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UNCERTAINTY, LONG-RUN, AND MONETARY POLICY RISKS IN A TWO-COUNTRY MACRO MODEL*

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We study the currency risk premium and the forward premium bias in a two-country New Keynesian model with production, no physical capital, and recursive utility. Monetary policy follows an interest rate feedback rule and exogenous total factor productivity (TFP) growth follows a long-run risk process with stochastic volatility, which we estimate from data. With cross-country heterogeneity in TFP and monetary policy, reasonable currency risk premia emerge under complete and incomplete markets but the forward premium bias is trivial. We diagnose the challenge faced by this fairly standard production model to explain the forward premium bias.

1. INTRODUCTION

We study the currency risk premium (deviation from uncovered interest rate parity) and the forward premium bias in a two-country dynamic stochastic general equilibrium New Keynesian model with production, no physical capital, and recursive utility. Monetary policy follows a Taylor-type interest rate feedback rule and exogenous total factor productivity (TFP) growth follows a long-run risk process with stochastic volatility, which we estimate from Australian, Canadian, Japanese, and U.S. data. We examine complete market and incomplete market environments and export pricing conventions of local currency pricing (LCP), producer currency pricing (PCP), and dominant currency pricing (DCP). The model is driven by four exogenous shocks: an outright shock to productivity growth, a shock to long-run risk, a shock to stochastic volatility, and a monetary policy shock. In the article, country 1 is home and country 2 is foreign. The currency risk premium always refers to the expected excess return from borrowing currency 2 and lending in currency 1. The forward premium bias always refers to the deviation from 1 of the slope coefficient from regressing the future depreciation of currency 1 on the country 1 minus country 2 interest rate differential.

Under log-normality of the underlying shocks, the interest rate is linear in the log stochastic discount factor's (SDF) conditional mean and conditional variance. This conditional variance represents the precautionary saving component driving interest rate dynamics and we refer to it as an uncertainty-risk factor.[1] An increase in country 2's uncertainty-risk factor will raise their precautionary saving and lower country 2's interest rate. A positive currency risk premium emerges to incentivize country 1 to borrow from country 2 to satisfy their excess precautionary saving. Cross-country heterogeneity in monetary policy rules and productivity growth are shown to produce systematic uncertainty-risk factor differentials.

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[1] We refer to "uncertainty" not in the Knightian sense, but in the modern macro sense of high dispersion in the underlying probability distributions. Our analysis of the uncertainty-risk factor pays particular attention to the conditional variance of the log nominal SDF. It serves as a useful, preference-based measure of economic uncertainty, that accounts for attitudes toward risk and the psychological ease of intertemporal consumption substitution.

Under complete markets, the currency risk premium is determined entirely by this uncertainty-risk factor differential. One might think that the country with low uncertainty risk would be safe and its currency, serving as a hedge asset, would earn a negative currency risk premium. Instead, it is the high uncertainty-risk country that "pays" the currency risk premium, as in Ready et al. (2017). Under complete markets, factors that drive differences in the uncertainty-risk factor alter the relative amount of precautionary saving, and hence, the interest rate differential, but have no effect on the expected exchange rate depreciation. That is, the risk premium is the interest rate differential induced by differences in precautionary saving. This suggests why the carry trade (the strategy of shorting the low interest rate currency and investing in the high interest rate currency) is profitable. The source of the profit comes from the interest rate differential, not from currency movements. Under incomplete markets, the risk premium also depends on the conditional mean of the exchange rate "wedge" of Lustig and Verdelhan (2019), but the uncertainty-risk factor differential is the dominant factor.

The conditional mean of the log SDF component of the interest rate is what drives interest rates and exchange rates in a deterministic world. Fluctuations reflect how agents adjust their valuation of future consumption due to nonuncertainty-risk–related economic fluctuations. Any effects that variations in these components have on the interest rate differential are exactly offset by the expected exchange rate depreciation. Hence, they have no effect on the risk premium, just as there is no risk premium in a deterministic world.

The forward premium bias/anomaly, on the other hand, is a phenomenon that is distinctly different from the risk premium.[2] There is a forward premium bias when the slope coefficient in the regression of the one-step-ahead depreciation of country 1's currency on the interest rate differential between countries 1 and 2 deviates from 1. There is a forward premium anomaly is when the slope coefficient is negative. This regression will be referred to as the Fama regression (Fama, 1984), and the coefficient of interest will be denoted as β_F and referred to as the Fama coefficient. Under complete markets and log-normality, one of the terms in the numerator of the Fama coefficient is the covariance between the country 2 minus country 1 log SDF differential in conditional means and the country 2 minus country 1 log SDF differential in conditional variances. The forward premium anomaly will be present if the covariance between these differentials is a sufficiently large negative quantity. The forward premium anomaly/bias is a feature of covariances and not the risk premium per se. There can be a risk premium with no forward premium anomaly.

With heterogeneity in both monetary policy and productivity growth, the model can generate sizable risk premia with signs of mean values that correspond correctly to those found in the data. The model generates a modest forward premium bias, but no forward premium anomaly. This is the case under complete markets and incomplete markets and LCP, PCP, and DCP. Although endowment models under complete markets (Bansal and Shaliastovich, 2012; Backus et al., 2013) can generate the forward premium anomaly, we find this to be much more challenging in a conventionally specified two-country dynamic stochastic general equilibrium model with production. The challenge faced by our production model is, although the covariances between the "own" country conditional means and conditional variances are negative and sizable, the cross-country covariances are also negative and nearly the same size. When the cross-country covariances are subtracted from the "own" country covariances, the two effects largely offset each other and render the Fama coefficient near 1.

The remainder of the article is as follows. The next section discusses related literature. Section 3 provides a brief presentation of the model and Section 4 discusses how the currency risk premium and the forward premium bias emerges in our setup. Section 5 reports the model parameterization. The main results from the model are presented in Section 6 and Section 7. Section 8 concludes.

[2] The distinction between the forward premium bias and the currency risk premium was explored in a different context in Hassan and Mano (2019).

2. RELATED LITERATURE

Our article is part of an open economy modeling literature that features recursive utility in production models. In our model, productivity growth is subject to three shocks—a direct shock, a shock to a long-run risk component, and a shock to a stochastic volatility component. In contrast, productivity is a cointegrated random walk in Tretvoll (2018), Mumtaz and Theodoridis (2017), Berg and Mark (2019), and Kollmann (2019). Productivity growth in Colacito et al. (2018b) has long-run risk but no stochastic volatility. In Benigno et al. (2012), productivity growth has a stochastic volatility component in a common global productivity component but no long-run risk, and Gourio et al. (2013) have a disaster shock in productivity with recursive utility.

Research on international finance topics often work in endowment economies with recursive preferences and consumption growth processes that feature long-run risk and stochastic volatility. David et al. (2016) employ such a structure to study average returns to capital in emerging markets. Kollmann (2016) models a stochastic volatility component in consumption growth to study international risk sharing. Colacito et al. (2018a) is a multicountry endowment model where consumption growth featuring long-run risk and stochastic volatility are used to explain how the cross-section of currency risk premia emerge from cross-country variation in exposure to global endowment shocks.[3] Colacito et al. (2022) study the international transmission of volatility shocks in an endowment model. Bansal and Shaliastovich (2012) incorporate long-run risk and stochastic volatility in exogenous consumption growth and inflation. Backus et al. (2013) study the role of cross-country monetary policy heterogeneity in determining the currency risk premium with exogenous consumption growth but endogenous inflation. Both of these latter two studies are able to generate the forward premium anomaly.

Other research that studies the implications of real structural heterogeneity across countries with production models include Benigno et al. (2012) and Ready et al. (2017). Under complete markets, Benigno et al. (2012) assume recursive utility and embed stochastic volatility in a global productivity factor, but they do not have long-run risk. They employ their model to study the effect of stochastic volatility and monetary policy shocks on the forward premium bias. Ready et al. (2017) present a two-country model with asymmetries in production and trade structure to emphasize the role of macroeconomic instability and precautionary saving in driving the currency risk premium.

Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021) explain currency risk and the forward premium bias in general equilibrium with noise traders operating in segmented markets and financial frictions.[4] Chen et al. (2021) implement a reduced form version of the Itskhoki and Mukhin (2021) financial frictions with an exogenous shock that creates a wedge between exchange rate depreciation and the relative log SDFs in an otherwise complete markets specification. Like us, they are unable to generate a forward premium anomaly in a model with production.

3. THE MODEL

We consider both complete markets and incomplete markets in a two-country New Keynesian model. Labor is the only input into production and prices are sticky in the sense of Calvo (1983). The model is nonstationary due to a unit-root in the log level of productivity. The numerical solution, which we obtain by perturbation of a third-order approximation around a nonstochastic steady state, requires a station-

[3] Our article also makes contact with Dou and Verdelhan (2018), who also emphasize the importance of cross-country heterogeneity, but they do not study the currency risk premium or forward premium bias and do not consider long-run risk and stochastic volatility processes.

[4] Mark and Wu (1998) and Jeanne and Rose (2002) show how the forward premium anomaly emerges in partial equilibrium models with noise trading.

ary representation of the model. We do this by normalizing the nonstationary variables by the one-period lag of productivity. Simulations of the model are implemented after pruning. The details of the stationarity inducing transformation are suppressed from the text. Since models in this class are well known and familiar to most readers, the text provides only a sketch of the model.

The main presentation assumes LCP of exports. We also consider PCP and DCP, but provide only a quick sketch for the setting of export prices under those rules. Early research, branching from the Mundell (1963) to the Fleming (1962) tradition (e.g., Obstfeld and Rogoff, 1995), assumed both countries set export prices by PCP whereby the law-of-one price holds for every traded good. Questions about the appropriateness of this implication led to the development of models under LCP (Betts and Devereux, 2000). Recently, Gopinath et al. (2020) report evidence that the practice of DCP, with the U.S. dollar as the dominant currency, is widespread and pervasive. In our two-country setup, DCP results when country 1 (home) sets export prices by PCP and country 2 (foreign) sets by LCP.

3.1. *Households.* Let $C_{k,t}$ be household consumption in country k at time t and $L_{k,t}$ be labor input from country k at time t where $k \in \{1, 2\}$. Households have recursive utility,

$$V_{k,t} = (1-\beta)\left(\ln(C_{k,t}) - \Phi\frac{L_{k,t}^{1+\chi}}{1+\chi}\right) - \frac{\beta}{\phi}\ln\left[\mathrm{E}_t\left(e^{-\phi V_{k,t+1}}\right)\right], \tag{1}$$

where E_t is the conditional expectation operator, $\beta \in (0, 1)$ is the subjective discount factor, $\Phi > 0$, $\chi > 0$, and $\phi \in R$ are parameters. $1/\chi$ is the Frisch elasticity of labor supply. This utility function constrains the intertemporal elasticity of substitution to be one and was introduced by Swanson (2019). Swanson (2019) shows that relative risk aversion is $RRA = \phi + (1 + \Phi/\chi)^{-1}$. The intertemporal marginal rate of substitution, or equivalently, the real SDF is

$$M_{k,t+1} = \beta\left(\frac{C_{k,t}}{C_{k,t+1}}\right)\left(\frac{e^{-\phi V_{k,t+1}}}{\mathrm{E}_t\left(e^{-\phi V_{k,t+1}}\right)}\right). \tag{2}$$

The nominal SDF is $N_{k,t+1} = M_{k,t+1}e^{-\pi_{k,t+1}}$, where $\pi_{k,t+1}$ is the inflation rate in country k.

3.1.1. *Complete markets.* In order to lighten the notation, we suppress the functional dependence on the state of nature. Under complete markets, households in both countries have access to a full set of nominal state-contingent securities, each paying one unit of country 1's currency if the state occurs. Let $B_{k,t}$ be the number of these securities held by country k households with nominal price Λ_t. Households receive flow resources from real labor income, real firm profits, and state-contingent bond payoffs. Shares of firms are not internationally traded. Households spend their resources on consumption and a portfolio of state-contingent bonds. Let $S_{k,j,t}$ be the nominal country k currency price of a unit of country j currency (the exchange rate), $P_{k,t}$ be the price level, $W_{k,t}$ be the real wage, and $\Pi_{k,t}$ be real firm profits in country k. The household budget constraint in country k is

$$C_{k,t} + \frac{E_t(\Lambda_{t+1}B_{k,t+1})}{S_{1,k,t}P_{k,t}} = W_{k,t}L_{k,t} + \Pi_{k,t} + \frac{B_{k,t}}{S_{1,k,t}P_{k,t}}, \tag{3}$$

where $(S_{k,k,t} = 1)$. The optimality conditions for the household are the labor supply equation,

$$W_{k,t} = \Phi C_{k,t}L_{k,t}^{\chi}, \tag{4}$$

and the Euler equation for the nominal state-contingent bond, which when aggregated across each bond, gives the price of the nominal risk-free bond,

$$\frac{1}{1+i_{k,t}} = E_t\left[N_{k,t+1}\left(\frac{S_{1,k,t}}{S_{1,k,t+1}}\right)\right], \tag{5}$$

where $i_{k,t}$ is the nominal interest rate. It follows that the nominal exchange rate depreciation is

$$\frac{S_{1,2,t+1}}{S_{1,2,t}} = \frac{N_{2,t+1}}{N_{1,t+1}}. \tag{6}$$

3.1.2. *Incomplete markets.* Under incomplete markets, each country issues a nominal non-state contingent discount bond denominated in their own currency. The issue price is one unit of currency k and the time $t+1$ payoff is $1+i_{k,t}$ units of currency k. These are the only internationally traded assets. Let $B_{k,j,t} > 0$ be the number of currency j bonds held by country k agents ($k, j \in \{1, 2\}$). There are no short-sale constraints, so if country k agents have shorted the bond, then $B_{k,j,t} < 0$. Let $b_{k,j,t}$ be the country j real value of those bonds.

In order to keep bond holdings stationary, we employ the Schmitt-Grohe and Uribe (2003) method of imposing a small fee (τ) on residents on either their long or short positions on foreign currency denominated bonds. The real cost to a country k household for taking a position in the currency j bond is $\Gamma(b_{k,j,t}) = (\tau/2)(Q_{k,j,t}b_{k,j,t}/\sqrt{A_{k,t-1}})^2$, where $Q_{k,j,t} = (S_{k,j,t}P_{j,t})/P_{k,t}$ is the real exchange rate. In the steady state, for any $\tau > 0$, households will want $b_{k,j} = b_{j,k} = 0$. Because the level of productivity ($A_{k,t}$) is nonstationary, we normalize the model by the one-period lagged productivity level ($A_{k,t-1}$) to induce stationarity in the quantities. The term $A_{k,t-1}$ enters the bond tax formula in anticipation of the normalization.

Households own the firms of their own country but not of the foreign country. Household resources consists of real firm profits, real labor income, and real bond payoffs. These resources are spent on consumption and a new bond portfolio. Let $r_{k,t}$ be the real interest rate. The gross real bond return is $(1+r_{k,t-1}) = (1+i_{k,t-1})e^{-\pi_{k,t}}$. The real budget constraint for the country k household is

$$\begin{aligned} C_{k,t} + b_{k,k,t} + Q_{k,j,t}b_{k,j,t} + \Gamma(b_{k,j,t}) = {} & (1+r_{k,t-1})b_{k,k,t-1} + (1+r_{j,t-1})Q_{k,j,t}b_{k,j,t-1} \\ & + W_{k,t}L_{k,t} + \Pi_{k,t}. \end{aligned} \tag{7}$$

The bond choice Euler equations for a country k household are

$$\text{Domestic Bond}: \tfrac{1}{(1+i_{k,t})} = \mathrm{E}_t[N_{k,t+1}], \tag{8}$$

$$\text{Nondomestic Bond}: \left(\tfrac{1}{1+i_{j,t}}\right)\left(1 + \tfrac{\tau Q_{k,j,t}b_{k,j,t}}{A_{k,t-1}}\right) = \mathrm{E}_t\left[N_{k,t+1}\left(\tfrac{Q_{k,j,t+1}}{Q_{k,j,t}}\right)\right], \tag{9}$$

where $k \neq j$. The labor supply condition is unaffected by the change to incomplete markets and is described by Equation (4). In equilibrium, we require zero net bonds outstanding. Hence, for $k, j \in \{1, 2\}$ and $k \neq j$,

$$0 = b_{k,k,t} + b_{j,k,t}. \tag{10}$$

The nominal exchange rate depreciation is augmented by an incomplete markets wedge factor, η_{t+1}, as in Lustig and Verdelhan (2019),

$$\frac{S_{1,2,t+1}}{S_{1,2,t}} = \frac{N_{2,t+1}}{N_{1,t+1}}e^{\eta_{t+1}}. \tag{11}$$

The remainder of the model that follows holds under both complete markets and incomplete markets.

3.2. *Goods Demand.* In each country, a continuum of firms indexed by $f \in [0, 1]$ each produce a differentiated product with price setting by LCP. Let λ be the elasticity of substitution between varieties f. $C_{k,j,t}(f)$ are goods produced by firm f in country j and consumed in country k, and $P_{k,j,t}(f)$ is its currency k price. The index of imports ($k \neq j$) or domestic demand ($k = j$) and the associated price index are

$$C_{k,j,t} = \left[\int_0^1 C_{k,j,t}(f)^{\frac{\lambda-1}{\lambda}} df\right]^{\frac{\lambda}{\lambda-1}}, \tag{12}$$

$$P_{k,j,t} = \left[\int_0^1 P_{k,j,t}(f)^{1-\lambda} df\right]^{\frac{1}{1-\lambda}}. \tag{13}$$

Aggregate demand (AD) of country k and the associated price level are

$$C_{k,t} = \left(d^{\frac{1}{\mu}} C_{k,k,t}^{\frac{\mu-1}{\mu}} + (1-d)^{\frac{1}{\mu}} C_{k,j,t}^{\frac{\mu-1}{\mu}}\right)^{\frac{\mu}{\mu-1}}, \tag{14}$$

$$P_{k,t} = \left[d P_{k,k,t}^{1-\mu} + (1-d) P_{k,j,t}^{1-\mu}\right]^{\frac{1}{1-\mu}}, \tag{15}$$

where d is the degree of home bias, μ is the elasticity of substitution between the domestically and nondomestically produced goods, and $k \neq j$.

3.3. *Firms.* Firm $f \in [0, 1]$ can distinguish between domestic and nondomestic shoppers and is able to charge them different prices. The production function for a firm in country k, for $k \in \{1, 2\}$, is

$$Y_{k,t}(f) = A_{k,t} L_{k,t}(f), \tag{16}$$

where $A_{k,t}$ is the productivity level. The firm's real total costs are $W_{k,t} L_{k,t}(f)$. Output is demand determined, $Y_{k,t}(f) = C_{k,k,t}(f) + C_{j,k,t}(f)$, where $k \neq j$. Domestic and nondomestic demands are, respectively,

$$C_{k,k,t}(f) = d\left(\frac{P_{k,k,t}(f)}{P_{k,k,t}}\right)^{-\lambda}\left(\frac{P_{k,k,t}}{P_{k,t}}\right)^{-\mu} C_{k,t}, \tag{17}$$

$$C_{j,k,t}(f) = (1-d)\left(\frac{P_{j,k,t}(f)}{P_{j,k,t}}\right)^{-\lambda}\left(\frac{P_{j,k,t}}{P_{j,t}}\right)^{-\mu} C_{j,t}. \tag{18}$$

It follows that labor employed by firm f is

$$L_{k,t}(f) = \frac{C_{k,k,t}(f) + C_{j,k,t}(f)}{A_{k,t}}. \tag{19}$$

Prices are sticky in the sense of Calvo (1983). Each period, the firm is allowed to change its price with probability $1 - \alpha$. LCP means firms in country 1 set export prices in country 2 currency whereas firms in country 2 set export prices in country 1 currency. Price setting goes as follows. If a firm in country k ($k, j \in \{1, 2\}$ and $k \neq j$) is chosen to reset prices, it adjusts both the currency k price for the domestic market ($P_{k,k,t}(f)$) and the currency j price for exports ($P_{j,k,t}(f)$). Prices are set to maximize the expected present value of future real profits with prices fixed at the optimum. Let $M_{k,t,t+h} = \prod_{z=0}^{h} M_{k,t+z}$ be the h-period real SDF where $M_{k,t} = 1$. Formally, the problem for price resetting is to maximize

$$(20)\quad \mathrm{E}_t \sum_{h=0}^{\infty} (\alpha)^h M_{k,t,t+h} \left[\frac{P_{k,k,t}(f)}{P_{k,t+h}} C_{k,k,t+h}(f) + \frac{Q_{k,j,t+h} P_{j,k,t}(f)}{P_{j,t+h}} C_{j,k,t+h}(f) - W_{k,t+h} L_{k,t+h}(f) \right]$$

subject to the output demand equations (17) and (18) and the labor demand equation (19).

Under PCP, firms in country 1 set export prices in country 1 currency whereas firms in country 2 set export prices in country 2 currency, where $P_{k,j,t}$ is now denominated in country j's currency. The price level in Equation (15) becomes $P_{k,t} = [dP_{k,k,t}^{1-\mu} + (1-d)(S_{k,j,t} P_{k,j,t})^{1-\mu}]^{\frac{1}{1-\mu}}$. Domestic output demand $C_{k,k,t}(f)$ is again given by Equation (17), but nondomestic demand is

$$(21)\qquad C_{j,k,t}(f) = (1-d) \left(\frac{P_{j,k,t}(f)}{P_{j,k,t}} \right)^{-\lambda} \left(\frac{S_{j,k,t} P_{j,k,t}}{P_{j,t}} \right)^{-\mu} C_{j,t},$$

where $k \neq j$. The firm's price setting problem is to choose prices to maximize

$$(22)\quad E_t \sum_{h=0}^{\infty} (\alpha)^h M_{k,t,t+h} \left[\frac{P_{k,k,t}(f)}{P_{k,t+h}} C_{k,k,t+h}(f) + \frac{P_{j,k,t}(f)}{P_{k,t+h}} C_{j,k,t+h}(f) - W_{k,t+h} L_{k,t+h}(f) \right].$$

Under DCP, firms in country 1 set export prices in country 1 currency (they engage in PCP) and firms in country 2 also set export prices in country 1 currency (they engage in LCP).

3.4. *Monetary Policy.* The monetary authorities set the interest rate according to a Taylor (1993)–type feedback rule that responds to inflation deviations from its steady-state level and to the output gap. For country $k \in \{1, 2\}$, we follow Swanson (2019) by setting the natural (log) level of output to be an infinite-dimensional moving average of output,

$$(23)\qquad \ln(\bar{Y}_{k,t}) = \rho_{y_k} \ln(\bar{Y}_{k,t-1}) + (1-\rho_{y_k}) \ln(Y_{k,t}),$$

where ρ_{y_k} is a parameter. The output gap is then the deviation between $\ln(Y_{k,t})$ and $\ln(\bar{Y}_{k,t})$. The monetary authorities set the short-term interest rate, with interest rate smoothing, according to

$$(24)\quad i_{k,t} = (1-\delta_k)\bar{\imath}_k + \delta_k i_{k,t-1} + (1-\delta_k)\left[\xi_k(\pi_{k,t} - \bar{\pi}_k) + \zeta_k\left(\ln(Y_{k,t}) - \ln(\bar{Y}_{k,t})\right)\right] + e_{k,t},$$

where δ_k, ξ_k, and ζ_k are parameters, $\bar{\imath}_k$ is the steady-state interest rate, $\pi_{k,t}$ is the inflation rate, $\bar{\pi}_k$ is the steady-state inflation rate, and $e_{k,t} \overset{iid}{\sim} N(0, \sigma_{e_k}^2)$.

3.5. *Aggregation and Equilibrium.* AD for goods produced by country k is given by equating firm f's supply to demand,

$$(25)\quad A_{k,t} L_{k,t}(f) = d\left(\frac{P_{k,k,t}}{P_{k,t}}\right)^{-\mu} \left(\frac{P_{k,k,t}(f)}{P_{k,k,t}}\right)^{-\lambda} C_{k,t} + (1-d)\left(\frac{P_{j,k,t}(f)}{P_{j,k,t}}\right)^{-\lambda} \left(\frac{P_{j,k,t}}{P_{j,t}}\right)^{-\mu} C_{j,t},$$

then integrating Equation (25) to obtain,

$$(26)\qquad A_{k,t} L_{k,t} = C_{k,k,t} v_{k,k,t}^{p} + C_{j,k,t} v_{j,k,t}^{p},$$

where $L_{k,t} = \int_0^1 L_{k,t}(f)df$ is total country k employment,

$$C_{k,k,t} = d\left(\frac{P_{k,k,t}}{P_{k,t}}\right)^{-\mu} C_{k,t} = \left(\int_0^1 C_{k,k,t}(f)^{\frac{\lambda-1}{\lambda}} df\right)^{\frac{\lambda}{\lambda-1}} \tag{27}$$

is aggregate domestic demand, and

$$C_{j,k,t} = (1-d)\left(\frac{P_{j,k,t}}{P_{j,t}}\right)^{-\mu} C_{j,t} = \left(\int_0^1 C_{j,k,t}(f)^{\frac{\lambda-1}{\lambda}} df\right)^{\frac{\lambda}{\lambda-1}} \tag{28}$$

is aggregate export demand. In Equation (26), $v^p_{k,k,t} \equiv \int_0^1 (P_{k,k,t}(f)/P_{k,k,t})^{-\lambda} df$ is a measure of price dispersion for goods in the domestic market and $v^p_{j,k,t} \equiv \int_0^1 (P_{j,k,t}(f)/P_{j,k,t})^{-\lambda} df$ is import price dispersion in country j. The recursive representation for the price dispersion term $v^p_{k,j,t}$ ($k, j \in \{1, 2\}$) is obtained by noting that a fraction α of these firms are stuck with last period's price, $P_{k,j,t-1}(f)$. Since there are a large number of firms charging the same price as last period, it will also be the case that $\int_0^\alpha P_{k,j,t-1}(f)^{-\lambda} df = \alpha P^{-\lambda}_{k,j,t-1}$. The complementary measure of firms $(1-\alpha)$ is able to reset the prices for exports and for the domestic market. They all reset to the same price, $P^*_{k,j,t}$. The result is the recursive representation

$$v^p_{k,j,t} = (1-\alpha)\left(\frac{P^*_{k,j,t}}{P_{k,j,t}}\right)^{-\lambda} + \alpha\left(\frac{P_{k,j,t-1}}{P_{k,j,t}}\right)^{-\lambda} v^p_{k,j,t-1}. \tag{29}$$

4. CURRENCY RISK PREMIUM AND FORWARD PREMIUM BIAS

Under complete markets, Backus et al. (2001) show that the currency risk premium is the country 2 minus country 1 differential of a series expansion of the log nominal SDF's higher-ordered conditional cumulants. Under log-normality, variation in the log nominal SDF is generated in only the first two conditional cumulants, which coincide with the conditional mean and variance. Henceforth, we focus our discussion around them. We also illustrate the fundamental difference between the currency risk premium and the forward premium bias, and the particular challenge in explaining the forward premium bias/anomaly in a production model. Unless required to prevent confusion, we suppress the country-specific notation in this section. The presentation is for nominal risk premia and excess returns. Modifications for real risk premia and excess returns will be obvious.

4.1. *Uncertainty Risk.* From this point, except for interest rates, we use lower case letters to denote the logarithm of a variable. Letting $n_t = \ln(N_t)$ be the log nominal SDF, Backus et al. (2001) observed that $\ln(E_t e^{n_{t+1}})$ is the conditional cumulant generating function of $e^{n_{t+1}}$ evaluated at one.[5] It has the series expansion, $\ln(E_t e^{n_{t+1}}) = \sum_{j=1}^{\infty} \kappa_{jt}/j!$, where κ_{jt} is the j-th

[5] Let $\psi(t) = E(e^{tX})$ be the moment generating function of X. Then, the k-th derivative evaluated at $t = 0$, is the k-th moment of X, $\psi^k(0) = E(X^k)$. The logarithm of the moment generating function is the cumulant generating function, $\phi(t) = \ln(\psi(t)) = \ln(E(e^{tX}))$. Just as the moment generating function can be expanded, the cumulant generating function has the expansion

$$\phi(t) = \ln\left(E\left(e^{tX}\right)\right) = t\kappa_1 + \frac{t^2\kappa_2}{2!} + \frac{t^3\kappa_3}{3!} + \cdots$$

The k-th derivative evaluated at $t = 0$, $\phi^k(0)$ is the k-th cumulant of X. Letting $X = \ln(N_{t+1})$ gives the result in the text. The first three conditional cumulants of X are also its first three central moments. That is, $\kappa_1 \equiv \phi^1(0) = E(X)$, $\kappa_2 \equiv \phi^2(0) = E(X - E(X))^2$, and $\kappa_3 \equiv \phi^3(0) = E(X - E(X))^3$. Cumulants of order 4 and higher are complicated functions of the moments.

conditional cumulant of the log nominal SDF, n_{t+1}. Under log-normality, which we assume, only the first two cumulants, which are also the first two central moments, are nonzero. Let μ_t and σ_t^2 be the conditional mean and the conditional variance of the log nominal SDF, n_{t+1}. From the bond-pricing equation (5) we have,

$$i_t = -\mu_t - \frac{1}{2}\sigma_t^2. \tag{30}$$

μ_t is a nonuncertainty-risk factor and is key in setting national saving associated with consumption smoothing and intertemporal substitution motives. In a world of certainty, μ_t completely determines the interest rate since $\sigma_t^2 = 0$. An increase in $\mu_t = E_t(\ln N_{t+1})$ (say because of lower expected future consumption flows) means people want to move current consumption to the future through higher saving. This increases bond prices and lowers the interest rate.

The conditional variance, σ_t^2, is the agents' subjective assessments of uncertainty and represents the effect of precautionary saving on the interest rate. Higher uncertainty increases precautionary saving, increases bond prices, and lowers interest rates. It is convenient to think of the conditional variance as an uncertainty-risk factor.

4.2. *Currency Risk Premium.* The currency risk premium (deviation from uncovered interest rate parity) is the expected excess return from borrowing country 2's currency and lending country 1's currency. Using the log-approximation, the nominal risk premium is

$$rp_t = i_{1,t} - i_{2,t} - E_t \Delta s_{1,2,t+1} \tag{31}$$

$$= \tfrac{1}{2}\left(\sigma_{2,t}^2 - \sigma_{1,t}^2\right) + E_t \eta_{t+1}, \tag{32}$$

where $s_{1,2,t} = \ln(S_{1,2,t})$ and η_{t+1} is the Lustig and Verdelhan (2019) exchange rate wedge under incomplete markets. Equation (32) follows from substituting Equations (6) and (30) into Equation (31).

Under complete markets, $E_t \eta_{t+1} \equiv 0$ and from Equation (32), fluctuations in the currency risk premium are driven entirely by time variation in the foreign-home uncertainty-risk factor differential. Nonuncertainty-risk factors, $\mu_{1,t}$ and $\mu_{2,t}$, cancel out. Suppose $\mu_{2,t}$ increases thus increasing country 2's log nominal SDF. The future becomes more important to country 2 residents, increasing the interest rate differential $i_{1,t} - i_{2,t}$ by lowering $i_{2,t}$. At the same time, the increased $\mu_{2,t}$ generates an expected appreciation of country 2's currency (increase in $E_t \Delta s_{1,2,t+1}$) of the exact same amount.

The size and sign of rp_t under complete markets is determined by assessments of relative economic uncertainty, which in turn drive the relative strength of precautionary saving. Suppose $\sigma_{2,t}^2$ increases. This has no effect on the expected exchange rate depreciation, but precautionary saving increases in country 2, which drives down $i_{2,t}$, thus, increasing the risk premium through $i_{1,t} - i_{2,t}$. This is, perhaps, the explanation for why the carry trade strategy of borrowing the low interest rate currency and lending the high interest rate currency is profitable. The positive rp_t induces country 1 to borrow (short) the country 2 currency and to lend (go long) the country 1 currency.[6]

The analytics for the risk premium are not as straightforward under incomplete markets because the expected exchange rate wedge $(E_t(\eta_{t+1}))$ exhibits dependencies on $\mu_{k,t}$ and $\sigma_{k,t}^2$, for $k \in \{1, 2\}$.

[6] Although the mechanisms under complete and incomplete markets are different, we will investigate through simulations the extent to which the complete markets insights carry through to incomplete markets. In order to understand the currency risk premium under incomplete markets, particularly when there is heterogeneity across countries, it may be necessary also to take into account $\mu_{2,t} - \mu_{1,t}$.

4.3. *Forward Premium Bias.* The Fama coefficient from regressing the future currency 1 depreciation on the interest rate differential is

$$\beta_F = \frac{\operatorname{cov}(\Delta s_{1,2,t+1}, (i_{1,t} - i_{2,t}))}{\operatorname{var}(i_{1,t} - i_{2,t})}. \tag{33}$$

There is a forward premium bias when $\beta_F \neq 1$. There is a forward premium anomaly when $\beta_F < 0$. A decomposition of the numerator finds that the forward premium anomaly requires

$$\underbrace{\operatorname{Cov}\left((\mu_{2,t} - \mu_{1,t}), \frac{1}{2}(\sigma^2_{2,t} - \sigma^2_{1,t})\right)}_{(i)} + \underbrace{\operatorname{Cov}\left(E_t\eta_{t+1}, \left(\mu_{2,t} - \mu_{1,t} + \frac{1}{2}(\sigma^2_{2,t} - \sigma^2_{1,t})\right)\right)}_{(ii)} < -\operatorname{Var}(\mu_{2,t} - \mu_{1,t}).$$

Hence, the risk premium and the forward premium anomaly are distinctly different phenomena. The risk premium is the *difference* between log nominal SDF conditional variances and reflects the foreign–home relative strength of precautionary saving. The forward premium bias/anomaly, on the other hand, is determined by the *covariance* between relative log nominal SDF's conditional means and conditional variances, and possibly the exchange rate wedge.

Under complete markets, the term labeled (ii) vanishes, and the forward premium anomaly requires a very large negative covariance between $(\mu_{2,t} - \mu_{1,t})$ and $(1/2)(\sigma^2_{2,t} - \sigma^2_{1,t})$. This could arise if when the uncertainty-risk component of the interest rate $(\sigma^2_{k,t})$ is high, the nonuncertainty-risk component $(\mu_{k,t})$ is low, for $k \in \{1, 2\}$. Times of high uncertainty are associated with a high valuation of the present. Alternatively, the forward premium anomaly is helped along if there is a large positive covariance between country k's nonuncertainty-risk component $(\mu_{k,t})$ and country j's uncertainty-risk component $(\sigma^2_{j,t})$, for $k \neq j$ and $k, j \in \{1, 2\}$. Here, for example, times of high uncertainty in country 2 are associated with a high valuation of the future by country 1.

Empirically, some currencies show a positive forward premium bias $(\beta_F > 1)$. This would occur if the nonuncertainty-risk and the uncertainty-risk components ($\mu_{2,t}$ and $\sigma^2_{2,t}$) have positive covariance. The only case in which there is no forward premium bias is when the risk premium is always zero $(\sigma^2_{2,t} = \sigma^2_{1,t})$.

Obviously, under incomplete markets, the second term (ii) must be taken into account. A negative covariance between $E_t\eta_{t+1}$ and the interest differential helps push us toward a forward premium anomaly.

5. MODEL PARAMETERIZATION

In this section, we outline the parameterization of the model under symmetric monetary policy and productivity growth processes. We first describe the estimation of productivity growth, modeled as a long-run risk and stochastic volatility process, using data for the United States. We also report the remaining parameters in the model.

5.1. *Productivity Growth Process.* Typically, long-run risk in international macroeconomics and finance is used to model consumption growth in endowment models (Bansal and Yaron, 2004; Bansal and Shaliastovich, 2012; Backus et al., 2013; Colacito et al., 2018a). Productivity shocks with stochastic volatility are more extensively studied in closed economy macro models (see the review article by Fernández-Villeverde and Guerrón-Quintana, 2020). Our productivity growth specification largely mimics that used for consumption growth in long-run risk models of asset pricing with the stochastic volatility process following Fernández-Villeverde and Guerrón-Quintana (2020) to keep the stochastic volatility component (ω_t) positive.

Notation to differentiate parameter values across countries is suppressed. Let $a_t = \ln(A_t)$ be log productivity, x_t be the long-run risk component, and ω_t be the stochastic volatility

TABLE 1
POSTERIOR MEANS–PRODUCTIVITY GROWTH PROCESS FOR THE UNITED STATES

	Posterior Mean	5%	95%
μ_g	0.002	0.000	0.003
θ	1.131	0.648	1.684
ρ_x	0.726	0.647	0.809
ρ_ω	0.842	0.765	0.921
γ	−1.353	−3.855	1.167
μ_ω	−6.257	−6.711	−5.793

NOTE: The productivity growth process for the United States is governed by Equations (34)–(36). Posterior means and the upper and lower 5% bands from a Bayesian estimation are reported.

TABLE 2
VOLATILITY OF PRODUCTIVITY GROWTH AND COMPONENTS FOR THE UNITED STATES

	Data	Simulated					
		Unadjusted			Adjusted		
	$sd(\Delta a_t)$	$sd(\Delta a_t)$	$sd(x_t)$	$sd(\omega_t)$	$sd(\Delta a_t)$	$sd(x_t)$	$sd(\omega_t)$
United States	2.605	3.433	1.467	0.453	2.603	1.106	0.350

NOTE: $sd(\bullet)$ is the standard deviation (volatility) of the variable stated as percent per annum. a_t is log productivity, x_t is the long-run risk component, and ω_t is the stochastic volatility component.

component. Productivity growth is governed by

$$\Delta a_t = \mu_g + x_{t-1} + e^{\theta}\omega_{t-1}\epsilon_t, \tag{34}$$

$$x_t = \rho_x x_{t-1} + \omega_{t-1}u_t, \tag{35}$$

$$\ln(\omega_t) = (1-\rho_\omega)\mu_\omega + \rho_\omega \ln(\omega_{t-1}) + e^{\gamma}v_t, \tag{36}$$

where $\epsilon_t \overset{nid}{\sim} (0,1)$, $u_t \overset{nid}{\sim} (0,1)$, and $v_t \overset{nid}{\sim} (0,1)$.

We begin by studying the symmetric model where both countries have the same productivity growth parameters. In order to choose sensible parameter values, we estimate the productivity growth process using U.S. TFP. In order to construct TFP, we first use quarterly GDP, investment, and employment data from *Datastream* and FRED. The capital stock is constructed by the perpetual inventory method. From this, we construct quarterly TFP.

We employ the posterior means from Bayesian estimation as the parameter values in the long-run risk and stochastic volatility process for productivity growth. The posterior means and the upper and lower 5% bands are shown in Table 1.

Table 2 shows the implied standard deviations or volatility of the process components $(\Delta a_t, x_t, \omega_t)$. Volatility of simulated TFP growth generated by the estimated process (3.433) overstates volatility in the data (2.605). In the ensuing analysis, we scale the innovations (ϵ_t, u_t, v_t) in Equations (34)–(36) by 2.605/3.433 to match the volatility in the data. The volatility of simulated TFP growth, the long-run risk component, and the stochastic volatility component after adjustment are shown in Table 2 under "Adjusted." As can be seen, the adjustment produces a close match between the volatility of the simulated process and the data.

Figure 1 plots productivity growth, log levels from the data, and a realized simulation of the adjusted process. The model is seen to do a reasonable job of capturing major features of the productivity data.

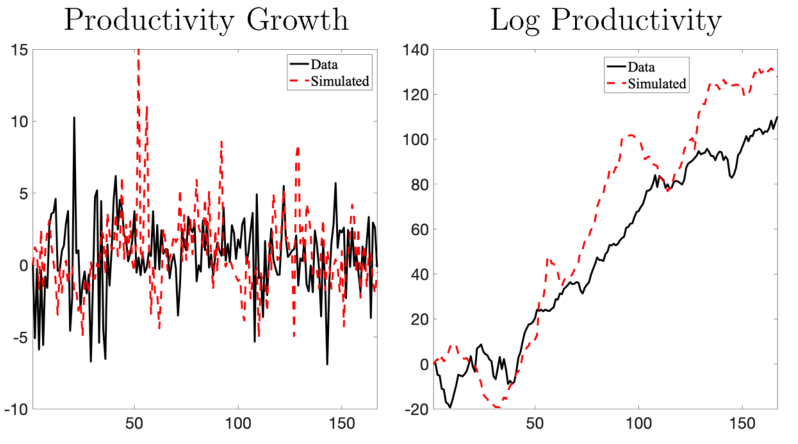


FIGURE 1

U.S. PRODUCTIVITY DATA AND REALIZED SIMULATION

Note that log TFP is modeled as a unit-root process. The solution of the two-country model requires that the two countries' log TFP be cointegrated. In order to achieve this, we employ the modified specification,

$$\Delta a_{1,t} = x_{1,t-1} + e^{\theta_1}\omega_{1,t-1}\epsilon_{1,t} + \psi(a_{1,t-1} - a_{2,t-1}), \tag{37}$$

$$\Delta a_{2,t} = x_{2,t-1} + e^{\theta_2}\omega_{2,t-1}\epsilon_{2,t} + \psi(a_{2,t-1} - a_{1,t-1}), \tag{38}$$

with $\psi = 0.0005$ and $\mu_g = 0$. This makes log productivity in countries 1 and 2 to be driftless and cointegrated, but not strongly so.

5.2. *Remaining Parameterization.* Monetary policy for both countries follow the benchmark Taylor rule where coefficients for the inflation response, the output gap response, and interest rate smoothing are $\xi = 1.5$, $\zeta = 0.5$, and $\delta = 0.7$, respectively. In regard to preference parameters, $\beta = 0.9925$, $\gamma = 0.545$, $\chi = 3$, and $\phi = 40$. We refer to ϕ as the risk-aversion coefficient, since it is the dominant parameter (for $\phi = 40$, relative risk aversion is $\phi + 1/(1 + \Phi/\chi) = 40.84$). Remaining parameters of the model are $d = 0.85$, $\tau = 0.001$, $\lambda = 10$, $\mu = 1.5$, $\alpha = 0.8$, and $\rho_y = 0.96$.

6. DYNAMICS OF UNCERTAINTY RISK AND CURRENCY RISK

In this section, we present the dynamics of the uncertainty-risk factor and the international currency risk implied by the model through impulse response analysis and variance decompositions. The analysis is done for complete markets and incomplete markets under symmetry in monetary policy and productivity growth across countries.[7]

One of our findings is that the currency in which export prices are set is not central for understanding systematic currency risk premium. Export pricing does matter for specific impulse

[7] It may be useful to classify the shocks as AD or aggregate supply (AS) based on their impact effect. If we say that, upon impact, AD shocks cause output and inflation to move in the same direction and AS shocks cause output and inflation to move in opposite directions, then productivity shocks are reliably and well known to be AS shocks and monetary policy shocks are reliably AD shocks. The Appendix shows that under all export pricing schemes, the stochastic volatility shock is an AD shock, as in Xu (2016) and Leduc and Liu (2016). Interestingly, the long-run risk shock is also classified as an AD shock.

TABLE 3
SIMULATED VARIANCE DECOMPOSITION UNDER LCP

	Volatility		Long-Run Risk		Productivity		Monetary		
	$\nu_{1,t}$	$\nu_{2,t}$	$u_{1,t}$	$u_{2,t}$	$\epsilon_{1,t}$	$\epsilon_{2,t}$	$e_{1,t}$	$e_{2,t}$	Total
Complete Markets									
$\mu_{1,t}$	46.226	36.817	3.002	1.567	1.771	1.518	4.482	0.056	95.442
$\sigma^2_{1,t}$	62.351	38.156	0.024	0.023	0.021	0.009	0.000	0.000	100.591
rp_t	48.492	48.283	0.533	0.439	1.106	0.464	0.000	0.000	99.319
er_t	2.413	2.422	15.108	15.207	17.815	18.029	7.413	7.561	85.968
$\Delta s_{1,2,t}$	2.088	2.091	14.865	14.961	17.811	18.028	7.739	7.906	85.485
Incomplete Markets									
$\mu_{1,t}$	95.741	0.014	1.107	0.020	0.318	0.034	1.231	0.033	98.499
$\sigma^2_{1,t}$	99.941	0.030	0.011	0.003	0.000	0.004	0.000	0.000	99.998
rp_t	3.349	3.356	6.776	6.749	29.843	30.269	0.015	0.016	80.371
er_t	1.766	1.786	12.577	12.518	18.280	18.588	9.208	9.299	84.024
$\Delta s_{1,2,t}$	1.653	1.671	13.091	13.007	16.943	17.196	10.407	10.551	84.520

NOTE: $\mu_{1,t}$ is the conditional mean of country 1's log nominal SDF, $\sigma^2_{1,t}$ is the conditional variance of country 1's log nominal SDF, $rp_t = E_t(i_{1,t} - i_{2,t} - \Delta s_{1,2,t+1})$ is the currency risk premium, $er_t = i_{1,t-1} - i_{2,t-1} - \Delta s_{1,2,t}$ is the ex post currency excess return, and $s_{1,2,t}$ is the log nominal exchange rate. Variables are multiplied by 400. Parameterization follows from Section 5. Averages over 10 replications of 5,000 periods. Numbers may not add up to 100 due to (i) nonzero correlation of simulated shocks in small samples and (ii) nonlinearity.

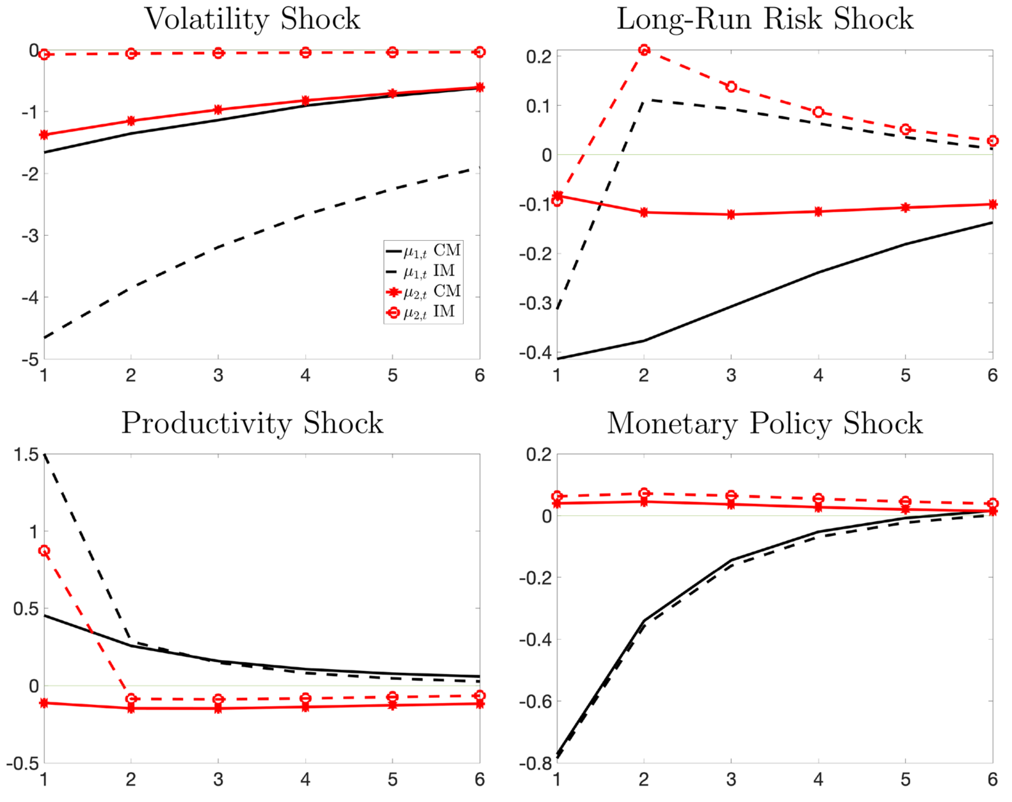
responses, especially with respect to trade-related variables, but the effect on aggregate economic uncertainty is unremarkable. As a result, unless otherwise noted, LCP will be the export pricing convention. Because the qualitative responses under DCP and PCP are approximately the same, these results are relegated to the Appendix.[8]

6.1. *Variance Decomposition.* Table 3 provides perspective on the sources of uncertainty and contributions to the currency risk premium. The table reports simulated variance decompositions for some key variables. Due to symmetry across countries, we suppress reporting on $\mu_{2,t}$ and $\sigma^2_{2,t}$.

Under complete markets, stochastic volatility shocks generate nearly all of the variation in the conditional variance of country 1's log nominal SDF, $\sigma^2_{1,t}$. The split is roughly 62%–38% between country 1 and country 2 stochastic volatility shocks. Likewise, most of the risk premium (differential in precautionary saving) variability is generated by stochastic volatility shocks with 48% of the variance generated by country 1 stochastic volatility shocks and roughly an equal amount from country 2 stochastic volatility shocks. Sources of excess return variance mirrors that of the exchange rate, with productivity growth and long-run risk shocks generating most of the variability, followed by monetary policy shocks and then stochastic volatility shocks.

Under incomplete markets, virtually all of the conditional variance of country 1's log nominal SDF is generated by its own-country stochastic volatility shock. Most of the risk premium variance is generated by productivity growth shocks and relatively little is due to stochastic volatility shocks. Most of the excess return and exchange rate variations are generated by productivity growth shocks (similar to complete markets), followed by long-run risk shocks and monetary policy shocks. As with complete markets, stochastic volatility shocks contribute very little to exchange rate variation.

[8] The export pricing convention matters primarily for trade-related variables in response to monetary policy shocks. This is why the DCP analysis in Gopinath et al. (2020) focuses on the effect of monetary policy shocks on trade variables.



NOTES: Impulse responses are to positive one standard deviation country 1 shocks under LCP and are reported in percent per annum. $\mu_{1,t}$ and $\mu_{2,t}$ are the conditional means of country 1's and country 2's log nominal SDF, respectively. CM represents complete markets and IM represents incomplete markets. Parameterization follows from Section 5.

FIGURE 2

IMPULSE RESPONSES OF $\mu_{1,t}$ AND $\mu_{2,t}$ UNDER COMPLETE MARKETS AND INCOMPLETE MARKETS

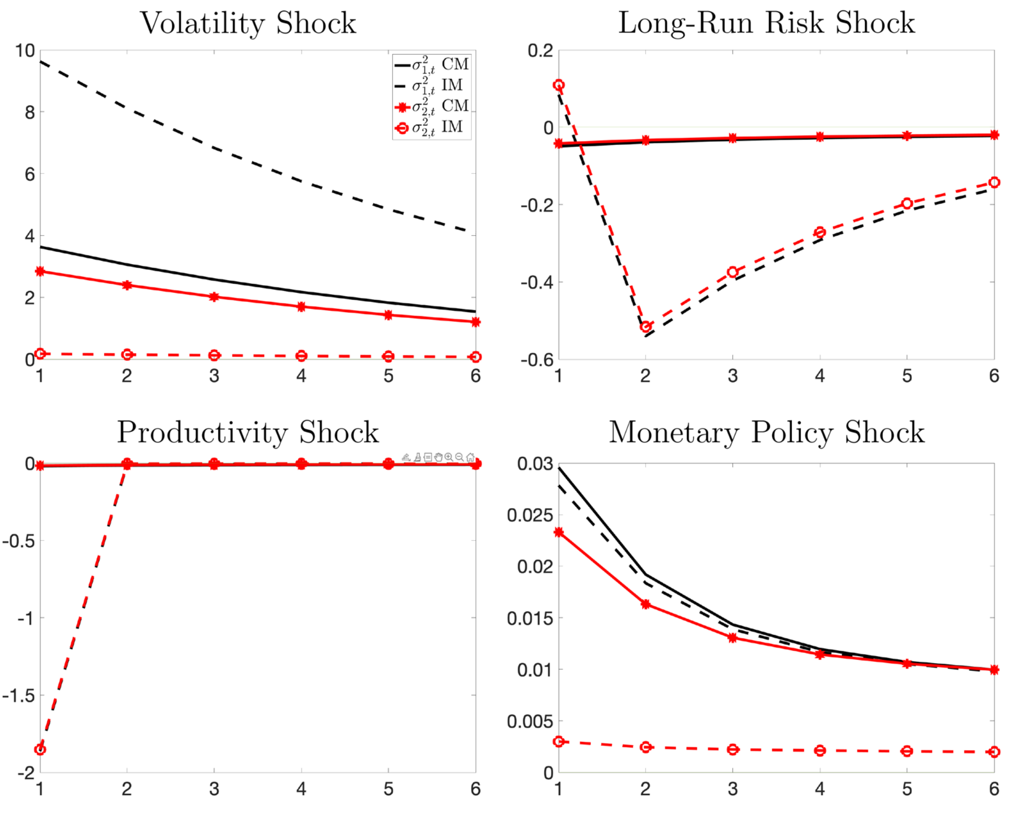
6.2. *Impulse Response Function Analysis.* The impulse responses are to a positive one standard deviation country 1 shock and are reported in percent per annum. We focus on key model variables that help to understand the currency risk premium.

6.2.1. *Transmission/sharing of nonuncertainty risk.* Figure 2 plots the impulse responses of the conditional mean of the log nominal SDF ($\mu_{k,t}$, for $k \in \{1,2\}$) under complete markets and incomplete markets to a positive one standard deviation country 1 shock. Solid lines are for complete markets and dashed lines are for incomplete markets. Black solid/dashed lines without symbols are for country 1 and red solid/dashed lines with symbols are for country 2.

A positive country 1 stochastic volatility shock lowers $\mu_{1,t}$ and $\mu_{2,t}$. This shock generates a bad state and people increase their concern for the present. There is a large effect on $\mu_{1,t}$ under incomplete markets. The high cross-country correlation between $\mu_{1,t}$ and $\mu_{2,t}$ under complete markets is evident. Under incomplete markets, the effect on $\mu_{2,t}$ is trivial.

Response magnitudes to the country 1 long-run risk shock are quite a bit smaller. The positive shock initially reduces $\mu_{k,t}$, for $k \in \{1,2\}$, under both complete markets and incomplete markets. Except for a level shift, response patterns for $\mu_{1,t}$ and $\mu_{2,t}$ are similar under incomplete markets. The initial effects of the country 1 productivity growth shock are opposite the initial effects of the country 1 long-run risk shock (except $\mu_{2,t}$ under complete markets).

A positive country 1 monetary policy shock generates nearly the same responses under complete markets and incomplete markets. It lowers $\mu_{1,t}$ and generates slight increases in $\mu_{2,t}$.



NOTES: Impulse responses are to positive one standard deviation country 1 shocks under LCP and are reported in percent per annum. $\sigma^2_{1,t}$ and $\sigma^2_{2,t}$ are the conditional variances of country 1's and country 2's log nominal SDF, respectively. CM represents complete markets and IM represents incomplete markets. Parameterization follows from Section 5.

FIGURE 3

IMPULSE RESPONSES OF $\sigma^2_{1,t}$ AND $\sigma^2_{2,t}$ UNDER COMPLETE MARKETS AND INCOMPLETE MARKETS

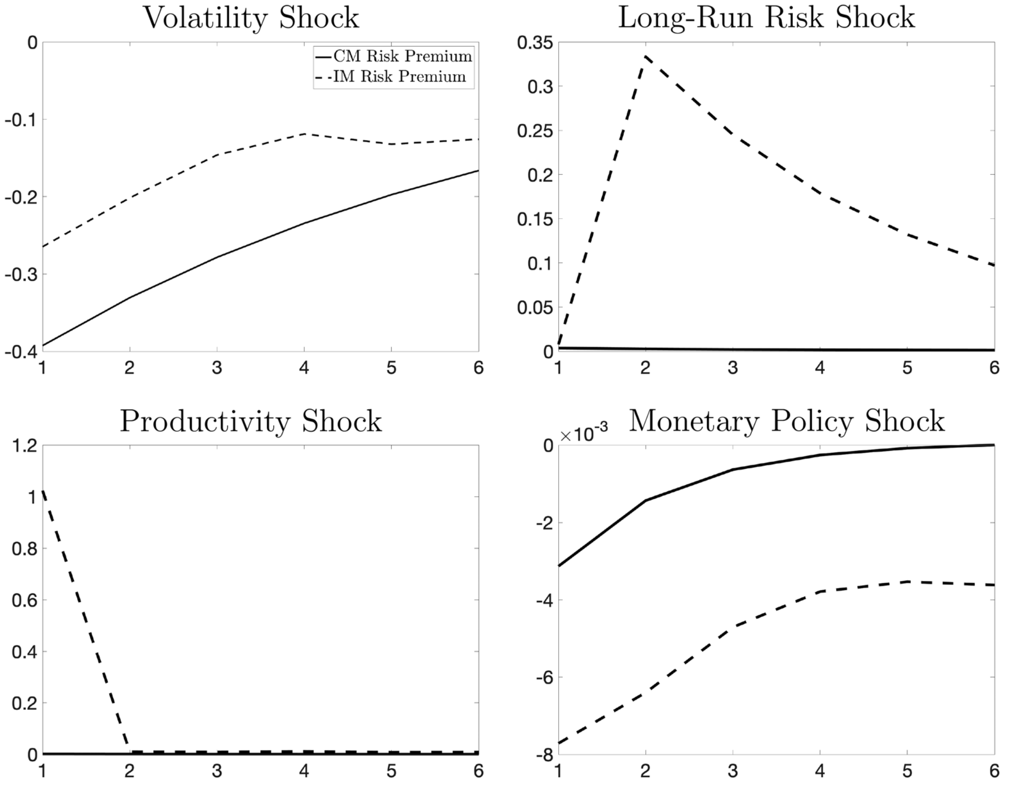
6.2.2. *Transmission/sharing of uncertainty risk.* Figure 3 plots the impulse responses of the conditional variance of the log nominal SDF ($\sigma^2_{k,t}$, for $k \in \{1, 2\}$) under complete markets and incomplete markets to a positive one standard deviation country 1 shock. Solid lines are for complete markets and dashed lines are for incomplete markets. Black solid/dashed lines without symbols are for country 1 and red solid/dashed lines with symbols are for country 2.

Under complete markets, positive country 1 stochastic volatility and monetary policy shocks raise $\sigma^2_{1,t}$ by more than $\sigma^2_{2,t}$. Positive country 1 long-run risk and productivity growth shocks reduce $\sigma^2_{1,t}$ and $\sigma^2_{2,t}$ by nearly the same amount, which consequently generates a trivial risk premium.

Under incomplete markets, positive country 1 stochastic volatility and monetary policy shocks increase $\sigma^2_{1,t}$ by more than $\sigma^2_{2,t}$. The incomplete markets financial structure is very effective at insulating (or ineffective at sharing) $\sigma^2_{2,t}$ from country 1 shocks. Positive country 1 long-run risk shocks initially increase $\sigma^2_{1,t}$ and $\sigma^2_{2,t}$ by nearly the same amount and positive country 1 productivity growth shocks decrease $\sigma^2_{1,t}$ and $\sigma^2_{2,t}$ by nearly the same amount.

Finally, in Figure 4, we plot impulse responses of the risk premium under complete markets and IMs to a positive one standard deviation country 1 shock. Solid lines are for complete markets and dashed lines are for IMs.

A positive country 1 stochastic volatility shock increases precautionary saving in country 1 relative to country 2, leading to a negative risk premium under both complete markets and incomplete markets. A positive country 1 monetary policy shock also increases



NOTES: Impulse responses are to positive one standard deviation country 1 shocks under LCP and are reported in percent per annum. CM represents complete markets and IM represents incomplete markets. Parameterization follows from Section 5.

FIGURE 4

IMPULSE RESPONSES OF THE RISK PREMIUM (rp_t) UNDER COMPLETE MARKETS AND INCOMPLETE MARKETS

precautionary saving in country 1 relative to country 2, leading to a negative risk premium under both complete markets and incomplete markets. Following positive country 1 long-run risk and productivity growth shocks, the complete markets risk premium barely moves in response. This is because the long-run risk and productivity growth shocks generate decreases in $\sigma_{1,t}^2$ and $\sigma_{2,t}^2$ of similar magnitudes so there is little difference between them.

7. UNCONDITIONAL MOMENTS

This section reports implied unconditional moments from simulations of the model. The purpose is to examine implied systematic international currency risk (nonzero meaned currency risk premia) and the forward premium bias. Although quantitative moment matching is not the objective of this article, it is useful to show those dimensions where the model performs well and where it falls short. We report results only under LCP and relegate PCP and DCP results to the Appendix.

7.1. *Symmetric Countries.* This section shows that, under symmetric monetary policy ($\xi = 1.5$ and $\zeta = 0.5$) and productivity growth (parameters set to estimates for the United States), the cross-country correlations and implied volatility of some of the model variables are reasonable and largely invariant to the degree of risk aversion. The main effect of increasing risk aversion is to raise the uncertainty-risk factor.

TABLE 4
UNCONDITIONAL MOMENTS AND RISK AVERSION COEFFICIENT (ϕ) UNDER LCP

	Complete Markets			Incomplete Markets		
Risk Aversion Coefficient	4	40	60	4	40	60
Panel A. Returns						
$\overline{rp}_t$	0.003	0.011	0.016	−0.034	0.035	−0.017
$sd(rp_t)$	0.129	1.032	1.549	1.648	1.775	1.944
$\max(rp_t)$	0.466	3.794	5.656	7.702	7.915	7.743
$\min(rp_t)$	−0.472	−3.745	−5.393	−7.622	−8.013	−8.575
$\rho(rp_t, rp_{t-1})$	0.841	0.843	0.844	0.137	0.261	0.345
$\overline{er}_t$	0.033	−0.051	0.075	−0.008	−0.020	−0.054
$sd(er_t)$	11.490	12.295	12.955	9.322	9.769	10.310
β_F	0.997	0.914	0.768	1.210	1.170	1.029
Panel B. Log Nominal SDF						
$sd(n_{1,t})$	13.776	94.736	141.507	14.197	116.072	179.388
$\overline{\mu}_{1,t}$	−2.973	−9.067	−18.123	−2.969	−13.782	−28.401
$sd(\mu_{1,t})$	1.699	4.167	8.883	1.344	8.653	19.357
$\overline{\sigma}^2_{1,t}$	0.365	16.194	35.649	0.383	24.905	55.559
$sd(\sigma^2_{1,t})$	0.222	8.501	18.908	0.266	17.946	40.063
$\rho(n_{1,t}, n_{2,t})$	0.634	0.991	0.996	0.180	0.317	0.349
Panel C. Exchange Rate, Interest Rate, Inflation						
$sd(\Delta s_{1,2,t})$	11.658	12.469	13.023	9.609	10.047	10.605
$sd(\Delta q_{1,2,t})$	10.473	11.287	11.755	8.228	8.770	9.265
$\rho(\Delta s_{1,2,t}, \Delta q_{1,2,t})$	0.987	0.987	0.987	0.946	0.951	0.952
$\bar{i}_{1,t}$	2.788	1.024	0.420	2.779	1.394	0.625
$sd(i_{1,t})$	1.696	1.773	1.905	1.331	1.418	1.532
$sd(\pi_{1,t})$	2.586	2.683	2.801	2.312	2.294	2.389
Panel D. Quantities						
$sd(\Delta y_{1,t})$	3.921	3.908	3.905	3.080	3.077	3.104
$sd(\Delta c_{1,t})$	4.433	4.313	4.397	3.616	3.492	3.483
$\rho(\Delta y_{1,t}, \Delta y_{2,t})$	0.337	0.401	0.412	0.207	0.236	0.271
$\rho(\Delta c_{1,t}, \Delta c_{2,t})$	0.017	0.121	0.142	−0.115	−0.048	0.003

NOTE: $\bar{\bullet}$ denotes the time-series mean of the variable stated, $sd(\bullet)$ is the standard deviation (volatility) of the variable stated, $\rho(\bullet)$ is the correlation of the variables stated, $\max(\bullet)$ is the maximum of the variable stated, and $\min(\bullet)$ is the minimum of the variable stated. $y_{1,t}$ and $y_{2,t}$ are log output in countries 1 and 2, $c_{1,t}$ and $c_{2,t}$ are log consumption in countries 1 and 2, $i_{1,t}$ is the nominal interest rate in country 1, $\pi_{1,t}$ is inflation in country 1, $n_{1,t}$ and $n_{2,t}$ are the log nominal SDFs in countries 1 and 2, $\mu_{1,t}$ is the conditional mean of country 1's log nominal SDF, $\sigma^2_{1,t}$ is the conditional variance of country 1's log nominal SDF, $q_{1,2,t}$ is the log real exchange rate, $s_{1,2,t}$ is the log nominal exchange rate, $rp_t = E_t(i_{1,t} - i_{2,t} - \Delta s_{1,2,t+1})$ is the currency risk premium, $er_t = i_{1,t-1} - i_{2,t-1} - \Delta s_{1,2,t}$ is the ex post currency excess return, and β_F is the Fama coefficient. Variables are multiplied by 400 to state in percent per annum. Parameterization follows from Section 5 except for the risk aversion coefficient (ϕ). Reported are median values from 10 replications of 5,000 periods.

Table 4 reports implied unconditional moments for risk aversion coefficient (ϕ) values of 4, 40, and 60. Due to symmetric treatment across countries, we generally report values only for country 1.

Panel A displays features of returns. We use overbars to denote the time-series mean of a variable. The unconditional mean of the risk premium ($\overline{rp}_t$) is trivial. Due to the model symmetry in this specification the mean risk premium should be exactly zero, but the slight non-zero values are due to finite sampling variation. Period-by-period however, the risk premium moves around quite a bit when risk aversion is large. It fluctuates from positive to negative and is fairly persistent under complete markets. For $\phi = 40$, the risk premium gets as large as 3.794% under complete markets and 7.915% under incomplete markets. Nominal interest rate mean and volatility are also reasonably sized (Panel C). The mean and volatility of the currency excess return mimics that of the exchange rate. As risk aversion increases, $\overline{\mu}_{1,t}$ decreases, which exerts an upward effect on the interest rate. At the same time, $\overline{\sigma}^2_{1,t}$ increases, which exerts a somewhat more than offsetting downward effect on the interest rate. The symmetric model struggles to generate a sizable forward premium bias, however.[9]

In Panel B, under complete markets, the correlations between log nominal SDFs ($\phi = 40, 60$) are close to 1. This correlation is much smaller under incomplete markets. Under incomplete markets, as the risk aversion coefficient increases, the perception of uncertainty is higher (see $\overline{\sigma}^2_{1,t}$) and the valuations of the future are lower (see $\overline{\mu}_{1,t}$). As risk aversion is increased, the log nominal SDF exhibits increasing variability with little effect on the interest rate or inflation. Presumably, this feature of the model contributes to the ability of Swanson (2019) to generate equity premia and returns volatility that match the data.

From Panel C, the correlation between the nominal and real exchange rate depreciation is over 0.9 under complete markets and incomplete markets. Even though country 1 and 2's log nominal SDFs are highly correlated under complete markets, the implied volatility of the exchange rate is not unreasonable.[10] Note that as Lustig and Verdelhan (2019) established, exchange rate volatility under incomplete markets is lower than under complete markets.

Panel D shows that implied volatility of the model's macroeconomic variables are not implausible. Under both complete markets and incomplete markets, cross-country consumption growth correlations are small and lie below output growth correlations.

Although the risk aversion coefficient has only a modest effect on the unconditional moments in Table 4, it has a large effect on the uncertainty-risk factor. Figure 5 shows, for a given risk aversion coefficient, perception of uncertainty is higher under incomplete markets than complete markets, but in both cases, $\overline{\sigma}^2_t$ is increasing in the risk aversion coefficient (ϕ).[11]

7.2. *Heterogeneous Countries.* The symmetric country model produces a time-varying risk premium that fluctuates from positive to negative, but does not produce systematic risk in the sense of a sizable nonzero meaned currency risk premium. In this section, we investigate how cross-country differences in monetary policy and productivity growth can affect systematic currency risk premia and the forward premium bias. We begin with heterogeneous monetary policy, then heterogeneous productivity growth, followed by a combination of heterogeneous monetary policy and productivity growth.

7.2.1. *Heterogeneous monetary policy.* Table 5 reports effects from heterogeneity in monetary policy with symmetric productivity growth. In this table, we suppress reporting of variables that are not substantially changed from the symmetric benchmark model to avoid redundancy. The specifications we consider include country 1 following the benchmark Taylor rule (inflation response coefficient $\xi_1 = 1.5$ and output gap response coefficient $\zeta_1 = 0.5$) and letting country 2 either deviate by targeting inflation ($\xi_2 = 4.0$ and $\zeta_2 = 0.1$), by setting very procyclical interest rates ($\xi_2 = 1.5$ and $\zeta_2 = 2.0$), or through a lack of monetary discipline (accommodating inflation and less responsiveness to the output gap, $\xi_2 = 1.3$ and $\xi_2 = 0.3$). We

[9] We note that $\beta_F > 1$ under incomplete markets. This is generally true also for the specifications we report below. A Fama coefficient greater than 1 does occasionally occur in the data. Ending the sample in 2007Q4, to avoid the zero-lower bound associated with the Global Financial Crisis, and letting the United States be the home country, we find a slope (t-ratio, beginning of sample in parentheses) of 1.27 (2.93, 1997Q2) for Chile, 1.22 (4.28, 1993Q3) for Hungary, and 1.35 (2.35, 1984Q1) for Taiwan.

[10] At a quarterly rate, the volatility of the point-sampled U.S. dollar-Euro rate from 1999Q1–2023Q2 is 15.58 and the volatility of the U.S. dollar-Canadian dollar rate is 16.33. Frequently, a challenge in international macro models is that too much risk sharing depresses exchange rate volatility. Under complete markets, the variance of the nominal exchange rate depreciation is, $\mathrm{Var}(\Delta s_{1,2,t+1}) = \mathrm{Var}(n_{2,t+1}) + \mathrm{Var}(n_{1,t+1}) - 2\mathrm{Cov}(n_{1,t+1}, n_{2,t+1})$. Brandt et al. (2006) identify the problem as a high covariance between the log nominal SDFs, $\mathrm{Cov}(n_{1,t+1}, n_{2,t+1})$, which depresses exchange rate depreciation volatility.

[11] Since the countries are symmetric, we only report the values for country 1.

TABLE 5
HETEROGENEOUS MONETARY POLICY UNDER LCP

	Complete Markets Country 2				Incomplete Markets Country 2			
	Inflation Targeter	Procyclical Interest Rate	Undis-ciplined	Accommo-dates Inflation	Inflation Targeter	Procyclical Interest Rate	Undis-ciplined	Accommo-dates Inflation
ξ_1	1.5	1.5	1.5	2.5	1.5	1.5	1.5	2.5
ζ_1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
ξ_2	4.0	1.5	1.3	1.3	4.0	1.5	1.3	1.3
ζ_2	0.1	2.0	0.3	0.5	0.1	2.0	0.3	0.5
Panel A. Returns								
$\overline{rp}_t$	0.456	−0.448	0.185	0.097	0.535	−0.406	0.112	−0.147
$sd(rp_t)$	1.271	0.869	1.085	1.095	1.906	1.750	1.785	1.793
$\max(rp_t)$	4.741	2.483	3.870	4.087	9.258	7.429	8.717	8.070
$\min(rp_t)$	−4.033	−3.405	−3.814	−3.896	−8.353	−8.192	−7.291	−8.146
$\overline{er}_t$	0.421	−0.422	0.112	−0.002	0.622	−0.405	0.119	−0.147
β_F	0.953	0.996	0.851	0.845	1.185	1.112	1.114	1.125
Panel B. Log Nominal SDF								
$\overline{\mu}_{1,t}$	−9.446	−9.140	−9.379	−10.090	−14.023	−13.805	−13.807	−14.340
$\overline{\mu}_{2,t}$	−10.284	−9.955	−8.016	−8.361	−14.776	−14.632	−13.037	−13.214
$\overline{\sigma}^2_{1,t}$	16.278	16.294	16.283	16.282	25.169	24.890	24.867	24.889
$\overline{\sigma}^2_{2,t}$	17.216	15.313	16.711	16.273	26.634	24.293	25.976	24.909
$\rho(n_{1,t}, n_{2,t})$	0.987	0.992	0.990	0.990	0.349	0.296	0.320	0.320
Panel C. Exchange Rate, Interest Rate, Inflation								
$sd(\Delta s_{1,2,t})$	15.226	11.933	13.244	13.072	12.900	8.908	10.705	10.581
$sd(\Delta q_{1,2,t})$	13.712	9.667	11.822	11.828	11.193	7.756	9.257	9.250
$\bar{\imath}_{1,t}$	1.327	0.933	1.120	1.946	1.438	1.344	1.444	1.896
$\bar{\imath}_{2,t}$	1.704	2.276	−0.370	0.042	1.518	2.506	0.106	0.716
$sd(i_{1,t})$	1.877	1.718	1.769	1.881	1.500	1.386	1.414	1.626
$sd(i_{2,t})$	2.651	6.896	1.911	2.006	2.418	1.812	1.511	1.543
$sd(\pi_{1,t})$	2.743	2.660	2.697	2.391	2.344	2.280	2.299	2.131
$sd(\pi_{2,t})$	2.560	7.899	3.094	2.973	2.252	2.675	2.526	2.466
Panel D. Quantities								
$sd(\Delta y_{1,t})$	3.979	3.784	3.898	4.206	3.160	3.037	3.083	3.220
$sd(\Delta y_{2,t})$	6.054	2.619	4.462	3.909	4.484	2.234	3.513	3.099
$sd(\Delta c_{1,t})$	4.216	4.241	4.262	4.922	3.548	3.541	3.481	3.648
$sd(\Delta c_{2,t})$	7.163	2.733	4.990	4.148	5.164	2.529	4.037	3.521
$\rho(\Delta y_{1,t}, \Delta y_{2,t})$	0.411	0.335	0.426	0.362	0.299	0.152	0.281	0.250
$\rho(\Delta c_{1,t}, \Delta c_{2,t})$	0.055	0.059	0.143	0.089	−0.040	−0.147	−0.024	−0.034

NOTE: $\overline{\bullet}$ denotes the time-series mean of the variable stated, $sd(\bullet)$ is the standard deviation (volatility) of the variable stated, $\rho(\bullet)$ is the correlation of the variables stated, $\max(\bullet)$ is the maximum of the variable stated, and $\min(\bullet)$ is the minimum of the variable stated. $y_{1,t}$ and $y_{2,t}$ are log output in countries 1 and 2, $c_{1,t}$ and $c_{2,t}$ are log consumption in countries 1 and 2, $i_{1,t}$ and $i_{2,t}$ are the nominal interest rates in countries 1 and 2, $\pi_{1,t}$ and $\pi_{2,t}$ are inflation in countries 1 and 2, $n_{1,t}$ and $n_{2,t}$ are the log nominal SDFs in countries 1 and 2, $\mu_{1,t}$ and $\mu_{2,t}$ are the conditional means of country 1's and country 2's log nominal SDF, $\sigma^2_{1,t}$ and $\sigma^2_{2,t}$ are the conditional variances of country 1's and country 2's log nominal SDF, $q_{1,2,t}$ is the log real exchange rate, $s_{1,2,t}$ is the log nominal exchange rate, $rp_t = E_t(i_{1,t} - i_{2,t} - \Delta s_{1,2,t+1})$ is the currency risk premium, $er_t = i_{1,t-1} - i_{2,t-1} - \Delta s_{1,2,t}$ is the ex post currency excess return, and β_F is the Fama coefficient. Variables are multiplied by 400 to state in percent per annum. Except for the monetary policy parameters, parameterization follows from Section 5. Reported are median values from 10 replications of 5,000 periods.

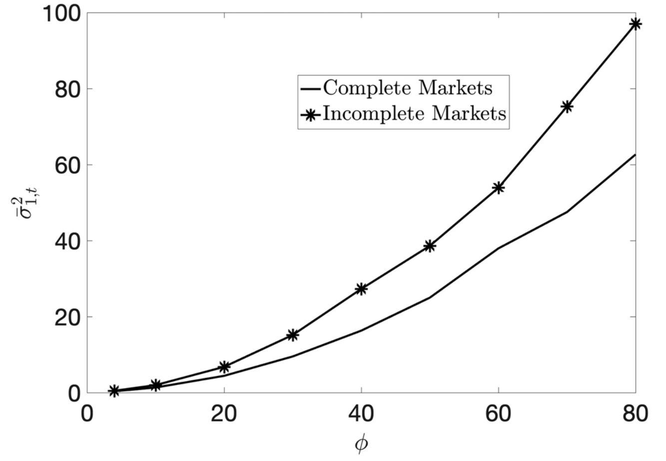


FIGURE 5

$\overline{\sigma}^2_{1,t}$ AND RISK AVERSION COEFFICIENT (ϕ) UNDER LCP

NOTES: $\overline{\sigma}^2_{1,t}$ IS THE TIME-SERIES MEAN OF COUNTRY 1'S LOG NOMINAL SDF CONDITIONAL VARIANCE, STATED IN PERCENT PER ANNUM. PARAMETERIZATION FOLLOWS FROM SECTION 5 EXCEPT FOR THE RISK AVERSION COEFFICIENT (ϕ).

also consider a specification labeled "Accommodates Inflation" where country 2 is relatively more accommodative to inflation ($\xi_1 = 2.5$, $\zeta_1 = 0.5$, $\xi_2 = 1.3$, and $\zeta_2 = 0.5$).

These variations in monetary policy generate a modestly larger average but not more volatile risk premium. When country 2 is either undisciplined or accommodating to inflation, there is a small forward premium bias under complete markets ($\beta_F \approx 0.85$), and an opposite effect under incomplete markets ($\beta_F > 1$).

Under both complete markets and incomplete markets, country 2 inflation targeting and procyclical interest rate policies cause country 2 to focus more on the present than country 1 ($\overline{\mu}_{2,t} < \overline{\mu}_{1,t}$). The result is less non-precautionary saving and higher interest rates in country 2. The opposite happens when country 2's monetary policy is either undisciplined or if it accommodates inflation.

In terms of how monetary policy affects relative uncertainty and precautionary saving, inflation targeting and undisciplined policy elevate $\overline{\sigma}^2_{2,t}$ over $\overline{\sigma}^2_{1,t}$, whereas procyclical interest rate policy suppresses $\overline{\sigma}^2_{2,t}$ relative to $\overline{\sigma}^2_{1,t}$. Accommodating inflation has almost no differential cross-country effect.

In none of these cases does pursuing different monetary policies create sizable differences in the conditional variances of the log nominal SDFs so the risk premium stays small. Similarly, monetary policy heterogeneity does not have a big effect on the covariances between the conditional means and conditional variances of the log nominal SDF, and thus, achieves only a modest forward premium bias. Inflation targeting does, however, generate too much output and consumption volatility in country 2 under complete markets. This effect on output and consumption volatility is not as strong under incomplete markets.

7.2.2. *Heterogeneous productivity growth.* This section maintains symmetric monetary policy ($\xi = 1.5$ and $\zeta = 0.5$) to isolate the effects of cross-country heterogeneity in productivity growth. In order to discipline the model with reasonable parameters for productivity growth across countries, we estimate the productivity growth with long-run risk and stochastic volatility processes (Equations (34)–(36)) for Australia, Canada, and Japan. The posterior means and the upper and lower 5% bands from a Bayesian estimation are shown in Table 6.

Table 7 shows that the simulated TFP growth generated by the estimated processes (labeled unadjusted, as was the case for the United States) overstate the volatility in the data.

TABLE 6
POSTERIOR MEANS—PRODUCTIVITY GROWTH PROCESSES FOR AUSTRALIA, CANADA, AND JAPAN

	Australia			Canada			Japan		
	Posterior Mean	5%	95%	Posterior Mean	5%	95%	Posterior Mean	5%	95%
μ_g	0.001	−0.001	0.002	0.001	−0.001	0.002	0.004	0.002	0.006
θ	1.634	1.108	2.117	0.618	0.197	1.020	1.288	0.843	1.838
ρ_x	0.731	0.652	0.817	0.742	0.664	0.823	0.744	0.666	0.824
ρ_ω	0.842	0.765	0.921	0.841	0.762	0.919	0.839	0.761	0.920
η	−1.753	−4.212	0.690	−1.159	−3.708	1.340	−1.859	−4.375	0.512
μ_ω	−6.376	−6.872	−5.910	−6.021	−6.334	−5.683	−5.864	−6.391	−5.458

NOTE: The productivity growth processes for Australia, Canada, and Japan are governed by Equations (34)–(36). Posterior means and the upper and lower 5% bands from a Bayesian estimation are reported.

TABLE 7
VOLATILITY OF PRODUCTIVITY GROWTH AND COMPONENTS FOR AUSTRALIA, CANADA, AND JAPAN

	Data	Simulated					
		Unadjusted			Adjusted		
	$sd(\Delta a_t)$	$sd(\Delta a_t)$	$sd(x_t)$	$sd(\omega_t)$	$sd(\Delta a_t)$	$sd(x_t)$	$sd(\omega_t)$
Australia	3.582	4.082	1.149	0.238	3.581	0.983	0.211
Canada	2.275	3.372	2.107	0.761	2.271	1.427	0.525
Japan	4.392	4.869	1.906	0.345	4.386	1.673	0.314

NOTE: $sd(\bullet)$ is the standard deviation (volatility) of the variable stated as percent per annum. a_t is log productivity, x_t is the long-run risk component, and ω_t is the stochastic volatility component.

The overstatement for Japan is minor and a bit more substantial for Australia and Canada. For all three countries, we apply an adjustment factor as we did for the United States. The panel labeled "Adjusted" shows the standard deviations of the simulated productivity growth ($sd(\Delta a_t)$) and its components.

Figure 6 plots productivity growth and log levels from the data along with a realized simulation of the adjusted process. As can be seen, the model does a reasonable job of capturing major features of the productivity data.

Table 8 shows the simulation results using adjusted country-specific productivity growth processes for Australia, Canada, Japan, and the United States. In each case, the United States is country 1. Monetary policy is symmetric with $\zeta = 1.5$ and $\xi = 0.5$. The table also includes available moments from the data. Though we are not conducting a quantitative moment matching exercise, the estimated TFP growth processes are used as guidance for a thought experiment on the effect of reasonable productivity asymmetries across countries, and the comparison to the data moments serves as a reality check on the model.

Asymmetric country-specific productivity is able to generate the correct sign on the risk premium/excess return for the United States against Japan and the United States against Canada. Japan is the risky country relative to the United States whereas Canada is the safe country. The correct sign on the excess return holds under both complete markets and incomplete markets. The Fama coefficient continues to lie modestly below 1 under complete markets and a bit above 1 under incomplete markets. In regard to uncertainty risk, due to historically low interest rates, the Japanese yen is typically thought of as the carry trade funding currency.[12]

Asymmetric productivity modestly raises exchange rate volatility. For Japan, it causes the average interest rate to become negative. The model does not incorporate a zero lower

[12] The carry trade is a trading strategy where you short the low interest rate currency and go long the high interest rate currency.

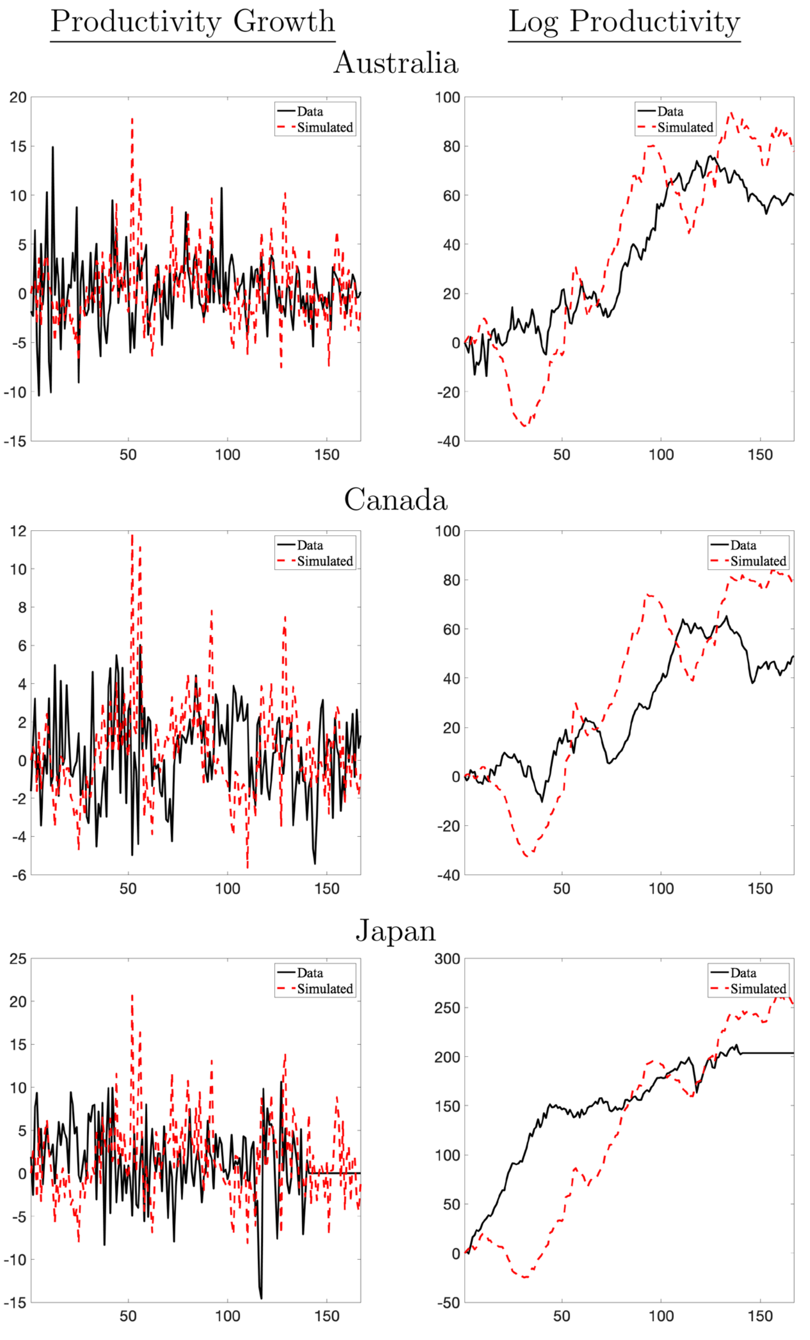


FIGURE 6

PRODUCTIVITY DATA AND REALIZED SIMULATION

bound. This feature could be remedied by incorporating a positive amount of steady-state inflation along with full indexation. Doing so raises average inflation and the average nominal interest rate by the steady-state amount but has little effect on anything else.

TABLE 8
HETEROGENEOUS PRODUCTIVITY GROWTH UNDER LCP (AUSTRALIA, CANADA, JAPAN, AND UNITED STATES)

	United States (1)–Japan (2)			United States (1)–Australia(2)			United States (1)–Canada (2)		
	Data	Complete Markets	Incomplete Markets	Data	Complete Markets	Incomplete Markets	Data	Complete Markets	Incomplete Markets
Panel A. Returns									
$\overline{rp}_t$		1.154	0.654		0.357	0.358		−0.151	−0.111
$sd(rp_t)$		1.122	2.071		1.104	2.075		0.986	1.620
$\max(rp_t)$		5.103	9.381		4.629	9.264		3.268	7.151
$\min(rp_t)$		−3.068	−8.142		−3.481	−9.205		−3.476	−7.473
$\overline{er}_t$	0.490	1.124	0.567	−4.366	0.447	0.337	−2.398	−0.170	−0.154
β_F	−2.820	0.843	1.211	−0.553	0.872	1.208	−0.074	0.880	1.044
Panel B. Log Nominal SDF									
$\overline{\mu}_{1,t}$		−14.646	−13.713		−10.754	−13.978		−9.024	−13.900
$\overline{\mu}_{2,t}$		−11.426	−25.987		−9.598	−19.268		−9.341	−13.208
$\overline{\sigma}^2_{1,t}$		24.471	24.655		19.398	25.208		15.790	25.160
$\overline{\sigma}^2_{2,t}$		26.783	52.810		20.216	37.810		15.599	23.105
Panel C. Exchange Rate, Interest Rate, Inflation									
$sd(\Delta s_{1,2,t})$	20.990	14.494	11.732	18.956	13.718	11.240	11.486	12.132	9.641
$sd(\Delta q_{1,2,t})$	21.204	13.028	10.339	19.448	12.502	9.923	11.434	10.859	8.375
$\bar{\imath}_{1,t}$	5.065	2.442	1.420		1.171	1.415		0.996	1.363
$\bar{\imath}_{2,t}$	0.268	−2.278	−0.367	8.060	−0.646	0.330	6.026	1.600	1.721
$sd(i_{1,t})$	2.098	3.326	1.491		2.178	1.498		1.861	1.388
$sd(i_{2,t})$	0.283	4.196	1.693	4.111	3.070	1.572	3.033	1.824	1.315
$sd(\pi_{1,t})$	2.623	3.817	2.303		2.939	2.341		2.710	2.318
$sd(\pi_{2,t})$	4.745	4.930	3.039	4.322	4.020	2.951	3.609	2.335	1.868
Panel D. Quantities									
$sd(\Delta y_{1,t})$	3.713	5.963	3.039		4.705	3.082		3.763	3.081
$sd(\Delta y_{2,t})$	3.986	4.391	3.718	4.128	4.179	3.637	3.378	3.448	2.777
$sd(\Delta c_{1,t})$	3.198	4.021	3.434		4.080	3.490		4.369	3.487
$sd(\Delta c_{2,t})$	3.841	6.170	4.076	3.118	5.740	4.040	2.709	3.708	3.189

NOTE: $\overline{\bullet}$ denotes the time-series mean of the variable stated, $sd(\bullet)$ is the standard deviation (volatility) of the variable stated, $\max(\bullet)$ is the maximum of the variable stated, and $\min(\bullet)$ is the minimum of the variable stated. $y_{1,t}$ and $y_{2,t}$ are log output in countries 1 and 2, $c_{1,t}$ and $c_{2,t}$ are log consumption in countries 1 and 2, $i_{1,t}$ and $i_{2,t}$ are the nominal interest rates in countries 1 and 2, $\pi_{1,t}$ and $\pi_{2,t}$ are inflation in countries 1 and 2, $\mu_{1,t}$ and $\mu_{2,t}$ are the conditional means of country 1's and country 2's log nominal SDF, $\sigma^2_{1,t}$ and $\sigma^2_{2,t}$ are the conditional variances of country 1's and country 2's log nominal SDF, $q_{1,2,t}$ is the log real exchange rate, $s_{1,2,t}$ is the log nominal exchange rate, $rp_t = E_t(i_{1,t} - i_{2,t} - \Delta s_{1,2,t+1})$ is the currency risk premium, $er_t = i_{1,t-1} - i_{2,t-1} - \Delta s_{1,2,t}$ is the ex post currency excess return, and β_F is the Fama coefficient. Variables are multiplied by 400 to state in percent per annum. Country-specific productivity growth process parameters are employed. Except for the productivity growth process parameters, parameterization follows from Section 5. Reported are median values from 10 replications of 5,000 periods.

7.2.3. *Heterogeneous monetary policy and productivity growth.* Finally, we combine heterogeneity in monetary policy and productivity growth to ask if there is a combination that is able to qualitatively, if not quantitatively, explain the currency risk premium. Here, we allow both countries to depart from the benchmark monetary policy, except for the United States in the United States–Japan specification. On the one hand, we make no claims that these monetary policies are necessarily tethered to reality. On the other hand, these policies are not necessarily unrealistic. Estimation of monetary policy rules are fraught with technical difficulties, result in estimates with large standard errors, vary over time, and typically do not validate the Taylor rule.

Here, we pare down reporting of the moments to focus on features of key interest. Table 9 shows the results. The message from Table 9 is an inflation targeting Japan produces an average risk premium of 2.1% under complete markets and 1.9% under incomplete markets. The

TABLE 9
HETEROGENEOUS MONETARY POLICY AND PRODUCTIVITY GROWTH UNDER LCP (AUSTRALIA, CANADA, AND JAPAN (COUNTRY 2) WITH THE UNITED STATES AS COUNTRY 1)

	Japan		Canada		Australia	
ξ_1	1.5		4.0		4.0	
ζ_1	0.5		0.1		0.1	
ξ_2	4.0		1.5		1.5	
ζ_2	0.1		2.0		2.0	
	Complete Markets	Incomplete Markets	Complete Markets	Incomplete Markets	Complete Markets	Incomplete Markets
$\overline{rp}_t$	2.103	1.878	−0.930	−0.995	−0.270	−0.730
$sd(rp_t)$	1.443	2.233	1.152	1.669	1.209	2.130
$\max(rp_t)$	7.285	11.065	3.185	5.427	4.025	8.158
$\min(rp_t)$	−2.860	−6.957	−4.898	−8.769	−4.197	−9.828
$\overline{er}_t$	2.020	1.889	−0.872	−1.038	−0.181	−0.808
β_F	0.938	1.220	0.993	1.111	1.007	1.149
$sd(\Delta s_{1,2,t})$	19.540	16.848	14.692	11.921	16.727	12.754
$sd(\Delta q_{1,2,t})$	17.164	14.802	11.921	10.297	13.719	11.124
$\bar{\imath}_{1,t}$	3.220	1.418	1.663	1.554	1.839	1.546
$\bar{\imath}_{2,t}$	0.008	−0.201	2.871	2.777	0.006	2.070
$sd(\Delta y_{1,t})$	6.730	3.250	5.539	4.402	7.040	4.345
$sd(\Delta y_{2,t})$	5.693	6.005	2.591	1.940	3.450	2.757
$sd(\Delta c_{1,t})$	4.435	3.517	7.216	5.179	6.712	5.119
$sd(\Delta c_{2,t})$	11.265	6.701	2.570	2.255	3.714	3.082

NOTE: $\bar{\bullet}$ denotes the time-series mean of the variable stated, $sd(\bullet)$ is the standard deviation (volatility) of the variable stated, $\max(\bullet)$ is the maximum of the variable stated, and $\min(\bullet)$ is the minimum of the variable stated. $y_{1,t}$ and $y_{2,t}$ are log output in countries 1 and 2, $c_{1,t}$ and $c_{2,t}$ are log consumption in countries 1 and 2, $i_{1,t}$ and $i_{2,t}$ are the nominal interest rates in countries 1 and 2, $q_{1,2,t}$ is the log real exchange rate, $s_{1,2,t}$ is the log nominal exchange rate, $rp_t = E_t(i_{1,t} - i_{2,t} - \Delta s_{1,2,t+1})$ is the currency risk premium, $er_t = i_{1,t-1} - i_{2,t-1} - \Delta s_{1,2,t}$ is the ex post currency excess return, and β_F is the Fama coefficient. Variables are multiplied by 400 to state in percent per annum. Country-specific productivity growth process parameters are employed. Except for the monetary policy and productivity growth process parameters, parameterization follows from Section 5. Reported are median values from 10 replications of 5,000 periods.

United States as an inflation targeter with Canada and Australia pursuing procyclical monetary policy gives a negative average risk premium of about −1% for Canada under incomplete markets and −0.73% for Australia under incomplete markets. The sign of the average risk premium coincide with the signs of currency excess returns in the data.

Finally, we address the challenge faced by the model in generating the forward premium anomaly, even when there is a sizable currency risk premium. We can follow Lustig and Verdelhan (2019) in representing the exchange rate (depreciation) as

$$\Delta s_{1,2,t+1} = n_{2,t+1} - n_{1,t+1} + \eta_{t+1},$$

where η_{t+1} is the "wedge" introduced by market incompleteness (with $\eta_{t+1} = 0$ under complete markets). Let $\Omega \equiv \mathrm{Var}(i_{1,t} - i_{2,t})$. Then, the Fama coefficient (β_F) has the decomposition

$$\begin{aligned}\frac{\mathrm{Cov}(\Delta s_{1,2,t+1}, (i_{1,t} - i_{2,t}))}{\Omega} &= \begin{bmatrix} (\mathrm{Var}(\mu_{2,t}) + \mathrm{Var}(\mu_{1,t})) \\ -2\mathrm{Cov}(\mu_{1,t}, \mu_{2,t}) + (1/2)\mathrm{Cov}(\mu_{1,t}, \sigma_{1,t}^2) \end{bmatrix} / \Omega \\ &+ \begin{bmatrix} (1/2)\mathrm{Cov}(\mu_{2,t}, \sigma_{2,t}^2) - (1/2)\mathrm{Cov}(\mu_{1,t}, \sigma_{2,t}^2) \\ -(1/2)\mathrm{Cov}(\mu_{2,t}, \sigma_{1,t}^2) \end{bmatrix} / \Omega \\ &+ \begin{bmatrix} \mathrm{Cov}(\eta_{t+1}, \mu_{2,t}) - \mathrm{Cov}(\eta_{t+1}, \mu_{1,t}) \\ +(1/2)\mathrm{Cov}(\eta_{t+1}, \sigma_{2,t}^2) - (1/2)\mathrm{Cov}(\eta_{t+1}, \sigma_{1,t}^2) \end{bmatrix} / \Omega.\end{aligned}$$

TABLE 10
FAMA COEFFICIENT (β_F) DECOMPOSITION

	Complete Markets	Incomplete Markets	Backus et al. (2013) Calibrated	Backus et al. (2013) Estimated
β_F	0.936	1.220	−29.837	0.994
$= \text{Var}(\mu_{2,t})/\Omega + \text{Var}(\mu_{1,t})/\Omega$	3.517	15.737	890.006	1.294
$-2\text{Cov}(\mu_{1,t}, \mu_{2,t})/\Omega$	−2.539	−1.781	0.237	−0.006
$+(1/2)\text{Cov}(\mu_{1,t}, \sigma^2_{1,t})/\Omega$	−0.997	−5.203	−409.631	−0.145
$+(1/2)\text{Cov}(\mu_{2,t}, \sigma^2_{2,t})/\Omega$	−1.153	−9.711	−510.204	−0.145
$-(1/2)\text{Cov}(\mu_{1,t}, \sigma^2_{2,t})/\Omega$	1.081	0.841	−0.122	−0.003
$-(1/2)\text{Cov}(\mu_{2,t}, \sigma^2_{1,t})/\Omega$	1.026	1.049	−0.122	−0.001
$+\text{Cov}(\eta_{t+1}, \mu_{2,t})/\Omega$	–	−7.644	–	–
$-\text{Cov}(\eta_{t+1}, \mu_{1,t})/\Omega$	–	−4.101	–	–
$+(1/2)\text{Cov}(\eta_{t+1}, \sigma^2_{2,t})/\Omega$	–	7.671	–	–
$-(1/2)\text{Cov}(\eta_{t+1}, \sigma^2_{1,t})/\Omega$	–	4.361	–	–

NOTE: β_F is the Fama coefficient, Var(•) is the variance of the variable stated, Cov(•) is the covariance of the variable stated, $\Omega \equiv \text{Var}(i_{1,t} - i_{2,t})$, $\mu_{1,t}$ and $\mu_{2,t}$ are the conditional means of country 1's and country 2's log nominal SDF, and $\sigma^2_{1,t}$ and $\sigma^2_{2,t}$ are the conditional variances of country 1's and country 2's log nominal SDF. United States and Japan productivity growth process parameters are employed under complete and incomplete markets. Except for monetary policy ($\xi_1 = 1.5$, $\xi_2 = 4.0$, $\zeta_1 = 0.5$, and $\zeta_2 = 0.1$) and productivity growth process parameters, parameterization follows from Section 5 under complete and incomplete markets. Results are from a realization from the United States and Japan production model under LCP and complete and incomplete markets and from a calibrated and estimated Backus et al. (2013) endowment model.

Note that the third line is identically zero under complete markets.

In Table 10, we decompose the Fama coefficient generated by a realization from the United States and Japan model under complete markets and under incomplete markets. In the complete markets case, the covariances between $\mu_{k,t}$ and $\sigma^2_{k,t}$, for $k \in \{1, 2\}$, are negative, which helps to work in the direction of a negative forward premium bias, but the covariances of $\mu_{k,t}$ and $\sigma^2_{j,t}$, for $j, k \in \{1, 2\}$ and $j \neq k$, are also negative and approximately the same size, which has an offsetting effect. In the incomplete markets model, the large variances of $\mu_{1,t}$ and $\mu_{2,t}$ relative to the variance of the interest rate differential cannot be overcome by the covariance terms.

We contrast these results with a decomposition from the endowment model of Backus et al. (2013), where preferences are recursive and the consumption growth process features long-run risk and stochastic volatility. The Appendix provides a sketch of their model and the details of their calibrated exogenous process and our estimation of the process. The column labeled "Calibrated" uses their benchmark calibration where both countries are completely symmetric and the risk aversion coefficient is 90.408. This gives a Fama coefficient (β_F) of −29.837. It is achieved by large negative covariances between $\mu_{k,t}$ and $\sigma^2_{k,t}$, for $k \in \{1, 2\}$. Producing the forward premium anomaly with their model is not automatic, however. If one estimates the consumption growth process from United States quarterly data from 1947 to 2019, the Fama coefficient (β_F) becomes 0.994, and the large negative covariances between $\mu_{k,t}$ and $\sigma^2_{k,t}$, for $k \in \{1, 2\}$, go away.

8. CONCLUSION

This article studies the currency risk premium and the forward premium bias in a two-country dynamic stochastic general equilibrium New Keynesian model. Exogenous productivity growth features long-run risk and stochastic volatility. Productivity growth process parameters are estimated using data from Australia, Canada, Japan, and the United States.

The currency risk premium and forward premium bias are two different phenomenon. Currency risk emerges from cross-country differences in precautionary saving. The forward premium bias emerges from the negative covariance between cross-country differences in nonprecautionary saving and precautionary saving. There can be a currency risk premium with no forward premium bias, but there cannot be a forward premium bias with no currency risk premium.

Cross-country heterogeneity in monetary policy and productivity growth allows our model to generate reasonably sized currency risk premia with signs (direction) that correspond to currency excess returns computed from the data. We find an important difference between endowment and production models. Endowment models have been successful in generating the forward premium anomaly, but this is not possible in our production model. Alternative frictions imposed by incomplete markets and alternative export pricing conventions of LCP, PCP, and DCP do not help to increase the forward premium bias.

Supporting Information

Additional supporting information may be found online in the Supporting Information section at the end of the article.

Online Appendix

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