

Why Are Pooled Panel-Data Regression Forecasts of Exchange Rates More Accurate than Time-Series Regression Forecasts?

Nelson C. Mark

University of Notre Dame and NBER

nmark@nd.edu

Donggyu Sul

University of Texas at Dallas

d.sul@utdallas.edu

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Abstract

Out-of-sample forecasts of exchange rates in the late 1990s and 2000s generated by time-series regression models have fared poorly. In mean-square error, these forecasts are typically dominated by the driftless random walk. On the other hand, pooled regression models estimated on panel data (and allowing for fixed effects) perform much better than forecasts generated by time-series regression models. In this essay we ask why is it that pooled regression forecasts generate more accurate exchange rate predictions than time-series regressions when the available econometric theory says that you should not pool. We offer an explanation that is based on finite sample considerations.

1 Introduction

Out-of-sample forecasts of exchange rates in the late 1990s and 2000s generated by time-series regression models have fared poorly. These forecasts are typically dominated (in mean-square error) by the driftless random walk. On the other hand, pooled regression models estimated on panel data (allowing for fixed effects) perform much better than forecasts generated by time-series regression models. The superior predictive performance of the pooled panel data models is a puzzle because the evidence also tells of significant underlying model heterogeneity, in which case econometric theory tells us that pooling is inappropriate. Groen (2005), Rapach and Wohar (2004), Westerlund and Basher (2010) address the question about whether or not it is appropriate to pool. In this essay we ask why is it that pooled regression forecasts generate more accurate exchange rate predictions than time-series regressions when the available econometric theory says that you should not pool.

The empirical literature upon which we focus traces its origin to, and is motivated toward overturning the findings of Meese and Rogoff (1983), who in studying floating exchange rates in the post Bretton Woods era, demonstrated that the driftless random walk model dominated economic theory based econometric models (e.g., purchasing-power parity models or simple monetary models) in out-of-sample forecast accuracy. Using time-series regression models, Mark (1995) and Chinn and Meese (1995) were able to overturn Meese and Rogoff by examination of long-horizon forecasts with error-correction models. The success of these papers was only temporary, however, because it was found that as time passed and more recent data from the 1990s and the 2000s were added to the time-series, the earlier findings of predictability became insignificant (Groen (1999), Cheung et al. (2005), Faust et al. (2003)). At the same time, other research (Lothian and Taylor (1996) and Rapach and Wohar (2002)) added observations to the front-end of the sample by constructing long historical time-series spanning over a hundred years or more. These studies found that simple monetary models and purchasing power parity based models have significant predictive ability. These papers are notable because their findings suggest that the inability to predict exchange rates with time-series regression models is a small-sample issue. The long time-span studies can be criticized, however, because they employed data that spanned across both fixed and flexible exchange rate regimes but they did not control for regime changes. The inter-

pretation of the ability to predict out of sample based on a forecasting model estimated across different regimes is not entirely obvious.

One reaction to this state of affairs was to restrict attention to the post Bretton Woods float and to increase sample size in the cross-sectional dimension and exploiting panel data, as in Mark and Sul (2001), Groen (2005), Cerra and Saxena (2010), and Ince (2010). These studies found that forecasts built from pooled regression models estimated on panel-data dominate those of time-series regression forecasts as well as random walk forecasts in mean square.¹ However, the data also tell us that there is significant heterogeneity across countries in the coefficients of the forecasting equation. Econometric theory instructs us in this case that one should not pool. But allowing for heterogeneous constants and slopes would generate little advantage over time-series regression, except to the extent that you could exploit correlated errors and do seemingly unrelated regression. Pooling when there is slope coefficient heterogeneity appears to commit, in the cross-sectional dimension, the same sort of error that failure to account for regime changes does in the time-series context.

The next section describes the methodology employed in the panel data exchange rate forecasting literature. In Section 3, we discuss the econometric theory that gives us some guidance in seeing the consequences from pooling when there is underlying heterogeneity. Section 4 contains a small Monte Carlo study that illustrates some of the ideas and predictions from the econometric theory. Section 5 is an application to data and Section 6 concludes.

2 Panel data exchange rate determination studies

In a data set of N countries indexed by $i \in [1, N]$ and T time-series observations indexed by $t \in [1, T]$, let x_{it} be a (scalar) prediction variable for $s_{i,t+k}$, the log exchange rate for country i . Mark and Sul (2001) and Cerra and Saxena (2010) investigate the empirical link between the monetary model fundamentals and the exchange rate. They set $x_{i,t}$ to be the deviation of today's exchange rate $s_{i,t}$ from the long-run equilibrium value predicted by economic theory. In the case of the monetary model, $x_{i,t} = f_{i,t} - s_{i,t}$,

¹Groen (), Husted and MacDonald (19xx) exploit panel data to test restrictions implied by the monetary model and find the evidence for the monetary model to be much stronger using panel data. These papers do not engage in out-of-sample prediction.

where $f_{i,t} = (m_{i,t} - m_{0,t}) - \lambda(y_{i,t} - y_{0,t})$, country ‘0’ is the base country, m is the log money stock and y is log real income. They used the panel exchange rate predictive regression

$$s_{i,t+k} - s_{it} = \beta x_{it} + \eta_{i,t+k}$$

where $\eta_{i,t+k} = \gamma_i + \theta_{t+k} + \epsilon_{i,t+k}$ has an error-components representation with individual (fixed) effect γ_i , common time effect θ_t and idiosyncratic effect $\epsilon_{i,t+k}$. Rapach and Wohar (2004) reject the null hypothesis of slope coefficient homogeneity at the one-percent level using Mark and Sul’s data set.

Groen (2005) uses a four-country panel and pools a vector error correction model (VECM). Letting $X_{i,t} = (s_{i,t}, (m_{i,t} - m_{0,t}), (y_{i,t} - y_{0,t}))'$ and $Z_{i,t-1} = (X'_{i,t-1}, 1)'$, the model he works with is

$$\Delta X_{i,t} = \alpha(1, -1, 3, c_i) Z_{i,t-1} + \sum_{j=1}^P \Gamma_{i,j} \Delta X_{i,t-j} + \epsilon_{i,t},$$

where the common cointegrating vector $(1, -1, 3)$ is imposed.² He finds that out-of-sample forecasts with the pooled VECM, beat the random walk and beat the unpooled (individual country) VECM forecasts.

In addition to forecasting studies, panel data has been used extensively in exchange rate research. Beginning with Frankel and Rose (1996), many studies have employed panel data to study long-run purchasing power parity (Papell (2006), Papell and Theodoridis (1998, 2001), Choi et al. (2006)), and cointegrating restrictions imposed by monetary models (Husted and MacDonald (1998), Groen (2000)). To the extent to which these papers pooled in the presence of cross-sectional heterogeneity, our comments may also have applicability. Engel, Mark and West (2010) employ factor models to predict the exchange rate. This is another use of panel data, but this sort of work is the topic of a separate paper.

3 Asymptotic consequences of pooling

Let $y_{i,t+k} = s_{i,t+k} - s_{i,t}$, where $s_{i,t}$ is the exchange rate between country i and the base country, in logarithms. Suppose that economic theory suggests to us that x_{it} contains

² c is the regression constant.

predictive information for future $s_{i,t}$. For example, we might think that the real exchange rate is mean reverting. In this case, we might let x_{it} be the deviation from purchasing power parity if we thought that the nominal exchange rate chases the relative price differential after a shock. Alternatively, we might let x_{it} be the deviation of s_{it} from a long-run specification of the equilibrium exchange rate. Simple monetary models suggest using some linear combination of the logarithm of country i 's money stocks, interest rates, and real GDP relative to those of the base country. Macroeconomic panel data sets typically have $T > N$. The existence of predictive ability has been investigated in two ways. The preferred method in the finance literature is to estimate a predictive regression for asset returns—that is a regression of future returns on currently observed data—and drawing inference on the slope coefficients using the full data set (Fama and French (1988), Daniel (2001), Stambaugh (1999)). The preferred method in research on exchange rates has been to employ out-of-sample prediction procedures and to examine the properties of the prediction errors.

3.1 Predictive regression estimated on full sample

Suppose that the truth is

$$y_{i,t+1} = \beta x_{it} + \epsilon_{i,t+1}, \quad (1)$$

where $\epsilon_t = (\epsilon_{1,t}, \dots, \epsilon_{N,t}) \stackrel{iid}{\sim} (0, \sigma_\epsilon^2)$. This is the case where the slope is identical across countries. In practice, one always includes a (possibly heterogeneous) constant in the regression but we will ignore it in the exposition. This is a predictive regression in the sense that future changes in the exchange rate are projected onto current values of x_{it} . We will assume that this is a well-behaved regression. This will be true if x_{it} is econometrically exogenous. If this assumption seems too strong, one may assume that an appropriate bias correction method is applied. The asymptotic results that we discuss will not be affected if this is the case. The existence of predictive ability can be tested by estimating the predictive equation (1) in the full sample ($i = 1, \dots, N, t = 1, \dots, T$) and doing a t-test of the null hypothesis $H_0 : \beta = 0$ against the alternative $H_A : \beta \neq 0$.

Asymptotic distribution theory tells us there are advantages to pooling if the truth is given by (1). An easy way to see this is by looking at the convergence rate of the

pooled least-squares estimator,

$$\sqrt{NT} \left(\hat{\beta} - \beta \right) \xrightarrow{d} N \left(0, \sigma_\epsilon^2 Q^{-1} \right) \quad (2)$$

where $Q = \text{plim} \left((NT)^{-1} \sum_i \sum_t x_{it}^2 \right)$. In contrast, the least-squares estimator for the time-series regression (not pooled) converges at rate \sqrt{T} . This suggests that asymptotically, there is an informational advantage to be gained by expanding the available observations in the cross-sectional dimension.

Now let us contrast this to the case where there is underlying heterogeneity in the regression slopes. In this case, let the truth be given by

$$y_{i,t+1} = \beta_i x_{i,t} + \epsilon_{i,t+1} \quad (3)$$

where the underlying heterogeneity is characterized by

$$\beta_i \stackrel{iid}{\sim} \left(\beta, \sigma_\beta^2 \right). \quad (4)$$

Evidence of underlying heterogeneity in the slope coefficients across individuals has been reported by Groen (2005), Rapach and Wohar (2004), and Westerlund and Basher (2010). Pooling in the presence of heterogeneous slopes distorts the error term

$$\begin{aligned} y_{i,t+1} &= \beta x_{it} + u_{i,t+1}, \\ u_{i,t+1} &= (\beta_i - \beta) x_{it} + \epsilon_{i,t+1}. \end{aligned}$$

As a result of this distortion, the asymptotic distribution of the pooled estimator is

$$\sqrt{NT} \left(\hat{\beta} - \beta \right) \xrightarrow{d} N \left(0, \left(\omega^2 Q^{-1} + \sigma_\epsilon^2 Q^{-1} \right) \right) \quad (5)$$

where ω^2 and σ_ϵ^2 are the asymptotic variances of $\left(\frac{1}{\sqrt{NT}} \sum_{i=1}^N \sum_{t=1}^T (\beta_i - \beta) x_{it}^2 \right)$ and $\left(\frac{1}{\sqrt{NT}} \sum_{i=1}^N \sum_{t=1}^T \epsilon_{i,t+1} \right)$ respectively.

To discuss the pros and cons of pooling, we compare the asymptotic distribution of the time-series regression (OLS) estimator with the pooled regression estimator. Under the null hypothesis of no predictability, $(\beta_i = \beta = 0)$, the asymptotic distribution of the OLS estimator in the time-series case is

$$\sqrt{T} \left(\hat{\beta}_i - 0 \right) \xrightarrow{d} N \left(0, \sigma_\epsilon^2 Q^{-1} \right).$$

To facilitate the comparison, we rewrite the asymptotic distribution of the pooled estimator (5) as

$$\sqrt{T} \left(\hat{\beta} - 0 \right) \xrightarrow{d} N \left(0, \frac{\omega^2 Q^{-1} + \sigma_\epsilon^2 Q^{-1}}{N} \right). \quad (6)$$

Now comparing (5) and (6), the asymptotic variance of the pooled regression can easily be smaller than that of the time-series regression estimator due to the N in the denominator. This suggests that under the null hypothesis, the test of the null may be better sized with the pooled regression. This is only a suggestion, however, because in practice there may be many nuisance parameter issues such as cross sectional and serial dependences that need to be dealt with and these complications can substantially distort the size of the test. These complications are assumed away here.

Perhaps a more concrete way to see potential advantages from pooling in the presence of slope heterogeneity is through an analysis of local alternative power. Let the sequence of local alternatives be given by

$$\beta_i = \frac{c_i}{\sqrt{T}}, \quad \beta_N = \frac{b}{\sqrt{T}}$$

where $b = \frac{1}{N} \sum_{i=1}^N c_i = O(1)$. Here we consider only the time T dimension under the local alternative in order to compare the power of the test between by pooling and not pooling. Under these circumstances, the estimators in the time-series regression and the pooled regression have the asymptotic distributions,

$$\frac{\sqrt{T} \left(\hat{\beta}_i - 0 \right)}{\sqrt{\sigma_\epsilon^2 Q^{-1}}} \xrightarrow{d} N \left(\frac{c_i}{\sqrt{\sigma_\epsilon^2 Q^{-1}}}, 1 \right), \quad (7)$$

$$\frac{\sqrt{NT} \left(\hat{\beta} - 0 \right)}{\sqrt{\Omega}} \xrightarrow{d} N \left(\frac{b}{\sqrt{\Omega}} N, 1 \right). \quad (8)$$

where $\Omega = \omega^2 Q^{-1} + \sigma_\epsilon^2 Q^{-1}$. Hence the power of the pooled test is potentially very powerful with even a moderately large N because the local alternative parameter is $bN/\sqrt{\Omega}$. This means as N increases, the alternative moves farther away from the null which increases the power of the test.

To summarize, if the goal is to test for predictive power, it might make sense to pool whether or not one believes that there is underlying heterogeneity in the slope coefficients. This is not to say that one can obtain more accurate forecasts using the pooled prediction equation, however. We are saying only that the test of the null hypothesis

of no predictability can be easier to reject. There are a different set of issues involved in whether or not the pooled forecasts are actually more accurate. We now turn to an analysis of this topic.

3.2 Out-of-sample prediction

In the exchange rate literature, it is common to assess predictive ability with an analysis of out-of-sample forecasts. We let the full sample size in the time dimension be $T = S + P$. The model is estimated on the sample $t \in [1, S]$ and the observations $t \in [S + 1, S + P]$ are reserved for assessing forecast accuracy. The asymptotic results discussed below assume that $S, P \rightarrow \infty$ so that the full sample size $S + P$ increases faster than the portion of the sample used for estimation. The standard practice is to generate the P forecasts with recursively updated model estimates or with models estimated on a rolling sample. To simplify the exposition in this section, we assume that the model is estimated once on the first S observations. However, the empirical work that follows below is done with rolling regressions.

Again, let us assume that the truth is given by (1) in which there is no underlying heterogeneity in the slopes and one uses the sample $t \in [1, S]$ to estimate the predictive equation by pooled regression. Let $\hat{\beta}_{NS}$ and $\hat{\beta}_S$ be the pooled and time-series estimators of the slope and $\hat{y}_{i,j}^{pool} = \hat{\beta}_{NS}x_{i,j}$, $\hat{y}_{i,j}^{time} = \hat{\beta}_Sx_{i,j}$, $j \in [S + 1, S + P]$ be the predictors. Then as $N, S \rightarrow \infty$, the forecast errors have the representation

$$\begin{aligned}\hat{y}_{i,j+1}^{pool} - y_{i,j+1} &= \left(\hat{\beta}_{NS} - \beta\right)x_{i,j} + \epsilon_{i,j+1} = \epsilon_{i,j+1} + O_p\left(\frac{1}{\sqrt{NS}}\right) \\ \hat{y}_{i,j+1}^{time} - y_{i,j+1} &= \left(\hat{\beta}_S - \beta\right)x_{i,j} + \epsilon_{i,j+1} = \epsilon_{i,j+1} + O_p\left(\frac{1}{\sqrt{S}}\right)\end{aligned}$$

So we have that the pooled forecast should always be more accurate than the time-series forecast in the finite sample. Asymptotically, however, makes no difference whether one pools or not because there is no difference in forecast accuracy between pooled and unpooled. As $S \rightarrow \infty$ regardless size of N ,

$$\text{plim}_{S,P \rightarrow \infty} \frac{1}{P} \sum_{t=S+1}^{S+P} \left(\hat{y}_{i,j+1}^{pool} - y_{i,j+1}\right)^2 = \text{plim}_{S,P \rightarrow \infty} \frac{1}{P} \sum_{t=S+1}^{S+P} \left(\hat{y}_{i,j+1}^{time} - y_{i,j+1}\right)^2 = \sigma_\epsilon^2.$$

Next, let us assume that the slope coefficients vary across countries and the truth is

given by (4). The prediction errors from the pooled regression forecast are

$$\hat{y}_{i,j+1}^{pool} - y_{i,j+1} = \left(\hat{\beta}_{NS} - \beta_i \right) x_{i,j} + \epsilon_{i,j+1},$$

whereas the time-series forecast errors are

$$\hat{y}_{i,j+1}^{time} - y_{i,j+1} = \left(\hat{\beta}_{iS} - \beta_i \right) x_{i,j} + \epsilon_{i,j+1}.$$

Asymptotically,

$$\text{plim}_{N,S,P \rightarrow \infty} \frac{1}{P} \sum_{t=S+1}^{S+P} \left(\hat{y}_{i,j+1}^{pool} - y_{i,j+1} \right)^2 = \sigma_\epsilon^2 + \sigma_x^2 \sigma_\beta^2 > \sigma_\epsilon^2 = \text{plim}_{S,P \rightarrow \infty} \frac{1}{P} \sum_{t=S+1}^{S+P} \left(\hat{y}_{i,j+1}^{time} - y_{i,j+1} \right)^2.$$

Therefore pooling in the presence of underlying slope heterogeneity asymptotically leads to less accurate predictions.

The question here is why do researchers find that in applications when slope heterogeneity is present, pooled regression forecasts have lower mean square prediction errors than time-series regression forecasts? First, it is important to note that the guidance on whether to pool or not is based on asymptotic theory, which may be inaccurate in any finite sample. To see why this may be the case, we get a clue by examining the difference in the squared prediction errors from the pooled and time-series regressions when there is underlying heterogeneity in the slopes. For any finite S and P , as $N \rightarrow \infty$, this difference is

$$\left(\hat{y}_{i,j+1}^{pool} - y_{i,j+1} \right)^2 - \left(\hat{y}_{i,j+1}^{time} - y_{i,j+1} \right)^2 = \left[\left(\hat{\beta}_{NS} - \beta_i \right)^2 - \left(\hat{\beta}_{iS} - \beta_i \right)^2 \right] x_{i,j}^2 + o_p(1). \quad (9)$$

Thus, in any finite sample, it is certainly possible for $\left(\hat{\beta}_{NS} - \beta_i \right)^2 < \left(\hat{\beta}_{iS} - \beta_i \right)^2$, in which case the pooled predictor will perform better. This is most likely the case when the underlying dispersion in the slope coefficients is not too large. In the next section, we explore these ideas further with a small Monte Carlo study.

4 Monte Carlo Study

In our data generating process (DGP), the cross-sectional dimension is $N = 10$, and the time-series dimension varies from $S + P = 100$ to 400. For each time-series length, estimation is performed over half of the sample and prediction over the second half so

that $S = P$. For example, the smallest sample size that we consider is of length 100. The first 50 observations are used to estimate the prediction model and the second 50 are reserved for forecast evaluation.

The data generating process (DGP) for our Monte Carlo work is as follows:

$$\begin{aligned} y_{i,t+1} &= \beta_i x_{it} + \epsilon_{i,t+1}, \\ x_{it} &\stackrel{iid}{\sim} N(0, 1), \\ \epsilon_{it} &\stackrel{iid}{\sim} N(0, 1). \end{aligned}$$

In terms of the slope heterogeneity, we consider the following three cases considered for assignment of the slope coefficients:

β_i assignment				
Cross-sectional unit	Case 1	Case 2	Case 3	
1	0.05	0.05	0.05	
2	0.06	0.10	0.15	
3	0.07	0.15	0.25	
4	0.08	0.20	0.35	
5	0.09	0.25	0.45	
6	0.10	0.30	0.55	
7	0.11	0.35	0.65	
8	0.12	0.40	0.75	
9	0.13	0.45	0.85	
10	0.14	0.50	0.95	

For each experiment, 2000 replications are performed. We compute the mean square prediction error from the pooled regression forecasts, the time-series regression forecast, and the driftless random walk forecast. Our discussion is limited to these point estimates. Inference issues, while very important, are addressed in a separate chapter of this volume (Corte and Tsiakas (2010)).

Table 1 shows the mean-square prediction error (MSPE) comparison between the pooled and time-series forecasts. Bold entries indicate instances in which the MSPE of the pooled forecasts dominate those of the time-series forecasts. We observe two very striking features, which corroborate the implications of the asymptotic analysis in the

previous section. The pooled forecasts tend to dominate the time-series forecasts when (i) the sample size is small in the time dimension and (ii) when the slope heterogeneity across individuals is not too large (i.e., case 1 as opposed to case 3).

Table 2 reports the MSPE of the time-series regression forecasts relative to the driftless random walk. Beginning with the work of Meese and Rogoff (1983), the random walk has been the standard benchmark against which econometric forecasts are judged in the empirical exchange rate literature. The message from Table 2 is that the alternative has to be fairly far away from the null (β_i has to be fairly sizable) before predictions from the time-series regression model can beat the random walk, even in sample sizes as large as 200. Typically, in exchange rate research, we have access to short time series and slope coefficient estimates tend to be small in magnitude.

In Table 3, the MSPE of the pooled forecasts are compared to those of the random walk. Again, the pooled forecast will generally perform well relative to the random walk when the underlying model heterogeneity is small and when the time series is short. Notice case 3. When there is substantial heterogeneity in the slope coefficients, the pooled forecasts dominate the random walk in cases far from the null (large β_i) and are beaten badly when the β_i are small.

5 An illustration with data

In this section, we illustrate these ideas using data for US dollar exchange rates against the Canadian dollar, Danish Krone, Japanese yen, Korean won, Norwegian krone Swedish krona, UK pound, and the euro. The dimensions of the data set are $N = 8$, $S + P = 132$. These are monthly observations that extend from January 1999 through January 2010. We use observations from January 1999 through December 2003 initially to estimate the prediction model and to form forecasts at horizons of 1 to 24 months ahead. We then recursively update the sample and repeat the estimation and forecast generation. Relative forecast accuracy is measured by Theil's U-Statistic, which is the ratio of the RMPSE of a particular model to that of the driftless random walk.

The predictive variable is based on the monetary model. For country i , let $m_{i,t}$ be the log money stock, $y_{i,t}$ be log industrial production, and $s_{i,t}$ be the log exchange rate,

stated as the value of the US dollar in terms of country i 's currency. Then we set

$$x_{i,t} = (m_{it} - m_{0,t}) - (y_{i,t} - y_{0,t}) - s_{i,t},$$

where the '0' subscript refers to the US. For the time-series regression, the k -month ahead predictive equation is

$$s_{i,t+k} - s_{i,t} = \alpha_i + \beta_i x_{i,t} + \epsilon_{i,t+k}.$$

For the pooled regression case with fixed-effects, the predictive equation is

$$s_{i,t+k} - s_{i,t} = \alpha_i + \beta x_{i,t} + \epsilon_{i,t+k}.$$

This is the model of Mark and Sul (2001) but without controls (time-specific dummy variables) for a common time effect. The regression errors are in fact correlated across i and if we were concerned with drawing inference, then it would be necessary to model the cross-sectional dependence in the regression error. However, since we are reporting only the point estimates, we can safely ignore this complication.³

The prediction results for the time-series model is shown in Table 4. The results here are as found by Groen (1999), Faust et al. (2003), Cheung et al. (2005), who showed that time-series regression exchange rate forecasts over the 1990s and 2000s rarely outperform the random walk. There is some evidence of predictive ability only for the UK pound and at horizons greater than 16 months. For the other seven exchange rates, there is no evidence of predictive ability at all.

Table 5 shows Theil's U-Statistics for the pooled (with fixed effects) regression forecasts. It is immediately clear that the pooled forecasts are much more accurate, in the mean square sense. The pooled forecasts outperform the random walk in many more instances than the time-series regression forecasts.

The reason for this (must be) that the heterogeneity in the slope coefficients across currencies is not too large. This is evidently the case in the data. Using the full sample, we obtain the point slope estimates,

Pool	Can	Den	Jpn	Kor	Nor	Swe	UK	euro
0.005	0.014	0.008	0.012	0.008	0.009	0.004	0.001	0.041

³The fundamentals for the euro are an aggregation of variables in the euro zone.

To push farther on this argument, we refer back to (9). Because the true β_i is not observable, we use the point estimates from the full sample, $\hat{\beta}_{S+P,i}$ in their place and we will treat them as if they are the true values. First, we note that the relative ranking of the mean square error for currency i depends on the distance between the particular estimator (pooled or unpooled) and the true slope value β_i ,

$$\begin{aligned} & \text{sign} \left\{ \frac{1}{P} \sum_{t=S+1}^{S+P} \left(\hat{y}_{i,j+1}^{pool} - y_{i,j+1} \right)^2 - \frac{1}{P} \sum_{t=S+1}^{S+P} \left(\hat{y}_{i,j+1}^{time} - y_{i,j+1} \right)^2 \right\}, \\ &= \text{sign} \left\{ \frac{1}{P} \sum_{p=1}^P \left\{ \left(\hat{\beta}_{NS+p} - \beta_i \right)^2 - \left(\hat{\beta}_{iS+p} - \beta_i \right)^2 \right\} \right\}. \end{aligned}$$

The value of the last expression, $\gamma_{P,i}$, can be approximated as

$$\begin{aligned} \gamma_{P,i} &= \frac{1}{P} \sum_{p=1}^P \left\{ \left(\hat{\beta}_{NS+p} - \hat{\beta}_{S+P,i} \right)^2 - \left(\hat{\beta}_{iS+p} - \hat{\beta}_{S+P,i} \right)^2 \right\} \\ &= \frac{1}{P} \sum_{p=1}^P \left(\hat{\beta}_{NS+p} - \beta_i \right)^2 - \frac{1}{P} \sum_{p=1}^P \left(\hat{\beta}_{iS+p} - \beta_i \right)^2 - 2 \frac{1}{P} \sum_{p=1}^P \left(\hat{\beta}_{NS+p} - \hat{\beta}_{iS+p} \right) \left(\beta_i - \hat{\beta}_{S+P,i} \right) \\ &= \frac{1}{P} \sum_{p=1}^P \left(\hat{\beta}_{NS+p} - \beta_i \right)^2 - \frac{1}{P} \sum_{p=1}^P \left(\hat{\beta}_{iS+p} - \beta_i \right)^2 + O_p \left(\frac{1}{\sqrt{S+P}} \right) \end{aligned}$$

as $P \rightarrow \infty$. The last step follows since $\left(\beta_i - \hat{\beta}_{S+P,i} \right) = O_p \left(\frac{1}{\sqrt{S+P}} \right)$ but $\left(\hat{\beta}_{NS+p} - \hat{\beta}_{iS+p} \right) = O_p(1)$. The preceding algebra predicts that the pooled forecast will dominate the time-series regression forecast when $\gamma_{P,i} < 0$.

Table 6 shows Theil's U of the pooled regression forecast relative to the time-series regression forecast and the associated $\gamma_{P,i}$ for each currency. First, we point out whenever $\gamma_{P,i} < 0$, MPSEs of pooled forecasts are smaller than MPSE of time-series forecasts. Bold entries indicate that the sign of $\gamma_{P,i}$ projects the wrong signal, which it does in 7 out of 48 cases. The incorrect signal arises because the comparison is only approximate due to the $O_p \left(\frac{1}{\sqrt{S+P}} \right)$ term.

6 Conclusions

The empirical literature finds that out-of-sample exchange rate forecasts over the 1990s and 2000s from (pooled) panel data models perform much better than those generated

from time-series regression models. This is true, even though there is substantial model heterogeneity across countries (or currencies). Asymptotic analysis tells us that pooling should be inappropriate in this case. We have offered a possible explanation based on small sample considerations. On the other hand, if one is interested in testing for predictive ability using the full sample, it might make sense to pool whether or not one believes that there is underlying slope heterogeneity.

References

- [1] Campbell, John Y. and Robert J. Shiller, (1988), ‘Stock Prices, Earnings, and Expected Dividends,’ *Journal of Finance*, July, 43, pp. 661-76.
- [2] Cerra, Valerie and Sweta Chaman Saxena, (2010). “The monetary model strikes back: Evidence from the world,” *Journal of International Economics*, 81:184-196.
- [3] Cheung, Yin-Wong, Menzie D. Chinn and Antonio Garcia Pascual, (2005). “Empirical exchange rate models of the nineties: Are any fit to survive?” *Journal of International Money and Finance*, 24: 1150-1175.
- [4] Chinn, M.D., Meese, R.A., (1995). “Banking on currency forecasts: how predictable is change in money?” *Journal of International Economics* 38, 161–178.
- [5] Choi, Chi-Young, Nelson C. Mark and Donggyu Sul, (2006). “Unbiased Estimation of the Half-Life to PPP Convergence in Panel Data,” *Journal of Money, Credit, and Banking*, 38: 921—938.
- [6] Corte, Pasquale Della and Ilias Tsiakas, (2010). “Statistical and Economic Methods for Evaluating Exchange Rate Predictability,” *mimeo*, University of Warwick.
- [7] Daniel, Kent, 2001. “The Power and Size of Mean Reversion Tests,” *Journal of Empirical Finance*, 8, 493-535.
- [8] Engel, Charles, Nelson C. Mark and Kenneth D. West, (2009). “Factor Model Forecasts of Exchange Rates,” *mimeo*, University of Wisconsin.
- [9] Fama, Eugene F. and Kenneth R. French, (1988). ‘Dividend Yields and Expected Stock Returns,’ *Journal of Financial Economics*, October, 22, pp.3—25.
- [10] Faust, J., Rogers, J.H., Wright, J., (2003). “Exchange rate forecasting: the errors we’ve really made.” *Journal of International Economics* 60, 35–59.
- [11] Frankel, Jeffrey A., and Andrew K. Rose (1996). “A Panel Project on Purchasing Power Parity: Mean Reversion within and between Countries.” *Journal of International Economics* 40, 209–224.
- [12] Goetzmann, William N. and Philippe Jorion, (1993). “Testing the Predictive Power of Dividend Yields,” *Journal of Finance*, 48, 663-79.

- [13] Groen, Jan J. J. (1999). “Long horizon predictability of exchange rates: Is it for real?” *Empirical Economics*, 24: 451-469.
- [14] Groen, Jan J.J. (2000). “The monetary exchange rate model as a long-run phenomenon,” *Journal of International Economics*, 52: 299-319.
- [15] Groen, Jan J.J. (2005). “Exchange Rate Predictability and Monetary Fundamentals in a Small Multi-Country Panel,” *Journal of Money, Credit, and Banking*, 37(3): 495-516.
- [16] Husted, Steven and Ronald Macdonald, (1998). “Monetary-based models of the exchange rate: a panel perspective,” *Journal of International Financial Markets, Institutions and Money*, 8: 1–19.
- [17] Ince, Omur, (2010). “Forecasting Exchange Rates Out-of-Sample with Panel Methods and Real-Time Data”, *mimeo*, University of Houston.
- [18] Lothian, James R. and Mark P. Taylor, (1996). “Real exchange rate behavior: the recent float from the perspective of the past two centuries,” *Journal of Political Economy*, 104: 488-510.
- [19] McCracken, Michael W. and Stephen Graham Sapp, (2005). “Evaluating the Predictability of Exchange Rates Using Long Horizon Regressions: Mind Your p’s and q’s!” *Journal of Money, Credit, and Banking*, 37(3): 473–494.
- [20] Meese, Richard and Kenneth Rogoff, (1983). “Empirical Exchange Rate Models of the 1980’s: Do They Fit Out of Sample?” *Journal of International Economics*, 14: 3–24.
- [21] Mark, Nelson C. (1995). “Exchange Rates and Fundamentals: Evidence on Long-Horizon Predictability,” *American Economic Review*, 85: 201-218.
- [22] Mark, Nelson C. and Donggyu Sul, (2001). “Nominal Exchange Rates and Monetary Fundamentals: Evidence from a Small Post-Bretton Woods Panel,” *Journal of International Economics*, 53: 29—52
- [23] Papell, David H. (2006). “The Panel Purchasing Power Parity Puzzle.” Manuscript, *Journal of Money, Credit, and Banking*, 447-467)
- [24] Papell, David H., and Hristos Theodoridis (1998). “Increasing Evidence of Purchasing Power Parity over the Current Float.” *Journal of International Money and*

Finance 17, 41–50.

- [25] Papell, David H., and Hristos Theodoridis (2001). “The Choice of Numeraire Currency in Panel Tests of Purchasing Power Parity.” *Journal of Money, Credit, and Banking* 33, 790–803.
- [26] Rapach, David E. and Mark E. Wohar, (2002). “Testing the monetary model of exchange rate determination: new evidence from a century of data,” *Journal of International Economics*, 58: 359–385.
- [27] Rapach, David E. and Mark E. Wohar, (2004). “Testing the monetary model of exchange rate determination: a closer look at panels,” *Journal of International Money and Finance*, 23: 867–895.
- [28] Stambaugh, Robert F. 1999. “Predictive Regressions,” *Journal of Financial Economics*, 54, 375—421.
- [29] Richardson, Matthew and James Stock, 1989. “Drawing Inferences from Statistics Based on Multiyear Asset Returns,” *Journal of Financial Economics*, 25, 343–48.
- [30] Westerlund, Joakim and Syed A. Basher, (2007). “Can Panel Data Really Improve the Predictability of the Monetary Exchange Rate Model?” *Journal of Forecasting*, 26: 365–383.

Table 1: $\text{MSPE}_{\text{pool}}/\text{MSPE}_{\text{time}}$

$P = S$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	$i = 8$	$i = 0$	$i = 10$
Case 1: $\beta = (0.05, 0.06, \dots, 0.14)$										
50	0.990	0.989	0.989	0.987	0.987	0.988	0.989	0.988	0.990	0.989
60	0.992	0.990	0.991	0.990	0.989	0.990	0.990	0.990	0.991	0.992
70	0.993	0.993	0.993	0.991	0.991	0.992	0.991	0.992	0.993	0.993
100	0.996	0.995	0.994	0.994	0.994	0.994	0.994	0.994	0.995	0.995
200	0.999	0.998	0.997	0.997	0.997	0.997	0.997	0.997	0.998	0.999
Case 2: $\beta = (0.05, 0.10, \dots, 0.50)$										
50	1.041	1.018	1.003	0.994	0.988	0.989	0.993	1.003	1.020	1.040
60	1.044	1.021	1.005	0.994	0.991	0.990	0.996	1.006	1.022	1.043
70	1.041	1.024	1.006	0.998	0.992	0.992	0.997	1.008	1.022	1.044
100	1.046	1.025	1.010	1.000	0.994	0.995	1.000	1.010	1.026	1.045
200	1.047	1.028	1.012	1.002	0.998	0.998	1.003	1.012	1.028	1.049
Case 3: $\beta = (0.05, 0.15, \dots, 0.95)$										
50	1.199	1.115	1.053	1.009	0.991	0.990	1.009	1.050	1.114	1.193
60	1.192	1.116	1.053	1.014	0.993	0.993	1.011	1.056	1.116	1.195
70	1.195	1.112	1.055	1.014	0.994	0.994	1.013	1.056	1.116	1.198
100	1.201	1.117	1.057	1.017	0.996	0.997	1.017	1.056	1.116	1.200
200	1.201	1.119	1.060	1.020	0.999	1.000	1.021	1.061	1.120	1.200

Table 2: $\text{MSPE}_{time}/\text{MSPE}_{r.w}$

$P = S$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	$i = 8$	$i = 9$	$i = 10$
Case 1: $\beta = (0.05, 0.06, \dots, 0.14)$										
50	1.012	1.011	1.010	1.009	1.007	1.005	1.001	1.001	0.999	0.997
60	1.010	1.010	1.009	1.004	1.005	1.002	1.000	0.999	0.996	0.992
70	1.008	1.008	1.005	1.004	1.004	1.000	0.998	0.996	0.993	0.992
100	1.005	1.004	1.002	1.000	1.000	0.998	0.995	0.993	0.992	0.989
200	1.001	1.000	0.999	0.997	0.996	0.994	0.992	0.989	0.988	0.984
Case 2: $\beta = (0.05, 0.10, \dots, 0.50)$										
50	1.012	1.006	0.994	0.977	0.958	0.932	0.908	0.879	0.848	0.817
60	1.009	1.003	0.991	0.972	0.957	0.931	0.903	0.877	0.844	0.814
70	1.008	1.001	0.987	0.973	0.954	0.930	0.903	0.873	0.845	0.812
100	1.005	0.998	0.985	0.970	0.949	0.925	0.897	0.870	0.838	0.808
200	1.001	0.994	0.981	0.965	0.948	0.920	0.893	0.867	0.835	0.802
Case 3: $\beta = (0.05, 0.15, \dots, 0.95)$										
50	1.012	0.992	0.957	0.906	0.852	0.787	0.726	0.660	0.597	0.545
60	1.009	0.991	0.953	0.906	0.850	0.784	0.724	0.652	0.595	0.543
70	1.007	0.988	0.951	0.901	0.845	0.781	0.719	0.651	0.594	0.538
100	1.004	0.986	0.948	0.898	0.841	0.775	0.711	0.651	0.589	0.534
200	1.001	0.982	0.945	0.895	0.835	0.771	0.706	0.643	0.586	0.530

Table 3: $MSPE_{pool}/MSPE_{r,w}$

$P = S$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	$i = 8$	$i = 9$	$i = 10$
Case 1: $\beta = (0.05, 0.06, \dots, 0.14)$										
50	1.001	0.999	0.997	0.996	0.993	0.992	0.988	0.988	0.987	0.985
60	1.001	0.999	1.000	0.994	0.993	0.991	0.990	0.988	0.986	0.984
70	1.001	1.000	0.997	0.995	0.994	0.991	0.989	0.987	0.986	0.984
100	1.001	0.999	0.996	0.995	0.993	0.991	0.989	0.987	0.986	0.984
200	1.000	0.998	0.996	0.994	0.992	0.991	0.989	0.987	0.985	0.983
Case 2: $\beta = (0.05, 0.10, \dots, 0.50)$										
50	1.053	1.024	0.998	0.971	0.946	0.920	0.900	0.878	0.860	0.843
60	1.054	1.025	0.996	0.966	0.948	0.921	0.897	0.879	0.859	0.844
70	1.050	1.025	0.994	0.972	0.947	0.922	0.899	0.877	0.860	0.843
100	1.051	1.023	0.996	0.970	0.944	0.919	0.896	0.877	0.857	0.842
200	1.048	1.022	0.994	0.968	0.946	0.918	0.895	0.877	0.858	0.840
Case 3: $\beta = (0.05, 0.15, \dots, 0.95)$										
50	1.215	1.109	1.012	0.917	0.844	0.778	0.728	0.686	0.655	0.637
60	1.204	1.108	1.007	0.921	0.845	0.777	0.728	0.683	0.656	0.637
70	1.204	1.100	1.006	0.915	0.840	0.775	0.726	0.682	0.655	0.635
100	1.207	1.103	1.004	0.914	0.838	0.771	0.721	0.684	0.653	0.634
200	1.203	1.100	1.003	0.914	0.835	0.770	0.719	0.681	0.654	0.633

Table 4

Theil's U-Statistics: time-series Model

Horizon	CAN	DEN	JAP	KOR	NOR	SWE	UK	Euro
1	1.083	1.010	1.003	0.982	1.033	1.026	1.022	1.023
2	1.207	1.014	1.001	0.966	1.078	1.067	1.048	1.015
3	1.416	1.012	1.000	0.949	1.133	1.109	1.076	0.995
4	1.675	1.016	1.003	0.937	1.227	1.155	1.113	0.988
5	2.018	1.026	1.002	0.923	1.340	1.215	1.151	0.977
6	2.336	1.046	1.016	0.909	1.448	1.291	1.197	0.977

Table 5

Theil's U-Statistics: Pooled Case

Horizon	CAN	DEN	JAP	KOR	NOR	SWE	UK	Euro
1	1.014	1.003	0.992	0.974	0.989	1.008	1.098	0.991
2	1.026	0.998	0.978	0.954	0.989	1.014	1.242	0.976
3	1.042	0.998	0.962	0.929	0.988	1.018	1.403	0.960
4	1.065	1.002	0.950	0.916	1.003	1.018	1.566	0.947
5	1.089	1.010	0.936	0.902	1.031	1.015	1.733	0.931
6	1.108	1.024	0.925	0.891	1.057	1.017	1.919	0.918

Table 6

Theil's U-Statistics: Pooled/time-series and Comparison of Mean Square

Estimation Error								
Horizon	CAN	DEN	JAP	KOR	NOR	SWE	UK	Euro
Theil's U-Statistics: Pooled/Time Series								
1	0.936	0.994	0.990	0.992	0.957	0.983	1.074	0.970
2	0.850	0.985	0.977	0.987	0.918	0.950	1.185	0.962
3	0.736	0.986	0.962	0.979	0.872	0.918	1.304	0.965
4	0.636	0.987	0.946	0.977	0.818	0.881	1.407	0.958
5	0.540	0.984	0.934	0.977	0.769	0.835	1.505	0.953
6	0.474	0.979	0.911	0.981	0.730	0.788	1.603	0.939
Mean Square Estimation Error, $\gamma_{P,i}$								
1	-0.001	0.000	0.000	0.000	0.000	-0.001	0.000	-0.006
2	-0.008	-0.001	0.000	0.000	-0.002	-0.005	0.001	-0.032
3	-0.026	-0.002	0.000	0.001	-0.004	-0.012	0.001	-0.075
4	-0.055	-0.003	0.001	0.001	-0.008	-0.025	0.002	-0.135
5	-0.104	-0.004	0.002	0.003	-0.014	-0.047	0.004	-0.218
6	-0.168	-0.006	0.004	0.004	-0.021	-0.077	0.006	-0.331