatility (without high risk aversion). Sticky price models have difficulty explaining persistence (without unrealistically sticky prices). Benigno breaks the link between real exchange rate persistence and the speed of nominal price adjustment.
<table>
<thead>
<tr>
<th>RS rate</th>
<th>$\rho(RS)(\text{CKM})$</th>
<th>$\sigma(RS)/\sigma(y^H)(\text{CKM})$</th>
<th>$\rho(RS)(\text{BF})$</th>
<th>$\sigma(RS)/\sigma(y^H)(\text{BF})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S./Fr</td>
<td>0.84</td>
<td>4.44</td>
<td>0.78</td>
<td>7.57</td>
</tr>
<tr>
<td>U.S./Ger</td>
<td>0.82</td>
<td>4.50</td>
<td>0.74</td>
<td>4.35</td>
</tr>
<tr>
<td>U.S./Ca</td>
<td>NA</td>
<td>NA</td>
<td>0.88</td>
<td>2.05</td>
</tr>
<tr>
<td>U.S./Japan</td>
<td>NA</td>
<td>NA</td>
<td>0.81</td>
<td>5.87</td>
</tr>
<tr>
<td>U.S./Spain</td>
<td>0.86</td>
<td>4.70</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>U.S./Nor</td>
<td>0.77</td>
<td>3.39</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>U.S./Italy</td>
<td>0.83</td>
<td>4.35</td>
<td>0.79</td>
<td>4.26</td>
</tr>
<tr>
<td>U.S./UK</td>
<td>0.81</td>
<td>4.40</td>
<td>0.79</td>
<td>4.79</td>
</tr>
</tbody>
</table>

Average: 0.82, 4.3, 0.8, 4.81

CKM refers to the data reported by Chari et al. (2002). Their statistics are based on logged and Hodrick–Prescott filtered data. Their data are quarterly series from International Monetary Fund and the Organisation for Economics Co-operation and Development, 1973:1–2000:1. BF refers to the data reported by Bergin and Feenstra (2001). All their data are quarterly series from International Financial Statistics, 1973:1–1997:3. Series are logged and Hodrick–Prescott filtered. For both contributions, $\rho(RS)$ represents the one-quarter autocorrelation of the real exchange rate while $\sigma(RS)/\sigma(y^H)$ represents the standard deviation of the real exchange rate, $\sigma(RS)$, relative to the standard deviation of output, $\sigma(y^H)$. 
Some useful calculations: Typical household problem in new Keynesian international macroeconomics

Household wants to maximize lifetime utility defined over consumption, leisure, and real money balances. Assume that period utility is separable.

\[ E_t \sum_{j=t}^{\infty} \beta^j \left[ U(C_{t+j}) + N \left( \frac{M_{t+j}}{P_{t+j}} \right) - V(L_{t+j}) \right] \]
where home goods are indexed as \( i \in [0, n) \) and foreign goods are \( i \in [n, 1] \) with consumption indices,

\[
C' = \left( a_1 x^{\mu} + a_2 y^{\mu} \right)^{\frac{1}{\mu}}
\]

\[
x = \left( b_1 \right)^{\frac{\rho(1-\theta)}{\theta}} \left( \int_0^n c(i)^{\theta} \, di \right)^{\frac{\rho}{\theta}}
\]

\[
y = \left( b_2 \right)^{\frac{\rho(1-\theta)}{\theta}} \left( \int_n^1 c(i)^{\theta} \, di \right)^{\frac{\rho}{\theta}}
\]

and \( \frac{1}{1-\mu} \) is the elasticity of substitution between home and foreign good indices, \( \frac{1}{1-\theta} \) is the elasticity of substitution between within country varieties.
Obtaining the price index: Cost minimization problem I.

For a two-good CES consumption index

\[ C = \left( a_1 x^\mu + a_1 y^\mu \right)^{\frac{1}{\mu}} \]

the elasticity of substitution between \( x \) and \( y \) is \( \frac{1}{1-\mu} \). The corresponding price index is

\[ P = \left( \frac{1}{a_1 (p_1)^{\frac{\mu}{\mu-1}} + a_2 (p_2)^{\frac{1}{1-\mu}} + \lambda} \right)^{\frac{\mu-1}{\mu}} \]

Proof: For a given expenditure \( PC \), we seek the best way to allocate it across \( x \) and \( y \). Form the Lagrangian

\[ L = p_x x + p_y y + \lambda \left[ C - (a_1 x^\mu + a_1 y^\mu)^{\frac{1}{\mu}} \right] \]
First-order conditions

\[ p_1 = \lambda a_1 x^{\mu - 1} (a_1 x^{\mu} + a_2 y^{\mu})^{\frac{1}{\mu} - 1} \]

\[ p_2 = \lambda a_2 y^{\mu - 1} (a_1 x^{\mu} + a_2 y^{\mu})^{\frac{1}{\mu} - 1} \]

Eliminate the multiplier

\[ \frac{p_1}{p_2} = \frac{a_1}{a_2} \left( \frac{x}{y} \right)^{\mu - 1} \]

From this relation, we can write

\[ x = \left( \left( \frac{p_1}{p_2} \right) \frac{a_2}{a_1} \right)^{\frac{1}{\mu - 1}} y \]

or

\[ y = \left( \left( \frac{p_2}{p_1} \right) \frac{a_1}{a_2} \right)^{\frac{1}{\mu - 1}} x \]
• Take the expression for $x$ and plug into the consumption index,

\[
C = (a_1 x^\mu + a_1 y^\mu)^{\frac{1}{\mu}}
\]

\[
C = \left( a_1 \left( \left( \left( \frac{p_1}{p_2} \right)^{\frac{\mu}{\mu-1}} \left( \frac{a_2}{a_1} \right)^{\frac{\mu}{\mu-1}} \right) y^\mu \right) + a_2 y^\mu \right)^{\frac{1}{\mu}}
\]

\[
= a_2^{\frac{\mu-1}{\mu}} p_2^{\frac{1}{1-\mu}} y \left[ \left( a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + (a_2)^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}} \right) \right]^{\frac{1}{\mu}}
\]

By symmetry of the consumption index, it must also be the case that

\[
C = a_1^{\frac{\mu-1}{\mu}} p_1^{\frac{1}{1-\mu}} x \left[ \left( a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + (a_2)^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}} \right) \right]^{\frac{1}{\mu}}
\]

Thus, we have

\[
C = \left( a_1^{\frac{1}{1-\mu}} x \left[ \left( a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + (a_2)^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}} \right) \right] \right)^{\frac{1}{\mu}}
\]

\[
C = \left( a_2^{\frac{1}{1-\mu}} y \left[ \left( a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + (a_2)^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}} \right) \right] \right)^{\frac{1}{\mu}}
\]

\[
C = a_1^{\frac{\mu-1}{\mu}} p_1^{\frac{1}{1-\mu}} x \left[ \left( a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + (a_2)^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}} \right) \right]^{\frac{1}{\mu}}
\]
which gives

\[
x = Ca_1^{1-\mu} p_1^{1-1} \left[ \left( a_1^{1-\mu} (p_1)^{\mu-1} + a_2^{1-\mu} p_2^{\mu-1} \right) \right]^{-\frac{1}{\mu}}
\]

\[
y = Ca_2^{1-\mu} p_2^{1-1} \left[ \left( a_1^{1-\mu} (p_1)^{\mu-1} + a_2^{1-\mu} p_2^{\mu-1} \right) \right]^{-\frac{1}{\mu}}
\]

- Use these expressions of \( x \) and \( y \) in the budget constraint and after some simplification, one obtains,

\[
p_1 x + p_2 y = \left( a_1^{1-\mu} (p_1)^{\mu-1} + a_2^{1-\mu} p_2^{\mu-1} \right)^{\mu-1} \mu \ C
\]

Therefore

\[
P = \left( a_1^{1-\mu} (p_1)^{\mu-1} + a_2^{1-\mu} p_2^{\mu-1} \right)^{\mu-1} \mu
\]
Demand functions:

\[ x = Ca_1^{\frac{1}{1-\mu}} p_1^{\frac{\mu}{1-\mu}} \left[ \left( a_1^{1-\mu} (p_1)^{\mu} + (a_2)^{1-\mu} p_2^{\frac{\mu}{1-\mu}} \right) \right]^{-\frac{1}{\mu}} = a_1^{\frac{1}{1-\mu}} \left( \frac{P}{p_1} \right)^{\frac{1}{1-\mu}} C \]

\[ y = a_2^{\frac{1}{1-\mu}} \left( \frac{P}{p_2} \right)^{\frac{1}{1-\mu}} C \]

Obtaining the price sub-indices for home and foreign goods: Cost minimization problem II.

For the home goods consumption index

\[ C_h = b_1^{\frac{1}{1-\theta}} \left( \int_0^n c(i)^\theta \, di \right)^{\frac{1}{\theta}} \]
the price sub-index is

\[ P_h = b_1^{\frac{(\theta-1)}{\theta}} \left( \int_0^n (p(i))^{\theta-1} \, di \right)^{1-\frac{1}{\theta}} \]

- Proof: The problem is how to allocate a given expenditure \( PC \) across \( i \in [0, n) \), where

\[ C_h = b_1^{\frac{(1-\theta)}{\theta}} \left( \int_0^n c(i)^{\frac{1}{\theta}} \, di \right)^{\frac{1}{\theta}} \]

- Note here that \( b_1^{\frac{(1-\theta)}{\theta}} \left( \int_0^n c(i)^{\frac{1}{\theta}} \, di \right)^{\frac{1}{\theta}} \) is shorthand for \( \left( \sum_i^{\infty} c(i)^{\theta} \right)^{\frac{1}{\theta}} \).

Since

\[ \frac{\partial}{\partial c(i)} \left( \sum_i^{\infty} c(i)^{\theta} \right)^{\frac{1}{\theta}} = c(i)^{(\theta-1)} \left( \sum_i^{\infty} c(i)^{\theta} \right)^{\frac{1-\theta}{1}}, \]

by analogy, we have the differentiantial rule,

\[ \frac{\partial b_1^{\frac{(1-\theta)}{\theta}} \left( \int_0^n c(i)^{\frac{1}{\theta}} \, di \right)^{\frac{1}{\theta}}}{\partial c(i)} = b_1^{\frac{(1-\theta)}{\theta}} c(i)^{(\theta-1)} \left( \int_0^n c(i)^{\theta} \, di \right)^{1-\theta} = c(i)^{(\theta-1)} b_1^{(1-\theta)} C_h^{(1-\theta)} \]
Form the Lagrangian

\[ L = \int_0^n p(i) c(i) \, di + \lambda \left( C_h - b_1^{(1-\theta)} \left( \int_0^n c(i)^\theta \, di \right)^{\frac{1}{\theta}} \right) \]

First-order conditions are

\[ p(i) = \lambda c(i)^{(\theta-1)} b_1^{(1-\theta)} C_h^{(1-\theta)} \]

which we use to write

\[
\frac{p(i)}{p(j)} = \left( \frac{c(i)}{c(j)} \right)^{\theta-1} \\
c(i) = \left( \frac{p(i)}{p(j)} \right)^{\frac{1}{\theta-1}} c(j)
\]
– Use the expression for \( c(i) \) in the sub-index,

\[
C_h = b_1^{\frac{(1-\theta)}{\theta}} \left( \int_0^n c(i)^\theta \, di \right)^{\frac{1}{\theta}} = b_1^{\frac{(1-\theta)}{\theta}} \left( \int_0^n \left( \frac{p(i)}{p(j)} \right)^{\frac{\theta}{\theta-1}} c(j)^\theta \, di \right)^{\frac{1}{\theta}}
\]

\[
= b_1^{\frac{(1-\theta)}{\theta}} p(j)^{\frac{1}{1-\theta}} c(j) \left( \int_0^n \left( p(i) \right)^{\frac{\theta}{\theta-1}} \, di \right)^{\frac{1}{\theta}}
\]

\[
\rightarrow c(j) = C_h b_1^{\frac{(\theta-1)}{\theta}} p(j)^{\frac{1}{\theta-1}} \left( \int_0^n \left( p(i) \right)^{\frac{\theta}{\theta-1}} \, di \right)^{\frac{-1}{\theta}}
\]

– Now use the expression for \( c(j) \) in the budget constraint.

\[
P_h C_h = \int_0^n p(j) c(j) \, dj = \left( \int_0^n \left( p(i) \right)^{\frac{\theta}{\theta-1}} \, di \right)^{\frac{-1}{\theta}} C_h b_1^{\frac{(\theta-1)}{\theta}} \int_0^n p(j)^{\frac{\theta}{\theta-1}} \, dj
\]

\[
P_h = b_1^{\frac{(\theta-1)}{\theta}} \left( \int_0^n \left( p(i) \right)^{\frac{\theta}{\theta-1}} \, di \right)^{\frac{-1}{\theta}} \int_0^n p(j)^{\frac{\theta}{\theta-1}} \, dj
\]

\[
\rightarrow P_h = b_1^{\frac{(\theta-1)}{\theta}} \left( \int_0^n \left( p(i) \right)^{\frac{\theta}{\theta-1}} \, di \right)^{1-\frac{1}{\theta}}
\]
– Use the price index back in the \( c(j) \) expression to write the good’s demand

\[
c(j) = p(j)^{\frac{1}{\theta-1}} b_1^{\frac{(\theta-1)}{\theta}} \left( \int_0^n (p(i))^{\theta-1} \, di \right)^{1-\frac{1}{\theta}} \left( \int_0^n (p(i))^{\theta-1} \, di \right)^{-1} C_h
\]

\[
= p(j)^{\frac{1}{\theta-1}} \frac{P_h}{b_1 (P_h)^{\theta-1}} C_h
\]

\[
= p(j)^{\frac{1}{\theta-1}} \frac{P_h}{b_1 (P_h)^{\theta-1}} a_1^{\frac{1}{1-\mu}} \left( \frac{P}{P_h} \right)^{\frac{1}{1-\mu}} C
\]

\[
\rightarrow c(j) = \frac{a_1^{\frac{1}{1-\mu}}}{b_1} \left( \frac{P_h}{p(j)} \right)^{\frac{1}{1-\theta}} \left( \frac{P}{P_h} \right)^{\frac{1}{1-\mu}} C
\]
Reparameterization for Benigno’s model

In Benigno’s JME paper, home consumer’s preferences for \( h \in [0, n] \) are

\[
U^h_t = E_t \sum_{j=t}^{\infty} \beta^{j-t} \left[ U(C^h_{t+j}) + N\left(\frac{M^h_{t+j}}{P_{t+j}}\right) - V(L^h_{t+j})\right]
\]
where for elasticity of substitution between foreign and home $\zeta$ and elasticity of substitution across within country goods $\sigma$,

\[
C = \left[ n^{\frac{1}{\zeta}} C_H^{\left(\frac{\zeta - 1}{\zeta}\right)} - (1 - n)^{\frac{1}{\zeta}} C_F^{\left(\frac{\zeta - 1}{\zeta}\right)} \right]^{\frac{\zeta}{(\zeta - 1)}}, \quad \zeta > 0
\]

\[
C_H = \left[ \left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c(z)^{\frac{\sigma - 1}{\sigma}} \, dz \right]^{\sigma/(\sigma - 1)}
\]

\[
C_F = \left[ \left(\frac{1}{1 - n}\right)^{\frac{1}{\sigma}} \int_n^1 c(z)^{\frac{\sigma - 1}{\sigma}} \, dz \right]^{\sigma/(\sigma - 1)}
\]

\[
P = \left[ n P_H^{1 - \zeta} + (1 - n) P_F^{1 - \zeta} \right]^{1/(1 - \zeta)}
\]

\[
P_H = \left[ \left(\frac{1}{n}\right) \int_0^n p(z)^{1 - \sigma} \, dz \right]^{1/(1 - \sigma)}
\]

\[
P_F = \left[ \left(\frac{1}{1 - n}\right) \int_n^1 p(z)^{1 - \sigma} \, dz \right]^{1/(1 - \sigma)}
\]

Parameter correspondence:
\[
\mu = \frac{\zeta - 1}{\zeta} \\
\Rightarrow \quad \zeta = \frac{1}{1 - \mu} \\
\Rightarrow \quad \frac{\mu}{\mu - 1} = 1 - \zeta \quad \Rightarrow \quad \frac{1}{\mu} = \frac{\zeta}{\zeta - 1} \\
\]

\[a_1 = n^{\frac{1}{\zeta}}\]

For the general indices,\[
C = (a_1 x^\mu + a_2 y^\mu)^{\frac{1}{\mu}} \rightarrow C = \left[n^{1/\zeta} x^{(\zeta - 1)/\zeta} + (1 - n)^{1/\zeta} y^{(\zeta - 1)/\zeta}\right]^{1/(\zeta - 1)}
\]

\[P = \left[a_1^{\frac{1}{1-\mu}} p_x^{\frac{\mu}{\mu-1}} + a_2^{\frac{1}{1-\mu}} p_y^{\frac{\mu}{\mu-1}}\right]^{\frac{\mu-1}{\mu}} \rightarrow P = \left[n p_1^{1-\zeta} + (1 - n) p_y^{1-\zeta}\right]^{1/(\zeta - 1)}\]

For sub-price indices,
3.1 Benigno’s model

- Households

  - Notation

    * Home: [0, n]. Foreign: (n, 1]. Continuum of goods and agents. Each agent owns one firm and supplies labor to that firm.

    * $h$ denotes home agent. $f$ denotes foreign agent.

    * $s_t$ an event in period $t$.

    * $s^t = (s_t, s_{t-1}, ..., s_0)$ is the history of events.

    * $B^h(s_{t+1})$: home agent’s holdings of one-period nominal state-contingent bonds.
* $Q(s_{t+1}|s^t)$ is the home-currency nominal price of the state-contingent bond.

- Preferences: Home consumer: For $h \in [0, n]$,

$$U^h_t = E_t \sum_{j=t}^{\infty} \beta^{(j-t)} \left[ U \left( C^h_{t+j} \right) + N \left( \frac{M^h_{t+j}}{P_{t+j}} \right) - V \left( L^h_{t+j} \right) \right]$$

where for elasticity of substitution between foreign and home $\zeta$ and
elasticity of substitution across within country goods $\sigma$,

$$C = \left[ n^{1/\zeta} C_H^{(\zeta-1)/\zeta} - (1-n)^{1/\zeta} C_F^{(\zeta-1)/\zeta} \right]^{\zeta/(\zeta-1)}, \quad \zeta > 0$$

$$C_H = \left[ \left( \frac{1}{n} \right)^{1/\sigma} \int_0^n c(z)^{(\sigma-1)/\sigma} \, dz \right]^{\sigma/(\sigma-1)}$$

$$C_F = \left[ \left( \frac{1}{1-n} \right)^{1/\sigma} \int_n^1 c(z)^{(\sigma-1)/\sigma} \, dz \right]^{\sigma/(\sigma-1)}$$

$$P = \left[ n P_H^{1-\zeta} + (1-n) P_F^{1-\zeta} \right]^{1/(1-\zeta)}$$

$$P_H = \left[ \left( \frac{1}{n} \right) \int_0^n p(z)^{1-\sigma} \, dz \right]^{1/(1-\sigma)}$$

$$P_F = \left[ \left( \frac{1}{1-n} \right) \int_n^1 p(z)^{1-\sigma} \, dz \right]^{1/(1-\sigma)}$$
- Budget constraint: Income comes from profits of firms owned by households, payoffs from state-contingent bonds, wages, and transfers from the government.

\[ C_t^h + \sum_{s_{t+1}} \frac{Q(s_{t+1}|s^t)}{P_t} B^h(s_{t+1}) \frac{M_t^h}{P_t} = \frac{B^h(s_t)}{P_t} + \frac{M_{t-1}^h}{P_t} + \frac{W_t^h L_t^h}{P_t} + \frac{\pi_t^h}{P_t} + \frac{T_{r_t}}{P_t} \]

- Government budget constraint: Transfers are helicopter drops of cash.

\[ \int_0^n (M_t^h - M_{t-1}^h) \, dh = \int_0^n T_{r_t} \, dh \]
For the intertemporal allocation problem, write down two terms from the Lagrangian:

\[
\beta_t \sum_{s_t} \pi(s_t) (C_t(s_t) + N)(M_t^h(s_t)) - V(L_t^h(s_t)) + N\left(\frac{M_{t+1}^h(s_t)}{P_t(s_t)}\right) - \sum_{s_t} \lambda_t(s_t)
\]

\[
\beta_t+1 \sum_{s_{t+1}} \pi(s_{t+1}) \left(\frac{M_{t+1}^h(s_{t+1})}{P_{t+1}(s_{t+1})}\right) - \sum_{s_{t+1}} \lambda_{t+1}(s_{t+1})
\]
* Obtain the first-order conditions for consumption,

\[ C_t^h : \beta^t \pi \left( s_t, s_t^{t-1} \right) U' \left( C_t^h (s^t) \right) = \lambda_t (s^t) \]

\[ B^h (s_{t+1}) : \lambda_t (s^t) \frac{Q \left( s_{t+1} \mid s^t \right)}{P_t (s^t)} = \lambda_{t+1} (s_{t+1}) \frac{1}{P_{t+1} (s_{t+1})} \]

Eliminating the multiplier gives the price of state contingent bonds

\[ Q \left( s_{t+1} \mid s^t \right) = \beta \pi \left( s_{t+1} \mid s^t \right) \frac{U' \left( C_{t+1}^h (s_{t+1}^{t+1}) \right)}{U' \left( C_t^h \right)} \frac{P_t}{P_{t+1}} \]

from which it follows that the price of one-period riskless nominal bonds are

\[ \frac{1}{1 + i_t} = \sum_{s_{t+1}} Q \left( s_{t+1} \mid s^t \right) = \beta E_t \left\{ \frac{U' \left( C_{t+1}^h \right)}{U' \left( C_t^h \right)} \frac{P_t}{P_{t+1}} \right\} \]
* Euler equations for labor and real money holdings:

\[
L : \rightarrow V_L (s^t) = U' \left( C_t^h (s^t) \right) \frac{W_t^h (s^t)}{P_t (s^t)}
\]

\[
M : \rightarrow N_M \left( \frac{M_t}{P_t} \right) = U' \left( C_t^h (s^t) \right) \frac{i_t}{1 + i_t}
\]

- In equilibrium, we can drop the \( h \) superscript on consumption. Due to complete markets, everyone’s consumption will be equal.

* Home consumer demand for the home good and the foreign good

\[
c^h (h) = \left( \frac{p (h)}{P_H} \right)^{-\sigma} \left[ \frac{P_H}{P} \right]^{-\zeta} C
\]

\[
c^h (f) = \left( \frac{p (f)}{P_F} \right)^{-\sigma} \left[ \frac{P_F}{P} \right]^{-\zeta} C
\]

with similar demand functions for the foreign guy.
• Complete international asset markets. One-period state contingent nominal bonds denominated in the home currency are traded internationally. Home guy's Euler equation has already been found. For foreign guy,

\[
C_t^f : \beta^t \pi \left( s^t \right) U' \left( C_t^f \left( s^t \right) \right) = \lambda_t^f \left( s^t \right)
\]

\[
B^h \left( s_{t+1} \right) : \lambda_t^f \left( s^t \right) \frac{Q \left( s_{t+1} | s^t \right)}{S_t P_t^* \left( s^t \right)} = \lambda_{t+1} \left( s_{t+1} \right) \frac{1}{S_{t+1} P_{t+1}^* \left( s_{t+1} \right)}
\]

\[
Q \left( s_{t+1} | s^t \right) = \beta \pi \left( s_{t+1} | s^t \right) \frac{U' \left( C_{t+1}^f \left( s_{t+1} \right) \right)}{U' \left( C_t^f \left( s^t \right) \right)} \frac{S_t P_t^* \left( s^t \right)}{S_{t+1} P_{t+1}^* \left( s_{t+1} \right)}
\]

\[
= \beta \pi \left( s_{t+1} | s^t \right) \frac{U' \left( C_{t+1}^h \left( s_{t+1} \right) \right)}{U' \left( C_t^h \right)} \frac{P_t}{P_{t+1}}
\]
By the law of one price,

\[
\frac{S_t P_t^*}{P_t} \frac{P_{t+1}}{S_{t+1} P_{t+1}^*} = \frac{U' \left( C_{t+1}^h \left( s^{t+1} \right) \right) U' \left( C_t^f \right)}{U' \left( C_{t+1}^f \left( s^{t+1} \right) \right) U' \left( C_t^h \right)}
\]

and after backwards recursive substitution,

\[
\frac{U' \left( C_t^f \right)}{U' \left( C_t^h \right)} = \kappa \frac{S_t P_t^*}{P_t} = \kappa R S_t
\]

where \( \kappa \) depends on initial conditions. This is the Backus–Smith condition. Benigno achieves real exchange rate dynamics through relative movements in consumption.
Monetary policy is conducted through interest-rate feedback rules (Taylor rules). Let $\varepsilon_t^H, \varepsilon_t^F$ be monetary policy shocks, $F$ and $F^*$ be a set of target variables. The general specification of the rules are

$$\frac{1 + i_t}{1 + \bar{i}} = \psi \left( F^t, s_t, \varepsilon^H_t \right)$$

$$\frac{1 + i^*_t}{1 + \bar{i}^*} = \psi^* \left( F^*_t, s_t, \varepsilon^F_t \right)$$

$$1 = (1 + i_t) \beta E_t \left\{ \frac{U' \left( C^h_{t+1} \right)}{U' \left( C^h_t \right)} \frac{P_t}{P_{t+1}} \right\} = (1 + i^*_t) \beta E_t \left\{ \frac{U' \left( C^f_{t+1} \right)}{U' \left( C^f_t \right)} \frac{P_t}{P^*_{t+1}} \right\}$$
• Production with local currency pricing. All produced goods are tradable. Deviations from PPP are due to international market segmentation. Prices are sticky in terms of the local currency.

– Sticky prices through the Calvo mechanism. Say the Calvo-lottery chooses home firm $h$ to set a new price at time $t$. The firm does so to maximize expected present value of net profits. Let $\alpha^s$ be the probability that the price will remain in effect from $t + 1$ through $t + s$. The owner’s discount factor is

$$\Phi_{t,t+s} = \beta^s \frac{U'(C_{t+s})}{U'(C_t)} \frac{P_t}{P_{t+s}}$$

and the production technology

$$c^h_t (h) + c^f_t (h) = y_t (h) = A_t L^h_t$$

where $A_t$ is a country-specific technology (productivity) shock.
The objective is to maximize

\[
E_t \sum_{j=0}^{\infty} \alpha^j \Phi_{t,t+j} \left[ p_{t+j}(h) y_{h}^{t+j}(h) + S_{t+j} p_{t+j}^*(h) y_{f}^{t+j}(h) - W_{t+j}^h L_{t+j}^h \right]
\]

subject to

\[
y_{h}^{t}(h) = \left( \frac{p_{t}(h)}{P_{H_t}} \right)^{-\sigma} \left[ \frac{P_{H_t}}{P_t} \right]^{-\zeta} \quad nC_t = \Lambda_{t}^h p_t(h)^{-\sigma}
\]

\[
y_{f}^{t}(h) = \left( \frac{p_{t}^*(h)}{P_{H_t}^*} \right)^{-\sigma} \left[ \frac{P_{H,t}^*}{P_t^*} \right]^{-\zeta} \quad (1 - n) C_t^* = \Lambda_{t}^* p_t^*(h)^{-\sigma}
\]

where

\[
\Lambda_{t}^h = \left( \frac{1}{P_{H_t}} \right)^{-\sigma} \left[ \frac{P_{H_t}}{P_t} \right]^{-\zeta} \quad nC_t
\]

\[
\Lambda_{t}^* = \left( \frac{1}{P_{H_t}^*} \right)^{-\sigma} \left[ \frac{P_{H,t}^*}{P_t^*} \right]^{-\zeta} \quad (1 - n) C_t^*
\]
The firm must set $p_t(h)$ and $p_t^*(h)$.

$$\Phi_{t,t} \left[ p_t(h) \Lambda_t^h p_t(h)^{-\sigma} + S_t p_t^*(h) \Lambda_t^* p_t^*(h)^{-\sigma} - W_t^h L_t^h \right]$$

$$+ \lambda_t \left( A_t L_t^h - \Lambda_t^h p_t(h)^{-\sigma} - \Lambda_t^* p_t^*(h)^{-\sigma} \right)$$

$$+ \alpha E_t \Phi_{t,t+1} \left[ p_{t+1}(h) \Lambda_{t+1}^h p_{t+1}(h)^{-\sigma} + S_{t+1} p_{t+1}^*(h) \Lambda_{t+1}^* p_{t+1}^*(h)^{-\sigma} - W_{t+1}^h L_{t+1}^h \right]$$

$$+ E_t \lambda_{t+1} \left( A_{t+1} L_{t+1}^h - \Lambda_{t+1}^h p_{t+1}(h)^{-\sigma} - \Lambda_{t+1}^* p_{t+1}^*(h)^{-\sigma} \right) + \ldots$$

* First, choose $p_t(h)$, with the expectation that it will stay in effect.

$$p_t(h) : (1 - \sigma) p_t(h)^{-\sigma} E_t \sum_{j=0}^{\infty} \alpha^j \Phi_{t,t+j} \Lambda_{t+j}^h \sigma p_t(h)^{-(1+\sigma)} E_t \sum_{j=0}^{\infty} \Lambda_{t+j}^h$$

and rearranging gives

$$p_t(h) = \frac{\sigma}{(1 - \sigma) E_t \sum_{j=0}^{\infty} \alpha^j \Phi_{t,t+j} \Lambda_{t+j}^h} \frac{E_t \sum_{j=0}^{\infty} \Lambda_{t+j}^h}{E_t \sum_{j=0}^{\infty} \alpha^j \Phi_{t,t+j} \Lambda_{t+j}^h}$$
* Similarly, the optimal way to set the foreign price is,

\[ p_t^*(h) = \frac{\sigma}{(1 - \sigma) E} \sum_{j=0}^{\infty} S_{t+j} \Lambda_{t+j}^* \]

* Evolution of price sub-indices. A measure \( \alpha \) of the firms have to keep the price as before. \((1 - \alpha)\) set to the new price \( p_t(h) \), therefore,

\[ P_{H,t} = \alpha_h P_{H,t-1} + (1 - \alpha_h) p_t(h) \]
\[ P_{H,t}^* = \alpha_h^* P_{H,t-1}^* + (1 - \alpha_h^*) p_t^*(h) \]

with similar equations holding for the foreign firm

\[ P_{F,t} = \alpha_f^* P_{F,t-1}^* + (1 - \alpha_f^*) p_t^*(f) \]
\[ P_{F,t} = \alpha_f P_{F,t-1} + (1 - \alpha_f) p_t(f) \]

Benigno allows for the possibility that the degree of price stickiness may differ across countries.
* Relative price indices of the imported good in terms of the domestically produced good in terms of the local currency

\[ T = \frac{P_F}{P_H} \]

\[ T^* = \frac{P_H}{P_F^*} \]
• Aggregate fluctuations.

  – Expand around a steady state with zero inflation and exchange rate depreciation, nominal interest rates equal to utility-based discount rate, and a real exchange rate of 1 and equal consumption across countries.

  – Highlights of the log-linearization.

    1. $\pi = \ln (1 + \pi)$
    
    2. $\widehat{RS}_t = \ln (RS_t)$
    
    3. $\widehat{T} = \ln \left( \frac{P_E}{P_H} \right)$, $\widehat{T}^* = \ln \left( \frac{P_H^*}{P_F^*} \right)$
    
    4. $\tilde{C}$, $\tilde{T}$ are flexible-price equilibrium values.
    
    5. $\eta = \frac{LV_{LL}}{V_L}$, $\rho = \frac{-CU_{CC}}{U_c}$
– Relative prices and the real exchange rate have unit roots. Obtain an IS-like curve, uncovered interest parity, and an expenditure switching effect.

\[ \begin{align*}
\hat{T}_t &= \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} \\
\hat{T}^*_t &= \hat{T}^*_t - \pi_{F,t} + \pi_{H,t} \\
\hat{R}S_t &= \hat{R}S_{t-1} + \pi^*_t - \pi_t + \Delta S_t \\
E_t\hat{C}_{t+1} &= \hat{C}_t + \frac{1}{\rho} [\hat{i}_t - E_t\pi_{t+1}] \\
E_t\Delta S_{t+1} &= \hat{i}_t - \hat{i}^*_t \\
y_{H,t} - y_{F,t} &= \zeta \left( n \left( \hat{T}_t - \hat{T}_t \right) - (1 - n) \left( \hat{T}^*_t - \hat{T}^*_t \right) \right)
\end{align*} \]
– Dynamics under sticky prices

* \( \alpha_h = \alpha^*_h = \alpha_f = \alpha^*_f \): Chari-Kehoe-McGrattan case.

* \( \alpha_h = \alpha_f, \ \alpha^*_h = \alpha^*_f \): Location of consumption determines degree of price stickiness.

* \( \alpha_h = \alpha^*_h, \ \alpha_f = \alpha^*_f \): Price stickiness is firm location specific and faces the same stickiness in the home and foreign market.

• Monetary policy rules reflect interest rate smoothing and exchange rate feedback.

\[
\begin{align*}
\hat{\pi}_t &= \gamma_H \hat{\pi}_{t-1} + \phi_H \pi_t + \psi_H y^H_t + \mu_H \Delta S_t + \xi_H \hat{R}S_t + \varepsilon^H_t \\
\hat{\pi}^*_t &= \gamma_F \hat{\pi}^*_{t-1} + \phi_F \pi^*_t + \psi_F y^F_t - \mu_F \Delta S_t - \xi_F \hat{R}S_t + \varepsilon^F_t \\
\varepsilon^j_t &= \rho_j \varepsilon^j_{t-1} + \nu_{j,t} \end{align*}
\]
Alternative policy rules

- Fixed exchange rates. Foreign country equates $i^*$ to $i$ with a reaction to deviations of the exchange rate from target. Let $\hat{S} = \ln \left( S/\bar{S} \right)$, where $\bar{S}$ is the target. Home country follows Taylor rule

\[
\begin{align*}
\hat{i}_t^* &= \hat{i}_t - \lambda \hat{S}_t \\
\hat{i}_t &= \phi \pi_t + \psi y_{H,t}
\end{align*}
\]

- Taylor rules

\[
\begin{align*}
\hat{i}_t &= \phi \pi_t + \psi y_{H,t} + \hat{\varepsilon}_H,t \\
\hat{i}_t^* &= \phi \pi_t^* + \psi y_{F,t} + \varepsilon_{F,t}
\end{align*}
\]

- Managed float

\[
\begin{align*}
\hat{i}_t &= \phi \pi_t + \psi y_{H,t} \\
\hat{i}_t^* &= \phi \pi_t^* + \psi y_{F,t} - \mu \Delta S_t
\end{align*}
\]
• Tradeoff between persistence and volatility as response to exchange rate $\mu$ increases.
• Interest rate smoothing induces additional inertia. $\gamma$ is the smoothing coefficient. For symmetric Taylor rules
Preliminary results

- Suppose $\alpha_H = \alpha^*_H = \alpha_F = \alpha^*_F$.

  * Then price dynamics are synchronized and the relative prices $T = P_F/P_H$ and $T^* = P^*_H/P^*_F$ are uncorrelated with monetary policy. Relative prices will be affected only by productivity shocks.

  * The real exchange rate displays no persistence following a monetary shock under inflation targeting or under the Taylor rule.

    - After a monetary shock, the nominal and real exchange rates return to equilibrium after one period, as in the Redux model.

    - There will be persistence in the exchange rates if monetary shocks are serially correlated.

  * Under inflation targeting, the real exchange rate is isolated from productivity shocks.
When there is no weight on output stabilization, productivity shocks will have no effect on the real exchange rate. Also, the inflation differential is independent of relative price changes. The link between productivity shocks and the real exchange rate is broken.
• Plausible examples of persistence generation in the real exchange rate:
  Calibration and simulations. $\beta = 0.99, \eta = 2, n = 0.5, \rho = 6, \sigma = 10, \zeta = 1.5$. The only shock is from domestic monetary policy with no autoregressive component.
– Interest rate smoothing

Fig. 5. Persistence with smoothing regime.
- Firm-specific price rigidity $\alpha_H = \alpha^*_H \neq \alpha_F = \alpha^*_F$

Fig. 6. Persistence of the real exchange rate with different degrees of rigidities.
\[- \alpha_H = 0.8, \alpha_H^* = 0.66, \alpha_F = 0.67, \alpha_F^* = 0.4, \gamma = 0.85, \phi = 0.225\]
Fig. 8. Persistence and volatility of the real exchange rate.
Takeaway: Previously thought: Persistence of real exchange rate increasing in degree of price stickiness, and unrealistically long period of price stickiness to match the persistence in real exchange rate. Now thought: Price rigidity is not sufficient by itself to generate persistence following a monetary shock.
4 Inflation News and the Exchange rate: Clarida and Waldman

Theoretical model for how bad news about inflation (higher than expected) is good news for the exchange rate (home currency strengthens). Interesting and nicely constructed empirical work.

Questions:

1. What is the correlation between inflation surprises and changes in the nominal exchange rate?

2. Is the sign different for inflation targeters and non-inflation targeters?

- Construct returns 10-minute percentage changes to capture behavior in a plus/minus 5 minute window of the announcement.

- Positive inflation surprise means higher than expected. Expectations is the median survey response from Bloomberg News Service, which surveys commercial and investment banks on macroeconomic announcements. Use consumer price inflation. Month-over-month (MoM) and year-over-year (YoY) inflation for headline and core inflation.

- Regression. $R_t$ is 10 minute return around the announcement. $R > 0$ means home currency appreciates. $S_t$ is the inflation surprise.

\[ R_{it} = \alpha + \beta S_{it} + u_{it} \]
– Normalize coefficients to interpret $\beta$ as an elasticity.

– Pool the data and run as stacked OLS regression.

<table>
<thead>
<tr>
<th></th>
<th>Headline</th>
<th>Core</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MoM</td>
<td>YoY</td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>T- Statistic</td>
<td>5.9</td>
<td>6.2</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td># Observations</td>
<td>394</td>
<td>387</td>
</tr>
</tbody>
</table>

Regression method: stacked OLS.
Percentage change in exchange rate resulting from a one percentage point upward surprise in inflation.
Positive coefficient indicates appreciation of domestic currency.
Countries: Australia, Canada, Euro area, Japan, New Zealand, Norway, Sweden, Switzerland, UK, and US.
Data: July 2001- December 2005. Some countries missing observations.
– **Inflation targeters versus non-inflation targeters.**

<table>
<thead>
<tr>
<th></th>
<th>Inflation Targeters</th>
<th>Non-Inflation Targeters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Headline</td>
<td>Core</td>
</tr>
<tr>
<td><strong>Coefficient</strong></td>
<td>MoM</td>
<td>YoY</td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>T-Statistic</td>
<td>6.1</td>
<td>6.7</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td># Observations</td>
<td>286</td>
<td>310</td>
</tr>
</tbody>
</table>

Regression method: stacked OLS.
Percentage change in exchange rate resulting from a one percentage point upward surprise in inflation.
Positive coefficient indicates appreciation of domestic currency.
Inflation targeters includes: Australia, Canada, Euro area, New Zealand, Norway, Sweden, Switzerland, and UK.
Non-inflation targeters includes: Japan and US.
Non-inflation targeters YoY includes only Japan.
Data: July 2001 - December 2005. Number of observations may be less than total months due to missing observations.

Table 6: UK and Norway Pre-Inflation Targeting

<table>
<thead>
<tr>
<th></th>
<th>UK</th>
<th>Norway</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Headline</td>
<td>Core</td>
</tr>
<tr>
<td></td>
<td>MoM</td>
<td>YoY</td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.006</td>
<td>-0.05</td>
</tr>
<tr>
<td>T-Statistic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>0.1</td>
<td>-0.5</td>
</tr>
<tr>
<td>White</td>
<td>0.1</td>
<td>-0.7</td>
</tr>
<tr>
<td>Newey-West</td>
<td>0.1</td>
<td>-1.1</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td># Observations</td>
<td>46</td>
<td>46</td>
</tr>
</tbody>
</table>

Percentage change in exchange rate resulting from a one percentage point upward surprise in inflation.
Positive coefficient indicates appreciation of domestic currency.
Number of observations may be less than total months due to missing observations.
White and Newey-West used to correct for potential heteroscedasticity.
5 Calibrated partial equilibrium models of the exchange rate and Taylor rules

Engel and West JMCB. Rational expectations model.

Mark (mimeo). Adaptive learning
Partial equilibrium setup. We assume

- The public believes Fed and Bundesbank use some form of the Taylor rule.

- Public views inflation and output gap are exogenously generated by a VAR(4).

- Economic model is real interest parity, a stochastic difference equation, whose solution gives the real exchange rate.

- Public employs least-squares learning rules to form beliefs about model’s coefficients since their true values are unknown and may change over time.

Feed historically observed data on inflation and the output gap into the model. Observe the implied learning equilibrium path of the real exchange rate and compare with historical real exchange rate path.
5.1 Can we assume that the Fed and Bundesbank (ECB) reacts the same to inflation and the output gap?

- German variables subscripted by ‘G’ U.S. variables subscripted by ‘U.’

- German–U.S. differentials have no special notation.

\[
\begin{align*}
\pi_t &= \left( \pi_{G,t} - \pi_{U,t} \right) \\
i_t &= \left( i_{G,t} - i_{U,t} \right) \\
x_t &= \left( x_{G,t} - x_{U,t} \right)
\end{align*}
\]

are German-U.S. differentials in inflation, short-term nominal interest rates and activity gaps, respectively. Activity gap defined such that the economy operates in excess of its potential when \( x_{tj} > 0, j = G, U. \) The log real exchange rate is \( q_t. \)
• Fed rule for target rate

\[ i_{U,t}^T = \bar{i}_U + \gamma_\pi \left( E_t \pi_{U,t+1} - \pi_U \right) + \gamma_x x_{U,t}. \]

Actual interest rate subject to exogenous and i.i.d. policy shock \( \eta_{U,t} \) and interest rate smoothing

\[ i_{U,t} = (1 - \rho)i_{U,t}^T + \rho i_{U,t-1} + \eta_{U,t}. \]

• Bundesbank target rule

\[ i_{G,t}^T = \bar{i}_G + \gamma_\pi \left( E_t \pi_{G,t+1} - \pi_G \right) + \gamma_x x_{G,t} + \gamma_q q_{t-1}. \]
• Impose homogeneity of the coefficients \((\gamma_\pi, \gamma_x)\) across countries and write interest differential as

\[
i_t = \delta + \rho i_{t-1} + (1 - \rho) \left( \gamma_\pi E_t \pi_{t+1} + \gamma_x x_t + \gamma_q q_{t-1} \right) + \eta_t,
\]

\[
- \rho \equiv (1 - \rho) \left( \left( i_G - \bar{i}_U \right) - \gamma_\pi \left( \bar{\pi}_G - \bar{\pi}_U \right) \right), \text{ and } \eta_t \sim iid (0, \sigma_\eta^2).
\]

- Add and subtract \((1 - \rho)\gamma_\pi \bar{\pi}_{t+1}\) on the right side of and rearrange

\[
i_t = \delta + (1 - \rho) \left[ \gamma_\pi \pi_{t+1} + \gamma_x x_t + \gamma_s q_{t-1} \right] + \rho i_{t-1} + \eta'_t,
\]

where

\[
\eta'_t = \eta_t - (1 - \rho) \gamma_\pi \left[ \pi_{t+1} - E_t \pi_{t+1} \right].
\]

is uncorrelated with date \(t\) information. Estimate by GMM. Instruments are a constant, the current value and three lags of the inflation differential, the current value and three lags of the output (alternatively unemployment) gap differential, four lags of the nominal interest differential, and four lags of the real exchange rate.
Table 2: GMM Estimates of Bundesbank–Fed Relative Interest-Rate Reaction Function with Lagged Real Exchange Rate Feedback. Bold indicates significance at the 5% level

<table>
<thead>
<tr>
<th>Source output gap</th>
<th>Sample</th>
<th>$\delta$ (t-ratio)</th>
<th>$\rho$ (t-ratio)</th>
<th>$\gamma_\pi$ (t-ratio)</th>
<th>$\gamma_x$ (t-ratio)</th>
<th>$\gamma_q$ (t-ratio)</th>
<th>J-statistic (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>61.2-79.2</td>
<td>-0.006 (-6.477)</td>
<td>0.666 (10.545)</td>
<td>0.034 (0.376)</td>
<td>0.134 (5.092)</td>
<td>0.019 (4.069)</td>
<td>9.684 (0.644)</td>
</tr>
<tr>
<td></td>
<td>79.3-05.4</td>
<td>-0.001 (-1.194)</td>
<td>0.895 (22.450)</td>
<td>1.318 (3.063)</td>
<td>0.376 (2.318)</td>
<td>0.011 (0.880)</td>
<td>4.330 (0.977)</td>
</tr>
</tbody>
</table>

Structural change test
- All coeffs. Test statistic 40.037 0.000
- Inflation coeff. Test statistic 8.525 0.004
<table>
<thead>
<tr>
<th></th>
<th>(\delta)</th>
<th>(\rho)</th>
<th>(\gamma_\pi)</th>
<th>(\gamma_x)</th>
<th>(\gamma_q)</th>
<th>J-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(t-ratio)</td>
<td>(t-ratio)</td>
<td>(t-ratio)</td>
<td>(t-ratio)</td>
<td>(t-ratio)</td>
<td>(p-value)</td>
</tr>
<tr>
<td>61.2-79.2</td>
<td>-0.003</td>
<td>0.745</td>
<td>0.121</td>
<td>0.045</td>
<td>0.011</td>
<td>9.370</td>
</tr>
<tr>
<td></td>
<td>-3.589</td>
<td>12.551</td>
<td>1.357</td>
<td>1.166</td>
<td>2.708</td>
<td>0.671</td>
</tr>
<tr>
<td>79.3-05.4</td>
<td>0.000</td>
<td>0.873</td>
<td>1.544</td>
<td>0.428</td>
<td>0.004</td>
<td>5.314</td>
</tr>
<tr>
<td></td>
<td>-0.202</td>
<td>24.480</td>
<td>4.406</td>
<td>3.239</td>
<td>0.558</td>
<td>0.947</td>
</tr>
</tbody>
</table>

Structural change test
- All coeffs. Test statistic 36.747 0.000
- Inflation coeff. Test statistic 15.501 0.000
<table>
<thead>
<tr>
<th>HP unemployment gap</th>
<th>$\delta$</th>
<th>$\rho$</th>
<th>$\gamma_\pi$</th>
<th>$\gamma_x$</th>
<th>$\gamma_q$</th>
<th>J-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(t-ratio)</td>
<td>(t-ratio)</td>
<td>(t-ratio)</td>
<td>(t-ratio)</td>
<td>(t-ratio)</td>
<td>p-value</td>
</tr>
<tr>
<td>61.2-79.2</td>
<td>-0.003</td>
<td>0.579</td>
<td>0.034</td>
<td>-0.644</td>
<td>0.005</td>
<td>7.746</td>
</tr>
<tr>
<td></td>
<td>-3.077</td>
<td>8.394</td>
<td>0.554</td>
<td>-4.847</td>
<td>1.873</td>
<td>0.805</td>
</tr>
<tr>
<td>79.3-05.4</td>
<td>0.000</td>
<td>0.860</td>
<td>1.154</td>
<td>-0.828</td>
<td>0.004</td>
<td>5.083</td>
</tr>
<tr>
<td></td>
<td>-0.163</td>
<td>22.028</td>
<td>2.812</td>
<td>-3.171</td>
<td>0.443</td>
<td>0.955</td>
</tr>
</tbody>
</table>

Structural change test

| All coeffs. | Test statistic | 25.042 | 0.015 |
| Inflation coeff. | Test statistic | 7.280 | 0.007 |
6 Modeling real exchange rate dynamics with learning

- Economic model is uncovered interest parity. For the log nominal exchange rate $s_t \ (\text{DM}/\$)$

$$s_t = E_t s_{t+1} - i_t.$$  

where $i$ is the German–US interest differential.

- Add and subtract $E_t \pi_{t+1}$ from the right side, rearrange

$$q_t = E_t q_{t+1} - i_t + E_t \pi_{t+1}$$

$$i_t = \delta + \rho i_{t-1} + (1 - \rho) \left( \gamma_\pi E_t \pi_{t+1} + \gamma_x x_t + \gamma_q q_{t-1}, \right) + \eta_t$$
Let $\pi$ and $x$ be exogenously given by VAR(4). Let $Y_t' = (\pi_t, \ldots, \pi_{t-3}, x_t, \ldots, x_{t-3})$, and $Z_{1,t}' = (Y_t', 1)$. Regression form of the VAR is

$$
\begin{align*}
\pi_t &= B_1'Z_{1,t-1}' + v_1t, \\
x_t &= B_2'Z_{2,t-1}' + v_2t,
\end{align*}
$$

$B_1$ and $B_2$ are $9 \times 1$ least-squares coefficient vectors.

Companion representation

$$
\begin{align*}
Y_t &= \alpha + AY_{t-1} + v_t, \\
x_t &= e_1 + (1, 0, 0, 0, 0, 0, 0) \\
&= e_1 + Y_t, \\
\pi_t &= e_2 + (0, 0, 0, 1, 0, 0, 0) \\
x_t &= e_2 + x_t, \\
E_t \pi_{t+1} &= e_1 (\alpha + AY_t).
\end{align*}
$$
The above gives a second-order stochastic difference equation in $q_t$,

$$q_t = \delta ((1 - \rho) \gamma_\pi - 1) e_1 \alpha + (1 - \rho) \gamma_q q_{t-1} + \rho i_{t-1} + ((1 - \rho) \gamma_x e_2 + ((1 - \rho) \gamma_\pi - 1) e_1 A) Y_t + \eta_t + E_t q_{t+1}$$

**Rational expectations solution**

$$q_t = a_0 + a_1 i_{t-1} + a_2 q_{t-1} + a_3 \eta_t + b Y_t$$

$$a_2 = \frac{1}{2} (1 - \rho) \pm \frac{\sqrt{(1 - \rho)^2 - (1 - \rho) 4 \gamma_q}}{2}$$

$$a_1 = \frac{-\rho a_2}{(1 - \rho) \gamma_q}$$

$$a_0 = \left( \left( \left( -1 + (\rho - 1) a_1 + (1 - \rho) \gamma_\pi \right) e_1 - b \right) \right) \alpha - \frac{(a_1 - 1) \delta}{a_2}$$

$$a_3 = \frac{1 - a_1}{1 - a_2}$$

$$b = (((1 + (a_1 - 1) (1 - \rho) \gamma_\pi) e_1) A + (a_1 - 1) (1 - \rho) \gamma_x e_2) ((1 - a_2) I - A)^{-1}.$$
Non-uniqueness. Choose solution with positive $a$.

- $\forall \gamma I$ in the post-1979 sample: Decline in $E_t$ and $r_t$ leads to real dollar appreciation.

- Expect increase in the $r_t$ and real dollar depreciation.

$\forall \gamma > 1$ in pre-Volcker sample: Decline in $E_t$ and $r_t$ leads public to depend on inflation response coefficient $q$.

- Non-uniqueness. Choose solution with positive $a$. 
Learning the rational expectations equilibrium. Solve real interest parity condition using expectations formed from perceived law of motion.

- At $t$, coefficient vectors $B_{1,t-1}, B_{2,t-1}$ are given (estimated in $t-1$). From regression form,

\[
\pi_t = B'_{1,t-1}Z_{1,t} + u_{1,t}, \\
x_t = B'_{2,t-1}Z_{2,t} + u_{2,t},
\]

form companion form

\[
Y_t = \alpha_{t-1} + A_{t-1}Y_{t-1} + v_t,
\]

Construct expected inflation

\[
E_t\pi_{t+1} = e_1 (\alpha_{t-1} + A_{t-1}Y_t).
\]

- Believe the rational expectation solution and use it for perceived law of motion

\[
q_t = a_{0,t-1} + a_{1,t-1}i_{t-1} + a_{2,t-1}q_{t-1} + a_{3,t-1} \eta_t + b_{t-1}Y_t \equiv B'_{3,t-1}Z_{3,t}.
\]
– Observe the policy shock $\eta_t$ from perceived law of motion for the interest differential

\[
i_t = \delta_{t-1} + \rho_{t-1}i_{t-1} + \theta_{\pi,t-1}e_1(\alpha_{t-1} + A_{t-1}Y_t) + \theta_{x,t-1}e_2Y_t + \theta_{q,t-1}q_{t-1} + \eta_t
\equiv B'_{4,t-1}Z_{4,t} + \eta_t.
\]

– The expected exchange rate

\[
E_{t+1} = a_{0,t-1} + a_{1,t-1}i_t + a_{2,t-1}q_t + b_{t-1}(\alpha_{t-1} + A_{t-1}Y_t).
\]

– Plug inflation forecast and $q$ forecast into real UIP gives the actual law of motion,

\[
q_t = \frac{1}{1 - a_{3,t-1}} \left[ a_{0,t-1} + (e_1 + b_{t-1})(\alpha_{t-1} + A_{t-1}Y_t) + (a_{2,t-1} - 1) i_t \right].
\]

– For next period, least-squares update the coefficients Let $y_{1,t} = \pi_t$, $y_{2,t} = x_t$, $y_{3,t} = q_t$, and $y_{4,t} = i_t$. For given $R_{j,t-1}$ ($j = 1, \ldots, 4$),


and a fixed gain $g$, the updating formulae are

$$
R_{j,t} = R_{j,t-1} + g \left( Z_{j,t-1}Z'_{j,t-1} - R_{j,t-1} \right), \quad (1)
$$

$$
B_{j,t} = B_{j,t-1} + gR_{j,t}^{-1}Z_{j,t-1}(y_{j,t} - B'_{j,t-1}Z_{j,t-1}). \quad (2)
$$

The learning path and coefficient updating is generated using observations of $\pi_t$, $x_t$, and $i_t$ from the data, but not with exchange rate data. The learning values of $q_t$ generated by the actual law of motion and employed in coefficient updating are generated solely as functions of $\pi$, $x$, and $i$. 
Calibrated Learning and Rational Exchange Rate Paths

The observations are standardized to highlight comovements between the series.
Figure 5. Learning path with source output gap.
The learning path shown in Figure 5 generally captures closer comovements with the data and does a good job of capturing the dollar cycle of the 1980s. The learning path exhibits the dollar appreciation and subsequent depreciation in the latter part of the sample although timing of the turning points are off a bit with the phase of the learning cycle leading the data.
Figure 7. Learning path with HP output gap.
The learning path in Figure 7 shows the dollar prematurely appreciating in 1979.2 and it does not generate quite the strength attained in the data by 1984.4. The learning path also leads the turning points in the dollar appreciation and depreciation beginning in 1995.2 but otherwise comoves with the data.
Figure 9. Learning path with HP unemployment gap.
Figure 9, it is seen that the comovements of the learning path with the data are also quite good. The learning path matches the timing of the 1980s dollar cycle very well. Except for a phase shift that leads the data, it also captures the cycle from 1995 to 2005.
Figure 4: Rational path with source output gap.
Using the source output gap, the rational path shown in Figure 4 misses a good deal of the real dollar appreciation from 1980.4 to 1984.4 and falsely predicts a dollar appreciation from 1989.3 to 1991.2. It also erroneously predicts a large dollar depreciation in 1981.1 Apart from these episodes, there is a close connection between the implied rational expectations path and the data. The rational expectations path does a good job of explaining the real dollar appreciation from 1996.2 to 2002.2 and the subsequent real dollar depreciation.
Figure 6: Rational path with HP output gap.
In Figure 6, the rational path generated with the HP output gap is quite similar to the path generated with the source output gap. Here, the rational path also misses the real dollar appreciation of the 1980s and signals a false appreciation in the early 1990s.
Figure 8: Rational path with HP unemployment gap.
Figure 8 plots the rational path using HP detrended unemployment. The comovements between the rational path and the data are generally quite close.
Figure 11. Learning path with contemporaneous exchange rate in Taylor rule, source output gap.
Figure 10 Rational path with contemporaneous exchange rate in Taylor rule, source output gap.
<table>
<thead>
<tr>
<th>Form</th>
<th>Activity variable</th>
<th>Corr</th>
<th>T-ratio</th>
<th>Volatility</th>
<th>Corr</th>
<th>T-ratio</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source gap</td>
<td>0.308</td>
<td>2.461</td>
<td>0.965</td>
<td></td>
<td>0.346</td>
<td>2.170</td>
<td>1.054</td>
</tr>
<tr>
<td>HP output</td>
<td>-0.029</td>
<td>-0.201</td>
<td>3.678</td>
<td>0.298</td>
<td>2.094</td>
<td>0.336</td>
<td>1.130</td>
</tr>
<tr>
<td>HP unemployment</td>
<td>0.484</td>
<td>3.426</td>
<td>0.644</td>
<td>0.340</td>
<td>1.834</td>
<td>1.130</td>
<td></td>
</tr>
<tr>
<td>1-qtr return</td>
<td>Source gap</td>
<td>0.030</td>
<td>0.700</td>
<td>2.085</td>
<td>-0.039</td>
<td>-0.596</td>
<td>1.254</td>
</tr>
<tr>
<td>HP output</td>
<td>0.019</td>
<td>0.536</td>
<td>7.918</td>
<td>0.033</td>
<td>0.601</td>
<td>0.458</td>
<td>1.130</td>
</tr>
<tr>
<td>HP unemployment</td>
<td>0.031</td>
<td>0.499</td>
<td>1.561</td>
<td>-0.008</td>
<td>-0.135</td>
<td>1.870</td>
<td></td>
</tr>
<tr>
<td>4-qtr return</td>
<td>Source gap</td>
<td>0.235</td>
<td>3.142</td>
<td>1.525</td>
<td>0.044</td>
<td>0.393</td>
<td>1.164</td>
</tr>
<tr>
<td>HP output</td>
<td>0.026</td>
<td>0.323</td>
<td>5.662</td>
<td>0.054</td>
<td>0.593</td>
<td>0.402</td>
<td>1.486</td>
</tr>
<tr>
<td>HP unemployment</td>
<td>0.459</td>
<td>5.575</td>
<td>1.050</td>
<td>-0.026</td>
<td>-0.265</td>
<td>1.486</td>
<td></td>
</tr>
<tr>
<td>8-qtr return</td>
<td>Source gap</td>
<td>0.308</td>
<td>2.699</td>
<td>1.264</td>
<td>0.157</td>
<td>1.023</td>
<td>1.014</td>
</tr>
<tr>
<td>HP output</td>
<td>0.066</td>
<td>0.639</td>
<td>4.374</td>
<td>0.037</td>
<td>0.338</td>
<td>0.357</td>
<td>1.142</td>
</tr>
<tr>
<td>HP unemployment</td>
<td>0.624</td>
<td>5.418</td>
<td>0.848</td>
<td>0.109</td>
<td>0.706</td>
<td>1.142</td>
<td></td>
</tr>
<tr>
<td>16-qtr return</td>
<td>Source gap</td>
<td>0.424</td>
<td>4.106</td>
<td>1.055</td>
<td>0.335</td>
<td>2.393</td>
<td>1.086</td>
</tr>
<tr>
<td>HP output</td>
<td>0.019</td>
<td>0.113</td>
<td>3.753</td>
<td>0.093</td>
<td>0.799</td>
<td>0.345</td>
<td>1.033</td>
</tr>
<tr>
<td>HP unemployment</td>
<td>0.691</td>
<td>4.381</td>
<td>0.746</td>
<td>0.093</td>
<td>0.799</td>
<td>1.033</td>
<td></td>
</tr>
</tbody>
</table>
To sum up, there were six major swings in the real DM-dollar rate in the sample. The real dollar depreciation from 1973.1 to 1979.4, the sharp appreciation (1980.1–1984.4) and subsequent depreciation (1985.1–1987.4), a more tempered dollar depreciation (1988.1 to 1995.1), a dollar appreciation (1995.2–2001.2), and the dollar decline (2001.3–2005.4). Each of the rational paths falsely predicted a strong real dollar appreciation in 1991 and none of them adequately matched the volatility in the data. The learning model provides a plausible description for the data. Regardless of the definition used for the activity gap, each of the learning paths captured the major swings in the real exchange rate.

Two obvious extensions. (1) relax homogeneity restrictions on coefficients across countries. (2) Have an economic model for the output gap and inflation.
7 Taylor-rule fundamentals and exchange rate forecasts

Molodtsova, Nikolsko-Rzhevskyy, and Papell, ‘Taylor Rules with Real-Time Data: A Tale of Two Countries and One Exchange Rate.’

- Real-time versus (revised) historical data.
  - Use the information (data) available to monetary authorities and other economic agents when they made their decisions.
  - Revision in output and output gap is more substantial than revision to price indices.
  - Alternative source of data: Greenbook forecasts of inflation and output gap observed by Fed but not the public until five years later.
- German real time data: Gerberding, Worms, and Seitz.


  - U.S. potential GDP constructed by Orphanides.
  - German output gap is deviation of GDP to quadratic time trend.
Find that Taylor rule coefficients are robust to choice of real-time or historical data, and choice of forward looking forecasts, current and lagged inflation/output gap data.

Table 1. Estimated Taylor Rules for the United States

<table>
<thead>
<tr>
<th></th>
<th>Revised Data</th>
<th>Real-Time Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Coefficient, $\lambda$</td>
<td>1.39</td>
<td>1.50</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.21)</td>
</tr>
<tr>
<td>Output Gap Coefficient, $\gamma$</td>
<td>0.11</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.20)</td>
</tr>
<tr>
<td>Smoothing Coefficient, $\varphi$</td>
<td>-</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(0.20)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.73</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Notes: The table reports nonlinear least squares estimates of the following equation, $i_t = (1-\rho)(\mu + \lambda \pi_t + \gamma y_t) + \rho \delta_{t-1} + \nu_t$, where $i$ is the Federal Funds Rate, $\pi$ is inflation, and $y$ is the output gap. The sample size is 1979:Q1 – 1998:Q4. Standard errors are in parentheses.
Table 2. Estimated Taylor Rules for Germany

<table>
<thead>
<tr>
<th></th>
<th>Revised Data</th>
<th>Real Time Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Coefficient, $\lambda$</td>
<td>1.36 0.84 0.84 0.88 0.86</td>
<td>1.37 1.19 1.19 1.19 1.20</td>
</tr>
<tr>
<td></td>
<td>(0.15) (0.51) (0.36) (0.43) (0.34)</td>
<td>(0.14) (0.48) (0.27) (0.48) (0.27)</td>
</tr>
<tr>
<td>Output Gap Coefficient, $\gamma$</td>
<td>0.09 1.13 1.27 1.02 1.20</td>
<td>0.23 0.78 0.99 0.78 1.00</td>
</tr>
<tr>
<td></td>
<td>(0.09) (0.77) (0.64) (0.67) (0.59)</td>
<td>(0.07) (0.35) (0.28) (0.38) (0.29)</td>
</tr>
<tr>
<td>Smoothing Coefficient, $\varphi$</td>
<td>- 0.89 0.85 0.88 0.85</td>
<td>- 0.87 0.80 0.87 0.80</td>
</tr>
<tr>
<td></td>
<td>(0.07) (0.07) (0.08) (0.07)</td>
<td>(0.06) (0.06) (0.06) (0.06)</td>
</tr>
<tr>
<td>Exchange Rate Coefficient, $\delta_1$</td>
<td>- - 0.09 - 0.08</td>
<td>- - 0.08 - 0.08</td>
</tr>
<tr>
<td></td>
<td>(0.05) (0.05)</td>
<td>(0.03) (0.03)</td>
</tr>
<tr>
<td>Money Growth Coefficient, $\delta_2$</td>
<td>0.23 0.12</td>
<td>0.01 0.01</td>
</tr>
<tr>
<td></td>
<td>(0.30) (0.14)</td>
<td>(0.37) (0.17)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.71 0.95 0.96 0.95 0.96</td>
<td>0.68 0.95 0.95 0.95 0.95</td>
</tr>
</tbody>
</table>

Notes: The table reports nonlinear least squares estimates of the following equation, $i_t = (\rho)(\kappa_0 \lambda_0 \gamma_0 \varphi_0 \delta_0 \delta_1 \delta_2 \pi_t + \delta_t)$, where $i$ is the Federal Funds Rate, $\pi$ is inflation, $\gamma$ is the output gap, and $\delta$ is a vector containing the real DM/Dollar exchange rate and the money growth deviation. The sample size is 1979:Q1 - 1998:Q4. Standard errors are in parentheses.
• Reduced form exchange rate forecasting equation. U.S. is home country

$$\Delta s_{t+1} = \alpha_0 + \alpha_1 \pi_t - \alpha_2 \pi^*_t + \alpha_3 y_t - \alpha_4 y^*_t + \rho_1 i_{t-1} + \rho_2 i^*_{t-1} + \alpha_4 q_t + \eta_t$$

Symmetric means homogeneity restrictions imposed on coefficients.
7.0.1 Clark-West test of equal forecast accuracy (in mean square sense).

1. Out-of-sample forecasting. Let the exchange rate return be

\[ y_t \equiv s_t - s_{t-1} \]

(a) RMSE, Theil’s U-statistic.

\[ H_0 : y_t = e_t, \quad E_{t-1}e_t = 0 \]
\[ H_a : y_t = Z_t^l \beta + e_t, \quad E_{t-1}e_t = 0 \]

Under \( H_a \),

\[ y_t = \hat{f}_t + \hat{e}_t, \quad \hat{f}_t = Z_t^l \hat{\beta} \]
Make $P$ one-step ahead predictions.

$$
\text{MSPE}(1) = \hat{\sigma}_1^2 = \frac{1}{P} \sum_{t=T-P+1}^{T} e_{t+1}^2
$$

$$
\text{MSPE}(2) = \hat{\sigma}_2^2 = \frac{1}{P} \sum_{t=T-P+1}^{T} \hat{e}_{t+1}^2 = \frac{1}{P} \sum_{t=T-P+1}^{T} (y_{t+1} - \hat{f}_{t+1})^2
$$

$$
U = \frac{\hat{\sigma}_2}{\hat{\sigma}_1}
$$

Can bootstrap Theil’s $U$. 
(b) Diebold-Mariano (1995)–West (1996) statistic. For non-nested hypotheses,

\[ F_{t+1} = \hat{e}_{t+1}^2 - \hat{\epsilon}_{t+1}^2 \]

\[ \bar{F} = \frac{1}{P} \sum_{t=T-P+1}^{T} F_{t+1} \]

\[ \hat{V} = \frac{1}{P} \sum_{t=T-P+1}^{T} (F_{t+1} - \bar{F})^2 \]

\[ DMW = \frac{\bar{F}}{\hat{V}} \sim N(0, 1) \]

To compute DMW: Regress \( F_{t+1} \) on a constant. DMW is the t-statistic for the constant. Asymptotics hold for non nested hypotheses.

i. DMW turns out not to be asymptotically normal for nested hypotheses. The random walk model is nested, causing DMR to be badly undersized.
ii. Under $H_0$, sampling variability in estimated coefficients under alternative cause

$E(\hat{\sigma}_1^2) < E(\hat{\sigma}_2^2)$

Therefore, one accepts the random walk too often.
(c) Clark-West (2005) test. A simple adjustment. Works reasonably well
for rolling or recursive regression. Both are slightly undersized but
offer large improvements over DMW. Recall $\hat{f}_{t+1} = x'_{t+1} \hat{\beta}$.

\[
F_{t+1}^a = e_{t+1}^2 - \hat{e}_{t+1}^2 + \hat{f}_{t+1}^2
\]

\[
CW = \frac{F_{t+1}^a}{\hat{V}} \sim N (0, 1)
\]

where a superscript stands for ‘adjusted.’

i. CW statistic: Regress $F_{t+1}^a$ on a constant. CW is the t-ratio from
regression output.

ii. For long-horizon forecasts.

\[
y_{t,k} = y_{t+1} + y_{2+2} + \cdots + y_{t+k}
\]

$H0 : y_{t,k} = e_{t+1,k}$

$Ha : y_{t,k} = x'_{t+1} \beta + e_{t+1,k} = \hat{f}_{t,k} + \hat{e}_{t+1,k}$
Compute $F_{t}^{a}, \tilde{F}_{t}^{a}$ as above.

$$F_{t+1,k}^{a} = e_{t+1,k} - \hat{e}_{t+1,k}^{2} - \hat{f}_{t,k}^{2}$$

Due to serial correlation induced by overlapping forecasts, need to adjust $\hat{V}$. Ken West suggests the Hodrick (1992) or West (1997) covariance matrix

$$\hat{g}_{t} = 2y_{t} \left( \hat{f}_{t,k} + \cdots + \hat{f}_{t-k+1} \right)$$

$$\hat{V} = \frac{1}{P - 2k + 2} \sum_{t=t-P+k}^{T-k+1} (\hat{g}_{t+k} - \bar{g})^{2}$$

7.0.2 Evaluate choice of real-time or historical data for exchange rate forecasts.

Typical results
Significant predictability observed in asymmetric model using Greenbook inflation forecasts and Survey of Professional Forecaster’s inflation forecasts.

### Table 5. CW Statistics: One-Quarter-Ahead USD/DM Exchange Rate Forecasts using Taylor Rules with Central Bank Output Gaps

<table>
<thead>
<tr>
<th></th>
<th>w/o Smoothing</th>
<th>w/ Smoothing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Symmetric</td>
<td>Asymmetric</td>
</tr>
<tr>
<td>Revised Data</td>
<td>0.358</td>
<td>0.548</td>
</tr>
<tr>
<td></td>
<td>(0.31)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>Real-Time Data</td>
<td>-0.936</td>
<td>1.979**</td>
</tr>
<tr>
<td></td>
<td>(0.77)</td>
<td>(0.02)</td>
</tr>
</tbody>
</table>

### Table 6. CW Statistics: One-Quarter-Ahead USD/DM Exchange Rate Forecasts using Taylor Rules with Quadratic Detrended Output Gaps

<table>
<thead>
<tr>
<th></th>
<th>w/o Smoothing</th>
<th>w/ Smoothing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Symmetric</td>
<td>Asymmetric</td>
</tr>
<tr>
<td>Revised Data</td>
<td>0.201</td>
<td>0.526</td>
</tr>
<tr>
<td></td>
<td>(0.36)</td>
<td>(0.37)</td>
</tr>
<tr>
<td>Real-Time Data</td>
<td>0.009</td>
<td>1.775**</td>
</tr>
<tr>
<td></td>
<td>(0.44)</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>
Inflation coefficients say bad news about inflation is good news for the exchange rate.

Increase in U.S. output gap predicts depreciation. German output gap coefficients never significant.
### 8 Conclusion

1. Explicit treatment of endogenous monetary policy and interest rate rules in exchange rate economics is relatively new.

2. The Taylor-rule approach might provide a resolution to the PPP puzzle.

3. The Taylor-rule approach identifies a very different set of macroeconomic fundamentals in discussions about exchange rate determination.

4. Calibrated partial equilibrium Taylor-rule models of the exchange rate seem to fit the data reasonably well, and evidently much better than PPP or traditional monetary approaches.
5. Taylor-rule fundamentals have statistically significant predictive power for the future exchange rate.

6. There is scope for more research in the area.