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Testing the CAPM with Time-Varying Risks and Returns

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ABSTRACT

This paper draws on Engle's autoregressive conditionally heteroskedastic modeling strategy to formulate a conditional CAPM with time-varying risk and expected returns. The model is estimated by generalized method of moments. A CAPM that allows mean excess returns to shift in January survives generalized method of moments specification tests for a number of omitted variables. However, a residual dividend yield component is found to remain in the excess returns of smaller firms. We find significant monthly and quarterly components in the risk premia and beta estimates.

RECENT EMPIRICAL WORK IN financial economics has attempted to determine the importance of changing risk premia and returns variability over time.[1] This paper models, estimates, and tests for the importance of time-varying risk premia and returns variability within the context of a *conditional* version of the Sharpe-Linter-Mossin CAPM.

The main contribution of this paper is its demonstration of how the generalized method of moments (GMM) can serve as a convenient alternative to maximum likelihood estimation of simultaneous equation systems of autoregressive conditionally heteroskedasticity (ARCH) models. The GMM is attractive in these applications because it allows the econometrician to avoid estimating a potentially large number of nuisance parameters. The opportunities for misspecification are also diminished because it is unnecessary to parameterize and estimate many features of the model that are only of incidental interest.

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[1]Research on a changing risk premia begins with Fama and Macbeth (1974). Work on the importance of changing returns variability includes Merton (1980), Christie (1982), Hasbrouck (1986), Bollerslev (1987), Fama and French (1988a), Lo and MacKinlay (1988), and Schwert (1989). Engle, Lilien, and Robins (1987), French, Schwert, and Stambaugh (1987), Campbell (1987), Genotte and Marsh (1987), and Bollerslev, Engle, and Wooldridge (1988) relate a market risk premium to changing returns variability.

The conditional CAPM provides a convenient way to incorporate the time-varying conditional variances and covariances that other researchers have found to be important in financial time series. An asset's beta in the conditional CAPM can be expressed as the ratio of the conditional variance between the forecast error in the asset's return and the forecast error of the market return and the conditional variance of the forecast error of the market return. We incorporate conditioning information by modeling these two components of the asset's beta as ARCH processes introduced by Engle (1982). Hansen and Richard (1987) have demonstrated that omission of conditioning information, as is done in tests of constant beta versions of the CAPM,[2] can lead to erroneous conclusions regarding the conditional mean-variance efficiency of a portfolio. Although a large literature reports the statistical violations of the unconditional CAPM, these results do not necessarily imply that the conditional CAPM is false. We adopt a pragmatic view regarding the usefulness of the CAPM. Most financial economists would agree that the CAPM is not literally true. One problem with the model is that it was originally derived in a static framework and can be shown to hold in an intertemporal setting only under restrictive assumptions.[3] No theory can provide an exact description of the real world, but in our view the CAPM might serve as a useful benchmark model of relative asset returns if it can be shown to be generally consistent with the data. Because the CAPM is both a simple and a practical theory, an investigation of the extent to which the conditional CAPM explains the data seems worthwhile.

We summarize our main results here. First, we find that the conditional CAPM and a purely statistical representation of conditional first and second moments cannot adequately explain the data. The GMM specification tests of orthogonality conditions not used in estimation find evidence of omitted variables in a model where the conditional second moments are modeled as ARCH processes and the market excess return is modeled as an autoregression. Current and lagged Treasury bill returns, dividend yields, low-grade corporate bond yields, lagged low-grade bond default premia, and lagged market conditional variances are likely to be important variables omitted

[2]This research is typified by Black, Jensen, and Scholes (1972), Blume and Friend (1973), and Fama and MacBeth (1974) and is elegantly summarized and critiqued in Roll (1977). Frankel and Dickens (1984), Frankel (1985a,b), Rayner (1985), Gibbons and Ferson (1985), and Ferson, Kandel, and Stambaugh (1987) have investigated the CAPM by relaxing the assumption of constant expected returns while maintaining the constant variance and covariance assumption. Engel and Rodrigues (1987), Bollerslev, Engle, and Wooldridge (1988), Mark (1988), Harvey (1989), Huang (1990), Shanken (1990), and Ng (1991) have further relaxed the assumption of constant covariances.

[3]The traditional CAPM can be shown to be consistent with optimal intertemporal investment decision making under any of the following three conditions: i) investors have logarithmic utility, ii) the return on the market portfolio and the riskless rate of interest are observable and conditionally efficient, and iii) the return on the market matches the r_m^* of Hansen, Richard, and Singleton (1982), which is equal to $m/(E(m^2 \mid I)$, where m is the marginal rate of intertemporal substitution and I, is the information set. Also see Merton (1973), Long (1974), Rubinstein (1976), and Breeden (1979).

from this statistical representation of the conditional CAPM. We find, however, that incorporating a January dummy variable in the mean excess returns goes a long way in accounting for the effects of the omitted variables. Allowing the means to shift in this fashion is admittedly not implied by the CAPM. Nevertheless, this ad hoc specification obviates the need to embark on a potentially exhausting specification search. We find, in our final specification of the conditional CAPM augmented by shifting mean excess returns in January, significant ARCH components in the betas and a previously undocumented quarterly component in the volatility of the market portfolio. We are also left with an unexplained dividend yield component in the smaller firm's mean excess returns.

The remainder of the paper is structured as follows. The conditional CAPM is presented in the first section. The econometric specification is discussed in Section II, and the empirical methodology is discussed in Section III. Section IV describes the data. The empirical results are reported in Section V, and some concluding remarks are contained in Section VI.

I. The Conditional CAPM

Let $R_{i,t}$ be the date t nominal return on asset i, $(i = 1, 2, \cdots, n)$, $R_{m,t}$ the date t nominal return on the market portfolio, and let $r_{i,t}$ and $r_{m,t}$ denote their returns in *excess* of the U.S. Treasury bill return. We begin by stating the conditional CAPM in excess returns form as

$$E(r_{it} \mid I_{t-1}) = \beta_{iI_{t-1}} E(r_{mt} \mid I_{t-1}), \tag{1}$$

where,

$$\beta_{iI_{t-1}} = \frac{\operatorname{cov}(R_{it}, R_{mt} \mid I_{t-1})}{\operatorname{var}(R_{mt} \mid I_{t-1})} = \frac{\operatorname{cov}(r_{it}, r_{mt} \mid I_{t-1})}{\operatorname{var}(r_{mt} \mid I_{t-1})}, \tag{2}$$

and $E(\mid I_{t-1})$ is the mathematical expectation conditioned on the information set available to investors at time $t - 1$, I_{t-1}. Expectations are rational in the sense of Muth (1961) so that mathematical expectations are interpreted as investor's subjective expectations. The second equality in equation (2) follows because the Treasury bill return (or the nominal risk-free rate) during period t is known at time $t - 1$ and hence is included in I_{t-1}. The conditional CAPM allows asset i risk premium to vary over time as a result of time-variation in three components: the market's conditional variance, the conditional covariance between the asset's return and the market's return, and/or the market's risk premium.

Let J be the information available to the econometrician. Most likely, J will contain less information than I. Our analysis draws on Hansen, Richard, and Singleton (1982) who show that if the CAPM holds conditioned on a subset J of the information set I, then the CAPM holds conditioned on I. This result implies that evidence in favor of the CAPM conditioned on I is obtained if the CAPM conditioned on J is not rejected. Unless additional assumptions are made (e.g., constant betas), however, the implication does not go in the other direction so that we need not reject the CAPM conditioned on I if the model conditioned only on J is rejected. For example, the unconditional CAPM obtains when J is the null set, but one cannot use the evidence against the unconditional CAPM to claim that the data also rejects the conditional CAPM.[4] For the remainder of the paper, the null hypothesis will be that the model conditioned on J is true. That is,

$$E(r_{it} \mid J_{t-1}) = \beta_{iJ_{t-1}} E(r_{mt} \mid J_{t-1}), \tag{3}$$

where,

$$\beta_{iJ_{t-1}} = \frac{\operatorname{cov}(R_{it}, R_{mt} \mid J_{t-1})}{\operatorname{var}(R_{mt} \mid J_{t-1})} = \frac{\operatorname{cov}(r_{it}, r_{mt} \mid J_{t-1})}{\operatorname{var}(r_{mt} \mid J_{t-1})}. \tag{3'}$$

Now decompose the return on asset i and the market into their forecastable and unforecastable components, to obtain

$$r_{it} = \beta_{iJ_{t-1}} E(r_{mt} \mid J_{t-1}) + u_{it}, \qquad i = 1, \cdots, n, \tag{4}$$

$$r_{mt} = E(r_{mt} \mid J_{t-1}) + u_{mt}. \tag{5}$$

The forecast errors u_{it} and u_{mt} are orthogonal to the information set J_{t-1}. Notice that the sequences $\{u_{it}u_{mt}\}$ and $\{u_{mt}^2\}$ can themselves be decomposed into forecastable and unforecastable components as

$$u_{it}u_{mt} = E(u_{it}u_{mt} \mid J_{t-1}) + \eta_{it}, \qquad i = 1, \cdots, n, \tag{6}$$

$$u_{mt}^2 = E(u_{mt}^2 \mid J_{t-1}) + \eta_{mt}. \tag{7}$$

The forecastable part of the sequence $\{u_{it}u_{mt}\}$ is the conditional covariance between r_{it} and r_{mt}, and the forecastable part of the sequence $\{u_{mt}^2\}$ is the conditional variance of r_{mt}

$$\operatorname{cov}(r_{it}, r_{mt} \mid J_{t-1}) = E(u_{it}u_{mt} \mid J_{t-1}), \tag{8}$$

$$\operatorname{var}(r_{mt} \mid J_{t-1}) = E(u_{mt}^2 \mid J_{t-1}). \tag{9}$$

Now, substitute equations (2), (8), and (9) into equation (4) to obtain

$$r_{it} = \frac{E(u_{it}u_{mt} \mid J_{t-1})}{E(u_{mt}^2 \mid J_{t-1})} [E(r_{mt} \mid J_{t-1})] + u_{it}. \tag{10}$$

Once the conditional expectations have been parameterized, equations (5), (6), (7), and (10) form an estimable system of four equations for any asset $i = 1, \cdots, n$. For any n assets considered simultaneously, the model implies a system of $2(n + 1)$ equations.

[4]This is not to say that inferences based on the unconditional CAPM necessarily lead to erroneous inferences regarding the validity of the conditional CAPM. If the data did not reject the unconditional CAPM, this could be evidence in support of the conditional CAPM. However, the unconditional CAPM has been rejected in the literature, and we find conditioning information to be important. See also Hansen and Richard (1987).

II. An Econometric Model

A. *Modeling the Conditional Variance and Conditional Covariances*

To proceed with estimation and inference, we must parameterize the conditional expectations appearing in equations (5), (6), (7), and (10). In general, these conditional expectations will be nonlinear functions of the information set, but the theory is vague on the functional form of the conditional expectations. The parameter set we work with, while necessarily *ad hoc*, is guided by a concern for computational feasibility and the ability of the chosen specifications to fit the data. To keep the estimation problem tractable, we first assume that the sequence of squares and cross products of the return forecast errors $\{u_{mt}^2\}$ and $\{u_{it}u_{mt}\}$ can be represented by autoregressions of low order. That is,

$$E(u_{mt}^2 \mid J_{t-1}) = \gamma_0 + \sum_{j=1}^{s} \gamma_j u_{mt-j}^2 \tag{11}$$

$$E(u_{it}u_{mt} \mid J_{t-1}) = \alpha_{i0} + \sum_{j=1}^{k} \alpha_{ij} u_{it-j} u_{mt-j} \tag{12}$$

This assumption reflects the idea that *own* past observations on a random variable provide useful information for predicting future observations and is in the spirit of the ARCH modeling strategy of Engle (1982). To reduce our estimation load, we set the order of the market excess return's variance process s equal to the order of the covariance process k and test for the appropriate k in a step-wise manner. This specification conforms to Bollerslev's (1986) generalized ARCH (GARCH) (0, k).

B. *Modeling the Market Excess Return*

A natural process governing the market excess return also is a finite-order autoregression. The implied prediction formula is simply

$$E(r_{mt} \mid J_{t-1}) = \pi_0 + \sum_{j=1}^{h} \pi_j r_{mt-j}. \tag{13}$$

We estimated an autoregression (3) for the market excess return and found the first and third lags to be significant at the 5% level. The residuals from the autoregression did not appear to be serially correlated, suggesting the appropriateness of the model.

An alternative representation for the market excess return is the ARCH in the mean (ARCH-M) specification that has been used in the work of Campbell (1987), Engle, Lilien, and Robins (1987), French, Schwert, and Stambaugh (1987), Engel and Rodrigues (1987), and Bollerslev, Engle, and Wooldridge (1988). These representations attempt to exploit the tradeoff between conditional mean returns with their conditional variability. The

ARCH-M model can be expressed as

$$E(r_{mt} \mid J_{t-1}) = \Psi_0 + \Psi_1 f[\mathrm{var}(r_{mt} \mid J_{t-1})]. \qquad (14)$$

Merton (1980) provides a theoretical motivation for these ARCH-M processes. Three variants for the function f have appeared in the literature. First, f has appeared as an affine function. This specification would be appropriate if the market price of risk, which is defined as the ratio of the expected excess return to its conditional variance, is constant. Second, f has been specified as the square root function, which is implied if the ratio of the expected market excess return to its conditional standard deviation, or the Sharpe risk measure, is constant. Third, the logarithmic function has also been investigated.[5]

To choose among these models for the market risk premium, we rely on Davidson and MacKinnon's (1981) C-test to discriminate between two non-nested alternatives. Elaboration on this test and the results is deferred until after we have discussed the estimation methodology.

III. Estimation Methodology

Let there be n assets under consideration, which implies $p = 2(n + 1)$ equations. There are n equations for the mean of each asset return, n covariance equations, plus an equation for the market excess return process and the market's conditional variance. Denote the q-dimensional parameter vector to be estimated by β, its true value by β_o, and the p-dimensional innovation vector by $\delta_t(\beta_o)$. Let $z^j_{1,t-1}(\beta_o) \subset J_{t-1}$ serve as instrumental variables for equation $j = 1, 2, \cdots, p$. We allow the instruments to vary across equations by including in $z^j_{1,t-1}(\beta)$ only the variables appearing on the right hand side of the jth equation. The variables include lagged market excess returns, lagged square residuals, and lagged cross-products of residuals. We selected the instruments in this way for two reasons. First, the actual regressors appearing in a particular equation are natural choices for instruments. Restricting the instruments in this way allows us to keep the estimation problem manageable. Second, we wanted to attenuate the bias in the GMM estimates that results in a proliferation of instrumental variables. Tauchen (1986) and Ferson and Foerster (1990) find in Monte Carlo experiments that the GMM estimator tends to be biased when the instrument set becomes large in samples of that size that we encounter in practice.

Since $\delta_t(\beta_o)$ is a vector of forecast errors, it follows that $E[f_{1t}(\beta_o)] = 0$, where

$$f_{1t}(\beta_o) = \begin{bmatrix} \delta_{1t}(\beta_o) z^1_{1,t-1}(\beta_o) \\ \delta_{2t}(\beta_o) z^2_{1,t-1}(\beta_o) \\ \vdots \\ \delta_{pt}(\beta_o) z^p_{1,t-1}(\beta_o) \end{bmatrix}.$$

[5]See French, Schwert, and Stambaught (1987).

The GMM estimator b_T of β_o is the minimizer of the quadratic criterion function

$$\phi(b_T) = g_{1T}(\beta)' S_{11,T}^{-1} g_{1T}(\beta), \tag{15}$$

where $g_{1T}(\beta) = 1/T \sum_{t=1}^{T} f_{1t}(\beta)$, $S_{11,T} = 1/T \sum_{t=1}^{T} f_{1t}(b) f_{1t}(b)'$, and b is a consistent estimate of β_o. A consistent estimate of the b_T covariance matrix is given by $1/T(D_T' S_{11,T}^{-1} D_T)^{-1}$, where $D_T = (\partial/\partial b) g_{1T}(b_T)$. We used the two-step procedure suggested by Hansen and Singleton (1982) to arrive at our estimates.[6]

We test restrictions implied by the theory using Hansen's (1982) test of the orthogonality conditions used in estimation. He shows that $T[\min \phi(b_T)]$ is asymptotically (central) chi-square distributed with N-q degrees of freedom under the null hypothesis that the model is correctly specified. $N = \dim[f_{1t}(\beta)]$ is the number of orthogonality conditions used in estimation, and q is the number of parameters estimated.

Since Hansen's test is a test against an unspecified alternative, it can have low power against specific alternatives. Also, the test could fail to reject because of its selective use of information. That is, the theory implies many more orthogonality conditions than those used in estimation. We address these concerns by also subjecting the model to GMM tests of orthogonality conditions not used in estimation following a suggestion by Newey (1985).

Let $f_{2t}(\beta)$ be an $(s \times 1)$ vector of orthogonality conditions not used in estimation but implied by the model. We form $f_{2t}(\beta)$ by multiplying the residual by variables in the information set at date $t-1$, say $z_{2,t-1}^{j}(\beta)$ $(j = 1, 2, \cdots, p)$, not used as instrumental variables. Now, consider the statistic

$$CS = T\left[L_T g_T(b_T)\right]' Q_T^{-1} \left[L_T g_T(b_T)\right], \tag{16}$$

where

$$g_{2T}(\beta) = \frac{1}{T} \sum_{t=1}^{T} f_{2t}(\beta), \qquad g_T(\beta) = \left[g_{1T}(\beta)' g_{2T}(\beta)'\right]',$$

$$L_T = \left[0_{s\times(k_1 p+s)} \vdots I_s\right], \qquad S_{ij,T} = \frac{1}{T} \sum_{t=1}^{T} f_{it}(b) f_{jt}(b)', \; (i = 1,2; j = 1,2),$$

$$B_T = \left(H_{1,T}' S_{11,T}^{-1} H_{1,T}\right) H_{2,T}', \qquad H_{i,T} = \sum_{t=1}^{T} \frac{\partial}{\partial b} f_{iT}(b_T), \; (i = 1,2),$$

$$Q_T = S_{22,T} - S_{21,T} S_{11,T}^{-1} H_{1,T} B_T - B_T' H_{1,T}' S_{11,T}^{-1} S_{12} + H_{2,T} B_T,$$

and b_T is the minimizer of equation (15).

[6]Ferson and Foerster (1990) computed Monte Carlo distributions of Hansen's statistic for models estimated by the two-step procedure and by continued iteration on the weighting matrix S_{11} until convergence. There is no theoretical reason to continue iterating in this way, but Ferson and Foerster find for the examples they studied that the test will reject the null more often in the two-step procedure. Although these results cannot be directly applied to our problem, they suggest that we may be accepting a larger probability of committing a type *I* error.

Using the results in Newey (1985), the statistics CS can be shown to have an asymptotic central chi-square distribution with s degrees of freedom under the null. The vector $g_T(\beta)$ stacks all the orthogonality conditions together, while the design matrix L_T selects those orthogonality conditions used for testing. If we take the functional form of the model as a maintained hypothesis, these tests can be viewed as specification tests for omitted variables.

IV. Data

We take monthly observations on total equity returns for firms listed on NYSE and monthly Treasury bill yields. The estimation period covers 1926–1985. In addition, the GMM tests on orthogonality conditions not used in estimation (the omitted variables tests) exploit data on both the excess yield and the default premium of low-grade corporate bonds over Treasury bonds and the dividend yield on the CRSP NYSE value-weighted index in excess of the Treasury bill return. The sources for this data are the CRSP tapes for the equity return and dividend series, Fama's U.S. Government issue file for the Treasury bill time series, and Ibbotson Associates for the corporate and Treasury bond series.

The model is highly nonlinear, and the computational burden involved in estimation is potentially quite high. We therefore restrict to five the number of equity returns that we model. We create time-series returns for five value-weighted portfolios as the assets priced by the CAPM. These portfolios are created by value-ranking the traded equity returns in each month, splitting these returns into value-ranked quintiles, and then forming five portfolio returns based on value weights within a quintile.[7] The benchmark or market return that we use is the CRSP value-weighted market return.

V. Empirical Results

A. An AR(3) Conditional CAPM

Our first task is to select a model for the market risk premium. We seek to choose between a third-order autoregression and an ARCH-M with a third-order ARCH process. We estimate each of the three variants of the ARCH-M model discussed in Section II.B. by GMM using a constant and three lags of the squared residual as instrumental variables.

Table I reports the estimation results and Davidson and MacKinnon's (1981) C-test in comparing the third-order autoregression against each of the three variants of the ARCH-M model. As can be seen, all of the ARCH-M models are rejected in favor of the autoregressive representation. The table

[7]To the extent that large returns in terms of absolute value cause shifts in particular equities across the value-weighted portfolios, we will miss some of the variability in actual returns due to our weighting procedure (Fama and French (1988a)).

Table I

Estimates of AR and ARCH-M Models

Estimates of an AR(3) and a third order ARCH-M for monthly CRSP value-weighted NYSE excess returns r_{mt} 1926–1985. The two models are as follows:

$$AR\colon r_{mt} = \pi_0 + \sum_{j=1}^{3} \pi_j r_{mt-j} + u_{mt} \text{ and } ARCH\text{-}M\colon r_{mt} = \Psi_0 + \Psi_1 f\left[\gamma_0 + \sum_{j=1}^{3} \gamma_j u^2_{mt-j}\right] + u_{mt}.$$

Asymptotic standard errors are in parentheses. The three specifications of the function f estimated are, the identity, the square root, and the natural log functions. The ARCH-M models were estimated by GMM with a constant and three lags of u_t^2 forming the instrument vector. The AR model was estimated by OLS. The test of non-nested alternatives is the C-test suggested by Davidson and MacKinnon (1981) which involves computing the t-ratio from a composite regression. Robust standard errors for the OLS estimates and the C-test statistics were computed using White's (1980) correction for conditional heteroskedasticity.

	ARCH-M f [Var]			AR (3)	
	Var	$\sqrt{(\text{Var})}$	ln (Var)		
Ψ_0	0.0053	−0.0001	0.0044	π_0	0.0066
	(0.0043)	(0.0013)	(0.0069)		(0.0023)
Ψ_1	0.3410	0.1430	0.0006	π_1	0.1138
	(1.464)	(0.2667)	(0.0011)		(0.0668)
γ_0	0.0017	0.0016	0.0016	π_2	−0.0046
	(5.3×10^{-4})	(5.3×10^{-4})	(5.3×10^{-4})		(0.0596)
γ_1	0.1283	0.1392	0.1169	π_3	−0.1243
	(0.0956)	(0.0961)	(0.0981)		(0.0641)
γ_2	0.1047	0.1392	0.1169		
	(0.0950)	(0.0098)	(0.0963)		
γ_3	0.2264	0.2541	0.2579		
	(0.1570)	(0.1537)	(0.1465)		
R^2	0.0004	0.0027	0.0031		0.0286

Tests of the null against a non-nested alternative: C-Test Statistic.

Null Hypothesis			
ARCH-M	2.428	2.548	2.521
AR (3)	0.072	0.182	0.165

also shows the ratio of the variance of the one-step-ahead prediction to the variance of the actual market excess returns (denoted by $R^2 = \text{var}[E(r_{mt} \mid J_{t-1})]\text{var}[r_{mt}]$) as a measure of the model's goodness of fit. Although the autoregressive model explains less than 3% of the variation in the data, the explanatory power of the ARCH-M models are modest by comparison. Based on these results, we adopt the autoregressive representation for the market excess return for the remainder of the empirical analysis.[8]

[8] A referee has pointed out that these results are consistent with the findings of Campbell (1987) and Harvey (1989) who reject that the market premium is proportional or linear in the market volatility.

We now move on to estimate the full model. The following instrumental variables were used to estimate the model. For each of the mean equations (i.e., the u residuals), a constant and three lags of the market excess return were used. For each of the conditional covariance equations and the conditional variance equation, a constant and k lags of the dependent variable were used. Using Hansen's test of the overidentifying restrictions, both $k = 1$ and $k = 2$ for these equations are rejected at very small significance levels. Setting $k = 3$, on the other hand, works much better, and these results are reported in Table II.

We make a number of comments in regard to Table II. First, the parameter estimates appear to be reasonable both in magnitude and in sign. The first lag on the market excess return, π_1, is significant and positive, while the third lag, π_3, is negative and marginally significant. These estimates are consistent with the estimated autoregression reported in Table I. Second, each of the lags in the estimate of the market excess return's conditional variance are significantly different from zero. The estimate for the third order lag, γ_3, is 0.2617 and suggests the presence of a quarterly component in market volatility. To our knowledge, this result has not previously been discussed in the literature. Third, estimates of the constant terms and the first and third order lag coefficients in the conditional covariance equations are all positive and estimated with a fair amount of precision.

Hansen's test of the orthogonality conditions used in estimation yields a chi-square statistic of 32.85. With 20 degrees of freedom, the p-value is 0.035.[9] Given that the model appears, loosely speaking, to *fit* the data, we proceed to investigate a number of additional features of the estimates.

First, a CAPM with constant betas is strongly rejected by the data. A Wald test of the restriction that the lag coefficients in the conditional covariance, conditional variance, and market excess return equations are jointly zero, (i.e., $(\pi_1, \cdots, \pi_3, \alpha_{11}, \cdots, \alpha_{53}, \gamma_1, \cdots, \gamma_3)' = 0$) yields a Wald Statistic of 98.7. With 18 degrees of freedom, the null hypothesis of no time variation in the CAPM is rejected at any reasonable significance level. We also investigate whether there is significant time variation of the market price of risk. A test that the lag coefficients in the market's excess return and conditional variance are jointly zero also rejects the null ($\chi^2_{(6)} = 14.4$, p-value $= 0.026$).

B. Residual Diagnostics

In this section, we study properties of the model's residuals by applying Newey's GMM tests of orthogonality conditions implied by the model but *not* imposed in estimation. The computed values of the CS statistic are reported in Table III. The orthogonality conditions for each equation are examined individually.

First, we test whether the residuals of each equation are orthogonal to six

[9]Since we have a system of 12 equations, this instrument set results in 48 orthogonality conditions. Because there are 28 parameters to estimate, there are 20 overidentifying restrictions and hence 20 degrees of freedom in the test of the orthogonality conditions.

Table II

AR(3) Conditional CAPM

Estimates of the conditional CAPM for monthly excess returns on five sized ranked NYSE portfolios (r_{it}, $i = 1, 2, \cdots, 5$), 1926–1985. The value-weighted CRSP index less the Treasury bill return r_{mt} serves as the market excess return.

$$r_{it} = \left[\alpha_{i0} + \sum_{j=1}^{3} \alpha_{ij} u_{it-j} u_{mt-j} / \gamma_0 + \sum_{j=1}^{3} \gamma_j u_{mt-j}^2\right]\left[\pi_0 + \sum_{j=1}^{3} \pi_j r_{mt-j}\right] + u_{it},$$

$$u_{it} u_{mt} = \alpha_{i0} + \sum_{j=1}^{3} \alpha_{ij} u_{it-j} u_{mt-j} + \eta_{it}, \qquad r_{mt} = \pi_0 + \sum_{j=1}^{3} \pi_j r_{mt-j} + u_{mt},$$

$$u_{mt}^2 = \gamma_0 + \sum_{j=1}^{3} \gamma_j u_{mt-j}^2 + \eta_{mt}.$$

The model is estimated by GMM. Hansen's (1982) statistic is used to test the orthogonality conditions used in estimation, and Wald statistics are constructed to test cross equation restrictions implied by the conditional CAPM. Numbers in parentheses are asymptotic standard errors. The instrumental variables used to estimate the model are as follows: for equations with the u residuals, a constant and three lags of the market excess return; for each of the conditional covariance equations and the conditional variance equation, a constant and 3 lags of the dependent variable.

Market Process Parameters				
	π_0	π_1	π_2	π_3
Conditional mean	0.0089*	0.0806*	0.0183	−0.0817
	(0.0016)	(0.0023)	(0.0022)	(0.0045)
	γ_0	γ_1	γ_2	γ_3
Conditional variance	0.0012*	0.2842*	0.1113*	0.2617*
	(2.4×10^{-4})	(0.0544)	(0.0463)	(0.0686)
Conditional Covariance Parameters				
Portfolio	α_0	α_1	α_2	α_3
1	0.0031*	0.5369*	−0.2579*	0.1671*
	(4.0×10^{-4})	(0.0818)	(0.0568)	(0.0739)
2	0.0021*	0.4698*	−0.1139*	0.2357*
	(3.0×10^{-4})	(0.0565)	(0.0421)	(0.0689)
3	0.0018*	0.4228*	−0.2013	0.2437*
	(2.8×10^{-4})	(0.0503)	(0.0450)	(0.0648)
4	0.0016*	0.4425*	0.0000	0.2175*
	(2.3×10^{-4})	(0.0634)	(0.0483)	(0.0698)
5	0.0011*	0.3474*	0.0729	0.2688*
	(2.3×10^{-5})	(0.0561)	(0.0484)	(0.0727)
Test of		χ^2 (d.f.)	d.f.	p-value
Orthogonality conditions		32.9*	20	0.035
Constant beta		98.7*	18	0.000
Constant market price of risk		14.4*	6	0.026

*Significant at the 5% level.

Table III

AR(3) Conditional CAPM Diagnostic Tests

GMM tests of the hypothesis that residuals from the AR (3) conditional CAPM are orthogonal to observations not used in estimation. The observations are, six lags of the own portfolio's excess return, six lags of the residual, the current and two lags of the T-bill return, low-grade corporate bond yield, dividend yield, three lags of the low-grade corporate bond default premium, one lagged market conditional variance, and monthly dummy variables. We use monthly observations for 1926–1985. The sources for this data are the CRSP tapes for the equity return and dividend series, Fama's U.S. Government issue file for the Treasury bill time series, and Ibbotson Associates for the corporate and Treasury bond series. The test statistic is distributed as a chi-square variate under the null hypothesis.

	Orthogonality Conditions							
Residual	(1) Lag return	(2) Lag residuals	(3) T-bill	(4) Default premium	(5) Low-grade yield	(6) Dividend yield	(7) Market variance	(8) Monthly dummies
returns								
u_1	3.664	4.123	9.678	4.579	5.429	18.020	3.660	28.425
u_2	3.373	3.347	9.889	4.050	4.815	13.101	2.436	23.208
u_3	3.369	3.842	8.716	3.637	4.581	9.031	1.539	20.771
u_4	4.956	5.132	9.489	2.860	3.594	5.542	1.219	17.248
u_5	6.612	7.071	10.489	1.749	2.424	4.685	0.126	18.171
u_{m}	5.986	6.468	10.801	1.891	2.741	4.997	0.304	17.585
covariances								
η_1	13.715	9.872	15.374	5.676	4.079	4.090	0.902	20.933
η_2	16.188	9.575	11.326	5.624	5.960	3.111	0.764	26.634
η_3	12.591	14.840	10.112	4.464	6.425	1.832	5.857	29.736
η_4	11.405	9.867	8.470	4.042	7.339	1.686	6.197	29.641
η_5	9.819	17.110	6.015	4.357	6.831	1.422	7.805	27.832
variance								
η_m	13.319	16.861	6.107	4.720	6.645	1.650	4.998	27.768
degrees of freedom	6	6	3	3	3	3	1	12
critical values								
5%	12.59	12.59	7.81	7.81	7.81	7.81	3.84	21.03
1%	16.81	16.81	11.3	11.3	11.3	11.3	6.63	26.22

lags of that portfolio's excess return. That is, we ask whether the residual on the return of portfolio j, u_{jt}, is orthogonal to six lags of portfolio j excess return, r_{jt}, and whether the residual from the conditional covariance between portfolio j and the market, η_{jt}, is orthogonal to six lags of the excess return on portfolio j. Column 1 of Table III reports CS statistics for the test of these six orthogonality conditions. It can be seen that marginal rejections of the null occur for the conditional covariances between the first three portfolios and the market.

Second, we investigate whether residuals from each equation are orthogonal to six of their own lags. CS statistics for this test are reported in column 2. Again, there is only mild evidence that the residuals are serially correlated.

A number of variables have been shown in the literature to help predict excess stock returns and returns volatility. The variables include Treasury bill returns (Fama and Schwert (1977)), dividend yields (Fama and French (1988b)), low-grade corporate bond yield premia (Keim and Stambaugh (1986)) and default premia (Fama (1990)), and lagged conditional second moments (Bollerslev, Engel, and Wooldridge (1988) and Giovannini and Jorion (1989).[10] The evidence from these studies raises the possibility that important information has been omitted from our conditioning set.

We investigate the significance of omitting these variables from our model by testing the orthogonality of each residual to the following variables: the current and two lags of the Treasury bill return, low-grade corporate bond yield premia, the market's dividend yield, three lags of low-grade corporate bond default premia, and one lag of the market's conditional variance.[11] These test results appear in columns 3 through 7. The default and yield premia on the low-grade corporate bonds do not appear to contain information beyond our conditional CAPM for predicting excess returns. However, there is evidence that each of the residuals is correlated with the Treasury bill returns. The residuals of the mean return equations for portfolios 1 through 3, (u_1, u_2, u_3), appear to be correlated with dividend yields, and the test rejects the orthogonality of the lagged market conditional variance to residuals of the covariance and market variance equations (η_3, η_4, η_5, η_m).

Finally, we investigate the possibility that a deterministic component remains in the data. It is important to do so because deterministic parts of the return series should be removed or otherwise accounted for in order to draw correct inference. Column 8 reports statistics that test whether each residual is orthogonal to a constant and 11 monthly dummies. These results strongly suggest the presence of a deterministic component in the data not captured by the model.

To summarize, this section finds evidence that the simple autoregression (3) model is misspecified. Current and lagged Treasury bill returns and dividend yields and lagged conditional second moments appear to contain information for predicting returns beyond that in our conditional CAPM. In addition, a deterministic component also appears to remain in the data. These results suggest that we augment the conditioning set of our model with some of these variables. However, the nonlinear nature of our conditional CAPM renders it an inconvenient vehicle for conducting a specification search among the potential omitted variables. Instead, we pursue an analysis that focuses on accounting for the deterministic component of the data. Beginning with Banz (1981), a large amount of attention has been devoted to

[10] See also, Campbell (1987), Harvey (1989), and Huang (1990).

[11] Date t values of the Treasury bill return, the dividend and corporate bond yields, are in J_{t-1}.

the month of January, and it is this aspect of the deterministic part of the returns that we concentrate on. Incorporating a January effect in the model may be an efficient way to explain the apparent omission of economic variables from the model of this section. In addition, failure to remove any deterministic components from the data may lead to a violation of the regularity conditions assumed in the statistical theory that we draw on.[12]

C. Accounting for Omitted Variables with a January Dummy Variable

This section introduces a variable that assumes a value of unity during January and zero otherwise. We are first confronted with deciding where in the model to include the January dummy. One possibility that preserves the basic CAPM structure is to incorporate a monthly shift in the conditional covariances, the market excess return, and the market conditional variance. Modeling a monthly shift in the excess return separate from the beta and/or the expected market excess return, on the other hand, is *ad hoc* and is inconsistent with the CAPM. We leave it to the data to determine the appropriate placement of the January dummy in the model by estimating a system of equations that introduces a January dummy in each of the aforementioned equations for the basic autoregression (3) model and testing exclusion restrictions on the different sets of dummy variables. The instrumental variables used to compute the values reported in Table IV are slightly different. For each of the equations, the constant was replaced by the January dummy. The remaining instruments are the same as those used in computing Table II.[13]

A Wald test that all the January coefficients in the conditional covariances, the market's excess return process, and the market's conditional variance are jointly zero yields a chi-square statistic (with 7 degrees of freedom) of 1.465. This result suggests that a January shift in these equations appears not to be an important characteristic of the data. A Wald test that the *ad hoc* January coefficients in the asset return equations are jointly zero yields a Wald statistic of 11.039 (p-value = 0.0506). These results suggest that the monthly shifting of returns in the data is better modeled by introducing an *ad hoc* January dummy that shifts mean returns than by allowing January shifts in the components of the beta or in the market risk premium. Although these results provide evidence against the model, it may still be of interest to examine the model with *only* the *ad hoc* January shifts in mean returns, even though these shifts are not implied by the CAPM. The estimates of this model are reported in Table IV.

[12]Specifically, the distribution theory assumes that the observations are indeterministic and covariance stationary. A referee has pointed out that the returns processes may fail to be stationary if the seasonal component is not properly accounted for. There is another form of nonstationarity that we do not investigate, that is, a switch in the probability law governing the observations during the sample period. While it is possible that we would obtain significantly different estimates across different sample periods, we did not pursue a subsample analysis.

[13]A set of 48 orthogonality condition still results; there are 33 parameters to estimate, which implies 15 degrees of freedom in the test of the orthogonality conditions.

Table IV

January Dummy Augmented AR(3) Conditional CAPM

Estimates of the conditional CAPM augmented by a January shift (Jan_i, $i = 1, 2, \cdots, 5$) in the means for monthly excess returns on five size ranked NYSE portfolios (r_{it}, $i = 1, 2, \cdots, 5$), 1926–1985. The value-weighted CRSP index less the Treasury bill return r_{mt} serves as the market excess return.

$$r_{it} = Jan_i + \left[\alpha_{i0} + \sum_{j=1}^{3} \alpha_{ij} u_{it-j} u_{mt-j} / \gamma_0 + \sum_{j=1}^{3} \gamma_j u_{mt-j}^2\right]\left[\pi_0 + \sum_{j=1}^{3} \pi_j r_{mt-j}\right] + u_{it},$$

$$u_{it} u_{mt} = \alpha_{i0} + \sum_{j=1}^{3} \alpha_{ij} u_{it-j} u_{mt-j} + \eta_{it}, \qquad r_{mt} = \pi_0 + \sum_{j=1}^{3} \pi_j r_{mt-j} + u_{mt},$$

$$u_{mt}^2 = \gamma_0 + \sum_{j=1}^{3} \gamma_j u_{mt-j}^2 + \eta_{mt}.$$

The model is estimated by GMM. Hansen's (1982) statistic is used to test the orthogonality conditions used in estimation, and Wald statistics are constructed to test cross equation restrictions implied by the conditional CAPM. Numbers in parentheses are asymptotic standard errors. The instrumental variables used to estimate the model are as follows: for equations with the u residuals, the January dummy and three lags of the market excess return; for each of the conditional covariance equations and the conditional variance equation, a constant and three lags of the dependent variable.

Market Process Parameters					
	π_0	π_1	π_2	π_3	
Conditional mean	0.0073	0.0227	0.0237	−0.1062*	
	(0.0018)	(0.0235)	(0.0321)	(0.0466)	
	γ_0	γ_1	γ_2	γ_3	
Conditional variance	0.0015*	0.3228*	0.0645	0.1773*	
	(3.0×10^{-4})	(0.0607)	(0.0594)	(0.0836)	
Conditional Covariance Parameters					
Portfolio	α_0	α_1	α_2	α_3	JAN
1	0.0027*	0.4036*	−0.0735	0.0843	0.0756*
	(4.9×10^{-4})	(0.0842)	(0.0494)	(0.0724)	(0.0125)
2	0.0022*	0.3759*	−0.0451	0.1510	0.0419*
	(3.9×10^{-4})	(0.0648)	(0.0503)	(0.0849)	(0.0089)
3	0.0018*	0.4032*	−0.0119	0.1700*	0.0267*
	(3.6×10^{-4})	(0.0597)	(0.0567)	(0.0784)	(0.0071)
4	0.0016*	0.4707*	0.0078	0.1533	0.0112
	(3.9×10^{-4})	(0.0761)	(0.0553)	(0.0815)	(0.0059)
5	0.0013*	0.3680*	0.0583	0.1861*	−0.0018
	(3.0×10^{-4})	(0.0676)	(0.0616)	(0.0863)	(0.0043)

Test of	χ^2 (d.f.)	d.f.	p-value
Orthogonality conditions:	13.8	15	0.542
January coefficients zero:	15.5	5	0.009
Constant beta:	73.3	18	0.000
Constant market price of risk:	10.9	6	0.092

*Significant at the 5 percent level.

We find that Table IV is qualitatively not much different from Table II. The parameter estimates appear largely unchanged but are less precise. The test of the orthogonality conditions used in estimation has a much lower marginal significance level, indicating that inclusion of the January dummies leads statistically to a better fit. The coefficients on the January dummy are largest for the smallest firm portfolio, as expected *a priori*. The coefficients are significantly different from zero only for the first three portfolios. The Wald test that the coefficients on the January dummies are jointly zero yields a Wald statistic of 15.5 (p-value 0.085). As was found in Table II, a CAPM with constant betas can still easily be rejected ($\chi^2_{(18)} = 73.3$). The hypothesis that the market price of risk is constant is not strongly rejected ($\chi^2_{(6)} = 10.9$, p-value $= 0.092$). We decided not to restrict the model any further, however, because the absolute size of the t-ratios for the estimates of π_3, γ_1, and γ_3 indicate that they are individually significant.

The GMM test results on the residuals are reported in Table V. As in Table III, we continue to find little evidence that the residuals are serially correlated. Nor do they appear correlated with past returns, the low-grade corporate bond yield premia, or the bond default premia. The January dummy appears to have accounted for the deterministic monthly component in returns, as none of the statistics in column 8 are significant at the 5% level. In addition, we no longer find evidence that the residuals are correlated with the Treasury bill returns or the lagged market conditional variance. The January-augmented CAPM leads to a significant improvement in fit, and goes a long way in explaining the data. It does not, however, fully explain the data, as it appears that dividend yields still contain information beyond the January-augmented CAPM for predicting the excess returns of portfolios 1 through 3.

We conclude this section by comparing the fit between the models with and without the January dummy variable as well as with the fit of a constant beta model. We continue to model the market risk premium as AR(3) in the constant beta model, and unconditional estimates were used to compute the betas. In this model, the variation in the one-step ahead forecasts of the model arise solely due to variations in the predicted market excess return. We compute pseudo R^2 values ($\mathrm{var}\{E[r_{it} \mid J_{t-1}]\}/\mathrm{var}[r_{it}]$) as the measure of the goodness of fit provided by the three models. Monthly asset returns are difficult to forecast. Although the proportion of the variation explained is somewhat modest, modeling time variation of the betas is an improvement over the constant beta model. A large improvement in prediction for the smaller firms is obtained with the January dummy variables (see Table VI).

VI. Conclusion

In the conditional CAPM, an asset's beta is the ratio of the conditional covariance between the asset and market returns and the conditional variance of the market return. This paper modeled these conditional covariances

Table V

January Dummy Augmented AR(3) Conditional CAPM Diagnostic Tests

GMM tests of the hypothesis that residuals from the January dummy augmented AR(3) conditional CAPM are orthogonal to observations not used in estimation. The observations are, six lags of the own portfolio's excess return, six lags of the residual, the current and two lags of the T-bill return, low-grade corporate bond yield, dividend yield, three lags of the low-grade corporate bond default premium, one lagged market conditional variance, and monthly dummy variables. We use monthly observations for 1926–1985. The sources for this data are the CRSP tapes for the equity return and dividend series, Fama's U.S. Government issue file for the Treasury bill time series, and Ibbotson Associates for the corporate and Treasury bond series. The test statistic is distributed as a chi-square variate under the null hypothesis.

	Orthogonality Conditions							
Residual	(1) Lag return	(2) Lag residuals	(3) T-bill	(4) Default premium	(5) Low-grade yield	(6) Dividend yield	(7) Market variance	(8) Monthly dummies
returns								
u_1	5.508	5.158	4.907	3.313	5.210	18.074	2.415	10.028
u_2	5.924	6.181	6.248	3.018	4.015	12.418	1.869	9.788
u_3	6.219	6.544	5.038	2.626	3.841	8.904	1.630	10.541
u_4	6.296	6.793	5.174	1.909	3.208	6.171	1.620	11.439
u_5	8.202	8.316	7.456	0.962	1.794	5.587	0.595	14.586
u_m	7.791	7.983	7.350	1.172	2.108	6.228	0.800	14.072
covariances								
η_1	12.472	3.653	8.569	5.889	5.735	5.182	0.942	14.714
η_2	13.675	3.352	7.471	5.515	5.717	4.986	0.519	17.288
η_3	11.022	3.998	6.425	4.381	4.626	3.497	0.714	17.836
η_4	9.830	6.328	5.574	3.850	5.719	2.494	3.171	19.428
η_5	9.207	5.616	5.121	4.469	5.092	2.464	2.804	19.071
variance								
η_m	11.091	4.393	5.250	4.609	4.872	2.829	1.202	19.067
degrees of freedom	6	6	3	3	3	3	1	12
critical values								
5%	12.59	12.59	7.81	7.81	7.81	7.81	3.84	21.03
1%	16.81	16.81	11.3	11.3	11.3	11.3	6.63	26.22

and variances as ARCH processes and the market risk premium as an autoregression. We showed how a large system of equations with ARCH features can be estimated in a straightforward way by GMM. The estimation strategy offers some concrete advantages over maximum likelihood methods in that it frees the investigator from having to parameterize and estimate many features of the ARCH model that are of only incidental interest. In addition to reducing the computational burden, the danger of misspecification is lessened.

Table VI

Proportion of Total Excess Returns Variation Explained by Fitted Values

This table reports the proportion of portfolio i excess return variability explained by the fitted values, $\mathrm{var}\{E[r_{it} \mid J_{t-1}]\}/\mathrm{var}[r_{it}]$. We estimate $E[r_{it} \mid J_{t-1}]$ by $r_{it} - u_{it}$ from each of the following three models. First, the January-augmented AR(3) CAPM is

$$r_{it} = Jan_i + \left[\alpha_{i0} + \sum_{j=1}^{3} \alpha_{ij} u_{it-j} u_{mt-j}/\gamma_0 + \sum_{j=1}^{3} \gamma_j u_{mt-j}^2\right]\left[\pi_0 + \sum_{j=1}^{3} \pi_j r_{mt-j}\right] + u_{it},$$

$$u_{it} u_{mt} = \alpha_{i0} + \sum_{j=1}^{3} \alpha_{ij} u_{it-j} u_{mt-j} + \eta_{it}, \quad (i = 1, 2, \cdots, 5), r_{mt} = \pi_0 + \sum_{j=1}^{3} \pi_j r_{mt-j} + u_{mt}$$

$$u_{mt}^2 = \gamma_0 + \sum_{j=1}^{3} \gamma_j u_{mt-j}^2 + \eta_{mt}.$$

Second, the conditional AR (3) CAPM is obtained by setting $Jan_i = 0$, $(i = 1, 2, \cdots, 5)$. Third, a constant beta CAPM is obtained by setting $\gamma_j = \alpha_{ij} = 0$, $(i = 1, 2, \cdots, 5;\ j = 1, 2, 3)$. These CAPM's were estimated by GMM using monthly observations for 1926–1985 on five size ranked portfolio returns in excess of the Treasury bill return $(r_{it},\ i = 1, 2, \cdots, 5)$. The value-weighted CRSP index less the Treasury bill return r_{mt} serves as the market excess return.

Portfolio	Conditional AR(3) CAPM	Conditional AR(3) CAPM with January dummies	Constant beta CAPM
1	0.0205	0.0469	0.0094
2	0.0167	0.0281	0.0111
3	0.0159	0.0212	0.0122
4	0.0170	0.0140	0.0129
5	0.0148	0.0124	0.0135

Relative to other recent tests of models with time-varying risk and/or returns, our results appear to be more supportive of the conditional CAPM. There are important differences between our paper and others that have appeared in the literature that give rise to the differing results. Our model also differs from Ng (1991), which appears in this volume, in a number of important respects.

We note first that both our model and Ng's extend the literature by allowing the market price of risk to vary over time. Since Ng models the market risk premium as an ARCH-M process while we model it as an autoregression, the two papers place different restrictions on the allowable dynamics for the market price of risk. Second, Ng uses market value weights as data and nests the model of Bollerslev, Engle, and Wooldridge (1988) and Harvey (1989), which assume a constant market price of risk, as a special case. In our model, we are unable to test the restrictions regarding how the model aggregates. The market return for each observation is a value-weighted average of observations of the five portfolio returns that we investigate. The effect of changing market-value weights is incorporated in the conditional covariance and market variance equations by construction. Introducing the associated weighting terms into the market excess return equation requires complicated recursions. Our approach leaves no tractable way to implement and, hence, test these additional value-weight restrictions. Third, Ng assumes that the innovations from her model follow a GARCH(1, 1) process while we adopt a third order ARCH process. The fourth major difference concerns the estimation strategy; Ng estimates her model by maximum likelihood while we adopt the GMM methodology.

As we have argued above, our approach is relatively parsimonious. However, our test results may be driven by a lack of power. This lack of power could arise from the limits that we set on the information set and the relatively small cross-section of assets considered. We accept these constraints for three reasons. First, because we simultaneously model time-varying covariances, market excess returns, and market variance, the resulting estimation problem is highly nonlinear and computationally intensive. Second, we supplement the usual GMM tests with residual diagnostics for alternatives that have been suggested in previous research. Third, the Monte Carlo analysis of Tauchen (1986) and Ferson and Foerster (1990) indicates that small-sample properties of the GMM estimator are better for smaller cross-sections and information sets.

We detected the presence of a deterministic component in returns and modeled it with a January dummy variable. Based on the test of the orthogonality conditions used in estimation, this model was found to fit the data relatively well. We found strong evidence of time variation in the conditional first and second moments of excess stock returns. The first- and third-order lags in the conditional variance of the market risk premium, as well as in the conditional covariance between the returns of five value-weighted portfolios and the market were found to be significant. These results suggest that monthly and quarterly variability components are priced in equity excess returns. The quarterly component may be evidence of an information effect corresponding to quarterly release of news in corporate and governmental statistical reporting. For example, the government makes GNP figures available on a quarterly basis. Corporation and investment managers may adjust their financial decisions for purposes of affecting their quarterly reports. Expected returns and returns variability might be affected to the extent that these factors influence market information and liquidity. The implications of our model held up relatively well under hypothesis testing. We temper our conclusions, however, as analyses of the residuals indicated that current and lagged dividend yields contain significant predictive information for the smaller sized firms beyond that contained in our conditional CAPM.

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