Learning, Monetary Policy Rules, and Real Exchange Rate Dynamics

Nelson C. Mark
University of Notre Dame
“Fundamentals given a bad rap”

Flood and Rose

“When it comes to understanding exchange rate volatility, macroeconomics – “fundamentals” – is irrelevant, except in high inflation countries or in the long run. The large differences in exchange rate volatility across countries and time are simply mysterious from an aggregate perspective.”
Standard Fundamentals

- Exchange rate fundamentals of old and many new macro models: $m, m^*, y, y^*, c, c^*$
- PPP fundamentals: a constant
  - Probable long-run connection
  - Tenuous short-run connections
Nominal Exchange Rate and PPP Fundamentals (US-Germany)
Traditional Fundamentals

Real Exchange Rate

Monetary Fundamentals

Real Levels Regression Monetary Fundamentals

- Actual
- Fitted
Inflation differentials, real dollar-DM rate
Inflation differentials, real dollar-DM rate
Inflation differentials, real dollar-DM rate
Inflation differentials, real $-pound rate

Inflation Differentials and Real Exchange Rate

- Log real dollar-pound
- Inflation differential

Inflation differentials, real dollar-yen rate

Inflation Differentials and Real Exchange Rate

- Log real dollar yen
- Inflation Differential

Inflation differentials, real dollar-CD rate

Inflation Differentials and Real Exchange Rate

Log real exchange rate

Inflation Differential

Alternative macro fundamentals through the lens of Taylor-rules

- Inflationary expectations
  - A rate of change
- Output gap
  - Deviation from natural level
Real exchange rate and real interest differentials

- Pricing equation is real interest parity: Expectations matter
- Alternative strategies for modeling interest differential
- Multivariate setting and monetary policy reaction functions contribute towards accurately modeling expectations of future interest rates
Model uncertainty and learning

- Is this a credible framework for understanding real exchange rate dynamics?
- Allow model uncertainty
  - Structural instability
    - Established FACT of life in international finance
    - Source of Meese-Rogoff’s failure to FIT out of sample
Model uncertainty and learning

- Public attempts to learn “true” values by recursive least squares
  - Coefficients of process that generates inflation, output gap, nominal interest rates
- Ask if long swings could have been explained by historical fundamentals data
  - Real depreciation of late 70s, the “great appreciation” and subsequent “great depreciation”
US-GERMANY: Interest rate reaction functions

- The Fed:

\[ i_t^T = \bar{i} + \gamma_{\pi}(E_t\pi_{t+1} - \bar{\pi}) + \gamma_{xx}x_t \]

\[ i_t = (1 - \rho)i_t^T + \rho i_{t-1} + \eta_t \]

- The Bundesbank:

\[ i_t^{*T} = \bar{i}^* + \gamma_{\pi}(E_t\pi^*_{t+1} - \bar{\pi}^*) + \gamma_{xx}x_t^* + \gamma_s(s_t - [p_t - p_t^*]) \]

\[ i_t^* = (1 - \rho)i_t^{*T} + \rho i_{t-1}^* + \eta_t^* \]
US-GERMANY: Interest rate reaction functions

- Impose homogeneity

\[
\tilde{i}_t = (1 - \rho) \tilde{i}_t^T + \rho \tilde{i}_{t-1} + \tilde{\eta}_t,
\]

\[
\tilde{i}_t^T = \zeta + \gamma_\pi E_t \tilde{\pi}_{t+1} + \gamma_x \tilde{x}_t + \gamma_s q_t,
\]

\[
\zeta \equiv (\bar{i}^* - \bar{i}) - \gamma_\pi (\bar{\pi}^* - \bar{\pi}),
\]

\[
\tilde{\eta}_t \overset{iid}{\sim} (0, \sigma^2_{\tilde{\eta}}).
\]
US-GERMANY: Interest rate reaction functions

- GMM estimable differential form

\[
\tilde{i}_t = \delta + (1 - \rho)\left[\gamma_\pi \tilde{\pi}_{t+1} + \gamma_x \tilde{x}_t + \gamma_s q_t \right] + \rho \tilde{i}_{t-1} + \eta'_t
\]

\[
\delta = (1 - \rho)\zeta \quad \eta'_t = \eta_t - (1 - \rho)\gamma_\pi [\tilde{\pi}_{t+1} - E_t \tilde{\pi}_{t+1}]
\]
Table 1: Bundesbank–Fed Relative Interest-Rate Reaction Function Estimates by GMM

<table>
<thead>
<tr>
<th>Output gap from source</th>
<th>$\gamma_\pi$ (s.e.)</th>
<th>$\gamma_x$ (s.e.)</th>
<th>$\gamma_q$ (s.e.)</th>
<th>$\rho$ (s.e.)</th>
<th>$\delta$ (s.e.)</th>
<th>J-statistic (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60.2-79.2</td>
<td>0.148 (0.482)</td>
<td>-0.126 (0.221)</td>
<td>-0.016 (0.015)</td>
<td>0.858 (0.063)</td>
<td>-0.439 (0.267)</td>
<td>2.571 (0.860)</td>
</tr>
<tr>
<td>79.3-03.4</td>
<td>1.987 (0.505)</td>
<td>0.573 (0.289)</td>
<td>-0.012 (0.013)</td>
<td>0.825 (0.068)</td>
<td>0.258 (0.108)</td>
<td>1.384 (0.967)</td>
</tr>
</tbody>
</table>

Structural Change Test

<table>
<thead>
<tr>
<th></th>
<th>Test statistic</th>
<th>p-value</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>All coeffs.</td>
<td>13.461</td>
<td>0.019</td>
<td></td>
</tr>
<tr>
<td>Inflation coeff.</td>
<td>5.680</td>
<td>0.017</td>
<td></td>
</tr>
</tbody>
</table>

Output gap estimated by HP filter

<table>
<thead>
<tr>
<th>Output gap estimated by HP filter</th>
<th>$\gamma_\pi$ (s.e.)</th>
<th>$\gamma_x$ (s.e.)</th>
<th>$\gamma_q$ (s.e.)</th>
<th>$\rho$ (s.e.)</th>
<th>$\delta$ (s.e.)</th>
<th>J-statistic (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60.2-79.2</td>
<td>-0.127 (0.580)</td>
<td>-0.556 (0.453)</td>
<td>-0.029 (0.020)</td>
<td>0.877 (0.062)</td>
<td>-0.516 (0.247)</td>
<td>2.336 (0.886)</td>
</tr>
<tr>
<td>79.3-03.4</td>
<td>2.048 (0.520)</td>
<td>0.016 (0.280)</td>
<td>0.001 (0.009)</td>
<td>0.795 (0.088)</td>
<td>0.119 (0.116)</td>
<td>1.287 (0.972)</td>
</tr>
</tbody>
</table>

Structural Change Test

<table>
<thead>
<tr>
<th></th>
<th>Test statistic</th>
<th>p-value</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>All coeffs.</td>
<td>9.968</td>
<td>0.084</td>
<td></td>
</tr>
<tr>
<td>Inflation coeff.</td>
<td>8.599</td>
<td>0.003</td>
<td></td>
</tr>
</tbody>
</table>
The REE path: Inflation, output gap differentials follow VAR(p)

- Forecasting: Companion form

\[ \tilde{Y}_t = (\tilde{\pi}_t, \ldots, \tilde{\pi}_{t-p+1}, \tilde{x}_t, \ldots, \tilde{x}_{t-p+1}) \]
\[ \tilde{Y}_t = \alpha + A\tilde{Y}_{t-1} + \tilde{\nu}_t \]
\[ E_t\tilde{\pi}_{t+1} = e_1 (\alpha + A\tilde{Y}_t) \]

- Estimation

\[ \tilde{Z}_{vt} = (1, \tilde{Y}_t) \]
\[ \tilde{\pi}_t = b'_\pi \tilde{Z}_{vt-1} + \tilde{\nu}_1t, \]
\[ \tilde{x}_t = b'_x \tilde{Z}_{vt-1} + \tilde{\nu}_2t, \]
The REE path: Pricing by real interest parity

\[ s_t = E_t s_{t+1} + \tilde{i}_t \]

\[ q_t = E_t q_{t+1} + \tilde{i}_t - E_t \tilde{\pi}_{t+1} \]

\[ q_t = E_t q_{t+1} + [\delta + (1 - \rho)(\gamma_\pi - 1)e_1 \alpha] \]
\[ + (1 - \rho)(\gamma_x e_2 + (\gamma_\pi - 1)e_1 A) \tilde{Y}_t + \rho \tilde{i}_{t-1} + \tilde{\eta}_t. \]
REE Solution

\[ q_t = \beta_0 + \beta_1'\tilde{Y}_t + \beta_2\tilde{i}_{t-1} + \beta_3\tilde{\eta}_t \]

\[ \beta_2 = \frac{\rho}{1 - \rho}, \]

\[ \beta_3 = \frac{1}{1 - \rho}, \]

\[ \beta_1' = ([\gamma_x e_2 + \gamma_{\pi e_1 A}] - e_1 A)(I - A)^{-1}, \]

\[ \beta_0 = -(\beta_1' + \beta_2 e_1)\alpha (I - A)^{-1}. \]
Learning Path

Given \((\alpha_{t-1}, A_{t-1})\) \((\delta_{t-1}, \rho_{t-1}, \gamma_{\pi,t-1}, \gamma_{x,t-1})\)

Generate \(\tilde{i}_t = (\delta_{t-1} + (1 - \rho_{t-1})\gamma_{\pi,t-1}e_1\alpha_{t-1})\)
\[+ (1 - \rho_{t-1})(\gamma_{x,t-1}e_2 + \gamma_{\pi,t-1}e_1A_{t-1})\tilde{Y}_t + \rho_{t-1}\tilde{i}_{t-1} + \tilde{\eta}_t\]
\(\bar{r}_t = i_t - e_1(\alpha_{t-1} + A_{t-1}\tilde{Y}_t)\)

Given \(\beta'_{t-1} = (\beta_{0t-1}, \beta'_{1t-1}, \beta_{2t-1}, \beta_{3t-1})\)

Perceived Law of Motion \(q_t = \beta_{0t-1} + \beta'_{1t-1}\tilde{Y}_t + \beta_{2t-1}\tilde{i}_{t-1} + \beta_{3t-1}\tilde{\eta}_t\)
\(E_tq_{t+1} = \beta_{0t-1} + \beta'_{1t-1}(\alpha_{t-1} + A_{t-1}\tilde{Y}_t) + \beta_{2t-1}\tilde{i}_t\).
Actual Law of Motion

\[ q_t = \tau_{0t} + \tau'_{1t} \tilde{Y}_t + \tau_{2t} \tilde{i}_{t-1} + \tau_{3t} \tilde{\eta}_t \]

\[ \tau_{0t} = \beta_{0,t-1} + (\beta_{1,t-1} - e_1 + \gamma_{\pi,t-1}(\beta_{2,t-1} + 1))\alpha_{t-1} + (1 + \beta_{2,t-1})\delta_{t-1} \]

\[ \tau'_{1t} = \gamma_{x,t-1}e_2 + \beta_{2,t-1}(\gamma_{x,t-1}e_2 + \gamma_{\pi,t-1}e_1A_{t-1}) \]
\[ + ((\gamma_{\pi,t-1} - 1)e_1 + \beta'_{1,t-1})A_{t-1} \]

\[ \tau_{2t} = \rho_{t-1}(1 + \beta_{2,t-1}) \]

\[ \tau_{3t} = 1 + \beta_{2,t-1} \]
Update coefficients

The VAR

\[ R_{v,t} = R_{v,t-1} + g_t \left( \tilde{Z}_{vt-1} \tilde{Z}_{vt-1}' - R_{v,t-1} \right) \]

\[ (b_{\pi,t}, b_{x,t}) = (b_{\pi,t-1}, b_{x,t-1}) + g_t R_{vt}^{-1} \tilde{Z}_{vt-1} \left[ (\tilde{\pi}_t, \tilde{x}_t) - \tilde{Z}'_{vt-1} (b_{\pi,t-1}, b_{x,t-1}) \right] \]

Interest Differential

\[ R_{i,t} = R_{i,t-1} + g_t \left( \tilde{Z}_{i,t-1} \tilde{Z}_{i,t-1}' - R_{i,t-1} \right) \]

\[ \phi_t = \phi_{t-1} + g_t R_{i,t-1}^{-1} \tilde{Z}_{i,t-1} \left( \tilde{i}_t - \phi_{t-1}' \tilde{Z}_{i,t-1} \right) \]

Real Exchange Rate

\[ R_{q,t} = R_{q,t-1} + g_t \left( \tilde{Z}_{q,t} \tilde{Z}_{q,t}' - R_{q,t-1} \right) \]

\[ \beta_t = \beta_{t-1} + g_t R_{q,t-1}^{-1} \tilde{Z}_{q,t} \left( q_t - \beta_{t-1}' \tilde{Z}_{q,t} \right) \]

Gain

\[ (g_t = 1/t) \]
Alternative gain specifications

**Gain type 0:** This is the specification of a constant gain of 0.02. This is the value assumed by Orphanides and Williams (2003), who calibrated the gain to the expectations provided by professional forecasters.

**Gain type 1:** This is the specification of a decreasing gain $g_t = 1/t$ throughout the entire sample.

**Gain type 2:** This is a decreasing gain specification which resets in 1979.3 to coincide with a new monetary policy regime.

**Gain type 3:** This is a decreasing gain specification which resets both in 1979.3 and at German reunification (1990.3).

**Gain type 4:** This is a decreasing gain specification that resets in 1992.3 to coincide with the European Monetary System crisis.
Table 3: Regressions of the real dollar-DM rate on the implied learning real exchange rate in log levels and percent changes. Output gap constructed at source.

<table>
<thead>
<tr>
<th>Gain Specification</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope</td>
<td>0.507</td>
<td>0.516</td>
<td>0.381</td>
<td>0.392</td>
<td>0.543</td>
</tr>
<tr>
<td>(s.e.)</td>
<td>(0.255)</td>
<td>(0.263)</td>
<td>(0.189)</td>
<td>(0.187)</td>
<td>(0.260)</td>
</tr>
<tr>
<td>corr</td>
<td>0.304</td>
<td>0.321</td>
<td>0.370</td>
<td>0.380</td>
<td>0.337</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.104</td>
<td>0.115</td>
<td>0.149</td>
<td>0.156</td>
<td>0.125</td>
</tr>
<tr>
<td>1-qtr return</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope</td>
<td>0.029</td>
<td>0.023</td>
<td>0.032</td>
<td>0.037</td>
<td>0.025</td>
</tr>
<tr>
<td>(s.e.)</td>
<td>(0.070)</td>
<td>(0.071)</td>
<td>(0.059)</td>
<td>(0.060)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>corr</td>
<td>0.049</td>
<td>0.039</td>
<td>0.064</td>
<td>0.072</td>
<td>0.040</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.003</td>
<td>0.002</td>
<td>0.005</td>
<td>0.006</td>
<td>0.002</td>
</tr>
<tr>
<td>4-qtr return</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope</td>
<td>0.118</td>
<td>0.150</td>
<td>0.183</td>
<td>0.188</td>
<td>0.159</td>
</tr>
<tr>
<td>(s.e.)</td>
<td>(0.094)</td>
<td>(0.088)</td>
<td>(0.087)</td>
<td>(0.087)</td>
<td>(0.088)</td>
</tr>
<tr>
<td>corr</td>
<td>0.115</td>
<td>0.148</td>
<td>0.231</td>
<td>0.238</td>
<td>0.156</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.014</td>
<td>0.023</td>
<td>0.055</td>
<td>0.058</td>
<td>0.026</td>
</tr>
<tr>
<td>8-qtr return</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope</td>
<td>0.331</td>
<td>0.373</td>
<td>0.323</td>
<td>0.326</td>
<td>0.379</td>
</tr>
<tr>
<td>(s.e.)</td>
<td>(0.221)</td>
<td>(0.212)</td>
<td>(0.172)</td>
<td>(0.173)</td>
<td>(0.210)</td>
</tr>
<tr>
<td>corr</td>
<td>0.242</td>
<td>0.281</td>
<td>0.346</td>
<td>0.350</td>
<td>0.286</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.060</td>
<td>0.080</td>
<td>0.122</td>
<td>0.125</td>
<td>0.084</td>
</tr>
<tr>
<td>16-qtr return</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope</td>
<td>0.756</td>
<td>0.750</td>
<td>0.484</td>
<td>0.500</td>
<td>0.787</td>
</tr>
<tr>
<td>(s.e.)</td>
<td>(0.304)</td>
<td>(0.296)</td>
<td>(0.207)</td>
<td>(0.199)</td>
<td>(0.275)</td>
</tr>
<tr>
<td>corr</td>
<td>0.510</td>
<td>0.528</td>
<td>0.543</td>
<td>0.556</td>
<td>0.549</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.273</td>
<td>0.292</td>
<td>0.308</td>
<td>0.323</td>
<td>0.315</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>One-quarter</th>
<th>Gain specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>return</td>
<td>Data    1   2   3   4   5</td>
</tr>
<tr>
<td>volatility</td>
<td>20.060  34.207  33.248  40.055  39.747  33.178</td>
</tr>
</tbody>
</table>
Table 2: Regressions of Real Exchange Rate Data on RE Rate

<table>
<thead>
<tr>
<th>Regression</th>
<th>slope</th>
<th>(s.e.)</th>
<th>corr</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source output gap</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level</td>
<td>0.214</td>
<td>(0.071)</td>
<td>0.486</td>
<td>0.248</td>
</tr>
<tr>
<td>1-qtr return</td>
<td>0.067</td>
<td>(0.024)</td>
<td>0.256</td>
<td>0.068</td>
</tr>
<tr>
<td>4-qtr return</td>
<td>0.111</td>
<td>(0.059)</td>
<td>0.312</td>
<td>0.100</td>
</tr>
<tr>
<td>8-qtr return</td>
<td>0.135</td>
<td>(0.072)</td>
<td>0.349</td>
<td>0.124</td>
</tr>
<tr>
<td>16-qtr return</td>
<td>0.168</td>
<td>(0.120)</td>
<td>0.411</td>
<td>0.180</td>
</tr>
<tr>
<td>HP filter output gap</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level</td>
<td>0.313</td>
<td>(0.097)</td>
<td>0.304</td>
<td>0.104</td>
</tr>
<tr>
<td>1-qtr return</td>
<td>0.083</td>
<td>(0.066)</td>
<td>0.151</td>
<td>0.024</td>
</tr>
<tr>
<td>4-qtr return</td>
<td>0.099</td>
<td>(0.123)</td>
<td>0.136</td>
<td>0.020</td>
</tr>
<tr>
<td>8-qtr return</td>
<td>0.087</td>
<td>(0.164)</td>
<td>0.108</td>
<td>0.012</td>
</tr>
<tr>
<td>16-qtr return</td>
<td>0.011</td>
<td>(0.243)</td>
<td>0.009</td>
<td>0.010</td>
</tr>
</tbody>
</table>
Rational and Learning Path Comparison (standardized)

Data

Rational

Fixed Gain

Conclusions

- Macro fundamentals approach not yet dead
  - Taylor rule fundamentals first cut.
  - Probably need to add a model of the risk premium