Changing Monetary Policy Rules, Learning, and Real Exchange Rate Dynamics.

Konstanz Seminar on Monetary Theory and Policy, May 2008
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1 What the paper does

- Real interest parity model of exchange rate determination for real DM–dollar rate.

- Short-term interest rates set by reaction function (the Taylor Rule).
  - Some evidence for this will be provided.

- People ignorant of exact values of Taylor Rule coefficients
  - People use adaptive (least-squares) learning rules to obtain coefficient values.

- Evaluate ability of formulation to explain (fit) real DM–dollar history.
2 Why do it?

- Fundamentals implied by standard open economy macro models \((m, m^*, y, y^*, p, p^*)\) do poorly at quarterly/annual horizons.

- If monetary policy is governed by a Taylor rule, fundamentals are inflation gaps and output gaps.

- Why Germany? Evidence that they follow something like a Taylor rule.

  - Taylor rule wasn’t “known” until 1993, not generally acknowledged until much later. Academic acceptance not necessarily the same as public beliefs.
- Model how public might have learned about the Taylor rule and assess the fit of the learning rules.
  - Learning allows for model uncertainty, structural instability
  - Instability is a generally accepted FACT in international finance
  - The source of pervasive failures to FIT out of sample

- Can long swings be explained by historical Taylor-rule fundamentals data? Real depreciation of late 70s, the “great appreciation” and subsequent “great depreciation”
Illustration of the problem: Deviations from PPP
Illustration of the problem: Nominal Exchange Rate and Monetary Fundamentals (in logs)
Dependent Variable: E  
Method: Least Squares  
Date: 05/05/08 Time: 09:03  
Sample: 1973Q1 2005Q4  
Included observations: 132  

$$E = C(1) + C(2) \cdot (M - MSTAR) + C(3) \cdot (Y - YSTAR)$$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>0.570824</td>
<td>0.106559</td>
<td>5.356865</td>
<td>0</td>
</tr>
<tr>
<td>C(2)</td>
<td>-0.23326</td>
<td>0.1317</td>
<td>-1.77112</td>
<td>0.0789</td>
</tr>
<tr>
<td>C(3)</td>
<td>0.883319</td>
<td>0.149536</td>
<td>5.907051</td>
<td>0</td>
</tr>
</tbody>
</table>

R-squared: 0.2136  
Mean dependent var: 0.675701  
Adjusted R-squared: 0.201407  
S.D. dependent var: 0.202651  
S.E. of regression: 0.181097  
Akaike info criterion: -0.55711  
Schwarz criterion: -0.49159  
Hannan-Quinn citer.: -0.53048  
Durbin-Watson stat: 0.119509  
Prob(F-statistic): 0
Inflation Differentials and Real Dollar–DM Rate
Inflation Differentials and Real Dollar–DM Rate

log real DM-dollar rate

DEU-USA inflation

Inflation Differentials and Real Dollar–DM Rate

DEU-USA inflation

USA-DEU inflation

log real DM-dollar

Inflation Differentials with ‘Poor-Man’s’ Structural Break and Real Dollar–DM Rate
3 The Data


- Imputed DM-dollar from 1998.1–2005.4

- Goods prices are real GDP deflator (IFS series code 13499BIRZF).

- Break in German price level at reunification (1990.4–1991.1) smoothed as in Engel and West (2006)

- Three definitions of activity gap.
  - Output gap constructed by source.
- Recursively HP detrended real GDP
- Recursively HP detrended unemployment rate
- Germany is the ‘home’ country. ↑ real exchange rate $\implies$ real DM depreciation.
4 Differentials in interest rate reaction functions

- Evidence that modeling differential Taylor rule under homogeneity is okay?

- Evidence of structural instability?
\[
\begin{align*}
\text{US Target} & \quad i^T_{U, t} = \tilde{i}_U + \gamma \pi (E_t \pi_t U_{t+1} - \tilde{\pi}_U) + \gamma_x \tilde{x}_U, \\
\text{Desired} & \quad i^{*}_{U, t} = (1 - \rho)i^T_{U, t} + \rho i^{*}_{U, t-1}, \\
\text{Gap} & \quad \eta_{U, t} + \text{Exog. policy shk}, \\
\text{Partial} & \quad \text{Adj}.
\end{align*}
\]

\[
\begin{align*}
\text{German Target} & \quad i^T_{G, t} = \tilde{i}_G + \gamma \pi (E_t \pi_t G_{t+1} - \tilde{\pi}_G) + \gamma_x \tilde{x}_G, \\
\text{Partial} & \quad \text{Adj}.
\end{align*}
\]
\[ i_t = \delta + \rho i_{t-1} + (1 - \rho) \left( \gamma_{\pi} E_t \pi_{t+1} + \gamma_x x_t + \gamma_q q_t, \right) + \eta_t, \]
\[ \delta \equiv (1 - \rho) \left( \left( \tilde{i}_G - \tilde{i}_U \right) - \gamma_{\pi} \left( \pi_G - \pi_U \right) \right), \]
\[ \eta_t \overset{iid}{\sim} (0, \sigma_\eta^2). \]

- Estimate by GMM

\[ i_t = \delta + (1 - \rho) \left[ \gamma_{\pi} \pi_{t+1} + \gamma_x x_t + \gamma_s q_t \right] + \rho i_{t-1} + \eta'_t, \]
\[ \eta'_t = \eta_t - (1 - \rho) \gamma_{\pi} \left[ \pi_{t+1} - E_t \pi_{t+1} \right] \]
Table 1: GMM Estimates of Bundesbank–Fed Relative Interest-Rate Reaction Function with Contemporaneous Real Exchange Rate Feedback. Bold indicates significance at the 5% level

<table>
<thead>
<tr>
<th>Source output gap</th>
<th>( \delta ) (t-ratio)</th>
<th>( \rho ) (t-ratio)</th>
<th>( \gamma_\pi ) (t-ratio)</th>
<th>( \gamma_x ) (t-ratio)</th>
<th>( \gamma_q ) (t-ratio)</th>
<th>J-statistic (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>61.2-79.2</td>
<td>-0.006 (-6.436)</td>
<td>0.724 (12.257)</td>
<td>-0.042 (-0.367)</td>
<td>0.144 (4.509)</td>
<td>0.023 (3.646)</td>
<td>12.482 (0.408)</td>
</tr>
<tr>
<td>79.3-05.4</td>
<td>-0.001 (-1.014)</td>
<td>0.892 (21.751)</td>
<td>1.332 (3.277)</td>
<td>0.382 (2.355)</td>
<td>0.010 (0.734)</td>
<td>4.237 (0.979)</td>
</tr>
</tbody>
</table>

Structural change test

<table>
<thead>
<tr>
<th>Test statistic</th>
<th>p-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>All coeffs.</td>
<td>39.645</td>
<td>0.000</td>
</tr>
<tr>
<td>Inflation coeff.</td>
<td>10.597</td>
<td>0.001</td>
</tr>
<tr>
<td>HP output gap</td>
<td>$\delta$ (t-ratio)</td>
<td>$\rho$ (t-ratio)</td>
</tr>
<tr>
<td>---------------</td>
<td>-----------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>61.2-79.2</td>
<td>-0.003</td>
<td>0.797</td>
</tr>
<tr>
<td></td>
<td>-2.894</td>
<td>15.971</td>
</tr>
<tr>
<td>79.3-05.4</td>
<td>0.000</td>
<td>0.873</td>
</tr>
<tr>
<td></td>
<td>-0.169</td>
<td>21.971</td>
</tr>
</tbody>
</table>

Structural change test

- All coeffs. Test statistic \textbf{34.191} p-value 0.000
- Inflation coeff. Test statistic \textbf{13.527} p-value 0.000
Table 1: GMM Estimates of Bundesbank–Fed Relative Interest-Rate Reaction Function with Contemporaneous Real Exchange Rate Feedback. Bold indicates significance at the 5% level.

<table>
<thead>
<tr>
<th>HP unemployment gap</th>
<th>$\delta$ (t-ratio)</th>
<th>$\rho$ (t-ratio)</th>
<th>$\gamma_\pi$ (t-ratio)</th>
<th>$\gamma_x$ (t-ratio)</th>
<th>$\gamma_q$ (t-ratio)</th>
<th>J-statistic (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>61.2-79.2</td>
<td>-0.002</td>
<td>0.593</td>
<td>0.014</td>
<td>-0.702</td>
<td>0.003</td>
<td>10.234</td>
</tr>
<tr>
<td></td>
<td>-2.365</td>
<td>9.546</td>
<td>0.224</td>
<td>-5.523</td>
<td>1.195</td>
<td>0.595</td>
</tr>
<tr>
<td>79.3-05.4</td>
<td>0.000</td>
<td>0.843</td>
<td>1.295</td>
<td>-0.793</td>
<td>0.001</td>
<td>8.444</td>
</tr>
<tr>
<td></td>
<td>0.167</td>
<td>18.196</td>
<td>2.329</td>
<td>-3.233</td>
<td>0.134</td>
<td>0.750</td>
</tr>
</tbody>
</table>

Structural change test

<table>
<thead>
<tr>
<th></th>
<th>Test statistic</th>
<th>p-value</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>All coeffs.</td>
<td>10.482</td>
<td>0.063</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation coeff.</td>
<td>4.250</td>
<td>0.039</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Data and Fitted Values
Data and Fitted Values

5 Modeling real exchange rate dynamics with learning

• The economic model

\[
 s_t = E_t s_{t+1} - i_t.
\]

\[
 q_t = E_t q_{t+1} - i_t + E_t \pi_{t+1}.
\]
• **VAR(4).**

\[
Y'_t = (\pi_t, \ldots, \pi_{t-3}, x_t, \ldots, x_{t-3}) \\
Z'_{1,t} = Z'_{2,t} = (Y'_t, 1).
\]

• **Companion form**

\[
Y_t = \alpha + AY_{t-1} + \nu_t,
\]

\[
e_1 = (1, 0, 0, 0, 0, 0, 0, 0) \\
e_2 = (0, 0, 0, 0, 1, 0, 0, 0)
\]

\[
x_t = e_2 Y_t \\
\pi_t = e_1 Y_t
\]

*Inflation differential forecast*

\[
E_t \pi_{t+1} = e_1 (\alpha + AY_t).
\]
• Plug into interest parity condition $\implies 2^o$ stochastic difference equation

$$q_t = \delta ((1 - \rho) \gamma_\pi - 1) e_1 \alpha + (1 - \rho) \gamma_q q_{t-1} + \rho i_{t-1}$$

$$+ ((1 - \rho) \gamma_x e_2 + ((1 - \rho) \gamma_\pi - 1) e_1 A) Y_t + \eta_t + E_t q_{t+1}$$
- The MSV rational expectations solution is,

\[ q_t = a_0 + a_1 i_{t-1} + a_2 q_{t-1} + a_3 \eta_t + b Y_t \]

\[ a_2 = \frac{1}{2} (1 - \rho) \pm \frac{\sqrt{(1 - \rho)^2 - (1 - \rho) 4 \gamma_q}}{2}, \]

\[ a_1 = \frac{-\rho a_2}{(1 - \rho) \gamma_q}, \]

\[ a_0 = \left( \frac{((-1 + ((\rho - 1) a_1 + (1 - \rho)) \gamma_\pi) e_1 - b)}{a_2} \right) \alpha - \frac{(a_1 - 1)}{a_2} \delta, \]

\[ a_3 = \frac{1 - a_1}{1 - a_2}, \]

\[ b = \frac{[((1 + (a_1 - 1)(1 - \rho) \gamma_\pi)e_1) A + (a_1 - 1)(1 - \rho) \gamma_x e_2) \times \frac{((1 - a_2) I - A)^{-1}}{}}{(1 - a_2) I - A)^{-1}}. \]

- Choose solution with positive \( a_2 \). Persistent exchange rate, rational agents know lagged exchange rate enters positively. Vector \( b \) depends on \( \gamma_\pi \). Shifting \( \gamma_\pi \) might explain divergent trends b/t \( \pi_t \) and \( q \) after 1979.
Learning the rational expectations equilibrium.

- In ‘real time,’ the public proceeds as econometrician, acquires knowledge with least-squares learning rules [Evans and Honkapojian (2001)].

\[ q_t = E_t q_{t+1} - i_t + E_t \pi_{t+1}. \]

- At \( t \), given \( \alpha_{t-1}, A_{t-1} \)

\[ Y_t = \alpha_{t-1} + A_{t-1} Y_{t-1} + v_t, \]

Expected inflation:

\[ E_t \pi_{t+1} = e_1 (\alpha_{t-1} + A_{t-1} Y_t) \]

- Perceived law of motion for exchange rate

\[ q_t = a_{0,t-1} + a_{1,t-1} i_{t-1} + a_{2,t-1} q_{t-1} + a_{3,t-1} \eta_t + b_{t-1} Y_t \equiv B_{3,t-1}' Z_{3,t} \]
• Obtain observations on $\eta_t$ from perceived law of motion for the interest differential

$$ i_t = \delta_{t-1} + \rho_{t-1} i_{t-1} + \theta_{\pi,t-1} e_1 (\alpha_{t-1} + A_{t-1} Y_t) + \theta_{x,t-1} e_2 Y_t + \theta_{q,t-1} q_{t-1} + \eta_t \\
\equiv B_{4,t-1} Z_{4,t} + \eta_t. $$

• Expected exchange rate $E_{t+1} = a_{0,t-1} + a_{1,t-1} i_t + a_{2,t-1} q_t + b_{t-1} (\alpha_{t-1} + A_{t-1} Y_t)$.

• Plug inflation forecast and $E_{t+1}$ into UIP equation for actual law of motion,

$$ q_t = \frac{1}{(1 - a_{3,t-1})} \left[ a_{0,t-1} + (e_1 + b_{t-1}) (\alpha_{t-1} + A_{t-1} Y_t) + (a_{2,t-1} - 1) i_t \right]. $$
Least-squares updating. \( y_{1,t} = \pi_t, y_{2,t} = x_t, y_{3,t} = q_t, y_{4,t} = i_t \) For given \( R_{j,t-1} \ (j = 1, \ldots, 4) \),

\[
R_{j,t} &= R_{j,t-1} + g \left( Z_{j,t-1} Z'_{j,t-1} - R_{j,t-1} \right), \\
B_{j,t} &= B_{j,t-1} + g R_{j,t}^{-1} Z_{j,t-1} (y_{j,t} - B'_{j,t-1} Z_{j,t-1}).
\]

Learning path, coefficient updating generated with \( \pi_t, x_t, \) and \( i_t \) from data, but no \( q_t \) data.
## Results

Table 3: Correlations and Relative Volatility

<table>
<thead>
<tr>
<th>Form</th>
<th>Activity variable</th>
<th>Corr</th>
<th>T-ratio</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>Source gap</td>
<td>0.346</td>
<td>2.170</td>
<td>1.054</td>
</tr>
<tr>
<td></td>
<td>HP output</td>
<td>0.298</td>
<td>2.094</td>
<td>0.336</td>
</tr>
<tr>
<td></td>
<td>HP unemployment</td>
<td>0.340</td>
<td>1.834</td>
<td>1.130</td>
</tr>
<tr>
<td>1-qtr return</td>
<td>Source gap</td>
<td>-0.039</td>
<td>-0.596</td>
<td>1.254</td>
</tr>
<tr>
<td></td>
<td>HP output</td>
<td>0.033</td>
<td>0.601</td>
<td>0.458</td>
</tr>
<tr>
<td></td>
<td>HP unemployment</td>
<td>-0.008</td>
<td>-0.135</td>
<td>1.870</td>
</tr>
<tr>
<td>16-qtr return</td>
<td>Source gap</td>
<td>0.335</td>
<td>2.393</td>
<td>1.086</td>
</tr>
<tr>
<td></td>
<td>HP output</td>
<td>0.093</td>
<td>0.799</td>
<td>0.345</td>
</tr>
<tr>
<td></td>
<td>HP unemployment</td>
<td>0.093</td>
<td>0.799</td>
<td>1.033</td>
</tr>
</tbody>
</table>
Figure 5. Learning path with source output gap.
Figure 7. Learning path with HP output gap.
Figure 9. Learning path with HP unemployment gap.
6 Compare to rational expectations

Table 3: Correlations and Relative Volatility

<table>
<thead>
<tr>
<th>Form</th>
<th>Activity variable</th>
<th>Corr</th>
<th>T-ratio</th>
<th>Volatility</th>
<th>Corr</th>
<th>T-ratio</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source gap</td>
<td></td>
<td>0.308</td>
<td>2.461</td>
<td>0.965</td>
<td>0.346</td>
<td>2.170</td>
<td>1.054</td>
</tr>
<tr>
<td>Level</td>
<td>Source gap</td>
<td>0.030</td>
<td>0.700</td>
<td>2.085</td>
<td>-0.039</td>
<td>-0.596</td>
<td>1.254</td>
</tr>
<tr>
<td></td>
<td>HP output</td>
<td>-0.029</td>
<td>-0.201</td>
<td>3.678</td>
<td>0.298</td>
<td>2.094</td>
<td>0.336</td>
</tr>
<tr>
<td></td>
<td>HP UE</td>
<td>0.484</td>
<td>3.426</td>
<td>0.644</td>
<td>0.340</td>
<td>1.834</td>
<td>1.130</td>
</tr>
<tr>
<td>1-qtr return</td>
<td>Source gap</td>
<td>0.019</td>
<td>0.536</td>
<td>7.918</td>
<td>0.033</td>
<td>0.601</td>
<td>0.458</td>
</tr>
<tr>
<td></td>
<td>HP output</td>
<td>0.031</td>
<td>0.499</td>
<td>1.561</td>
<td>-0.008</td>
<td>-0.135</td>
<td>1.870</td>
</tr>
<tr>
<td></td>
<td>HP UE</td>
<td>0.424</td>
<td>4.106</td>
<td>1.055</td>
<td>0.335</td>
<td>2.393</td>
<td>1.086</td>
</tr>
<tr>
<td>16-qtr return</td>
<td>Source gap</td>
<td>0.19</td>
<td>0.113</td>
<td>3.753</td>
<td>0.093</td>
<td>0.799</td>
<td>0.345</td>
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<tr>
<td></td>
<td>HP output</td>
<td>0.691</td>
<td>4.381</td>
<td>0.746</td>
<td>0.093</td>
<td>0.799</td>
<td>1.033</td>
</tr>
<tr>
<td></td>
<td>HP UE</td>
<td></td>
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</tr>
</tbody>
</table>
7 Conclude

- Macroeconomic fundamentals not necessarily dead in exchange rate economics

- For some exchange rates, Taylor rule fundamentals do better than monetary model fundamentals

- Taylor-rule exchange rate determination evidently learnable.