Radioactivity

Radioactive (unstable) nuclei are generally believed to be man made, however many unstable isotopes are produced in large quantities by “natural” occurring processes.

The decay always follows the same pattern described by radioactive decay law.
Radioactive Decay Laws

\[ \frac{dN}{dt} = -\lambda \cdot N \]

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\[ N(t) = N(t_0) \cdot e^{-\lambda t} \]

\( \lambda \): decay constant

\[ N(t_0) = N_0 \]: initial number of radio-isotopes
Half-Life of Radio-Isotope

\[ N(t) = N_0 \cdot e^{-\lambda \cdot t} \]

| time (days) | \(^{131}\text{I} (\%) | \]
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\( T_{1/2} = 8.04 \text{ d} \)
Half-Life & Decay Constant

\[ N\left(\frac{T_{1/2}}{2}\right) = \frac{1}{2} N_0 = N_0 \cdot e^{-\lambda \cdot T_{1/2}} \]

\[ \ln 2 = \lambda \cdot T_{1/2} \]

\[ \lambda = \frac{\ln 2}{T_{1/2}} \]

\[ T_{1/2} = \frac{\ln 2}{\lambda} \]

The decay constant $\lambda$ is inverse proportional to the half-life $T_{1/2}$.

Half-life of a radioactive substance determines a time-scale $\Rightarrow$ clock.
Example: If your 22920 year old sample (4 half-lives) originally had 1000 $^{14}\text{C}$ isotopes, how many $^{14}\text{C}$ isotopes are left today?

\[
N(22920 \text{[y]}) = 1000 \cdot e^{-\frac{\lambda}{22920} \cdot 22920} \\
\lambda = \ln 2 / 5730 \text{[y]} = 1.21 \cdot 10^{-4} \text{[y}^{-1}] \\
N(22920 \text{[y]}) = 62
\]
Units for scaling the decay

Classical Unit: 1 Curie [Ci]

$$1 \ [Ci] = \frac{dN}{dt} = 3.7 \cdot 10^{10} \left[ \frac{\text{decays}}{s} \right]$$

Modern Unit: 1 Becquerel [Bq]

$$1 \ [Bq] = \frac{dN}{dt} = 1 \left[ \frac{\text{decay}}{s} \right]$$

The so-called dosimetry units (rad, rem) determine the amount of damage radioactive radiation can do to the human body. They depend on the kind and nature of the incident radiation (X-rays, γ-rays, α-particles, β-particle, or neutrons). It also depends on the energy loss of the particular radiation and the associated ionisation effects in the human body material.
Units for measuring the impact

Dose:

\[ D = \frac{E}{m} \]

Amount of energy \( E \) deposited by radiation into body part of mass \( m \).

Unit: Rad or Gray

Equivalent Dose:

\[ H = Q \cdot D \]

Radiation independent dose

\( Q \) is normalization factor

Unit: Rem or Sievert

- Photons: \( Q = 1 \)
- Neutrons: \( E < 10 \text{keV} \) \( Q = 5 \)
- Neutrons: \( E > 10 \text{keV} \) \( Q = 15 \)
- Protons: \( Q = 5 \)
- Alphas: \( Q = 20 \)

Internal $\gamma$ Glowing

On average, 0.27% of the mass of the human body is potassium K of which 0.021% is radioactive $^{40}$K with a half-life of $T_{1/2}=1.25 \cdot 10^9$ [y]. Each decay releases an average of $E_{avg}= 0.5$ MeV $\beta$- and $\gamma$-radiation, which is mostly absorbed by the body but a small fraction escapes the body.

Calculate, how many radioactive $^{40}$K atoms are in your body system!
Example: $^{40}\text{K}$ in your body

* mass of the body: $m_{\text{body}}$
* mass of potassium K in the body: $m_K = 0.0027 \cdot m_{\text{body}}$
* mass of radioactive $^{40}\text{K}$ in the body: $m_{^{40}\text{K}} = 0.00021 \cdot m_K = 5.67 \cdot 10^{-7} \cdot m_{\text{body}}$

$40\text{g of }^{40}\text{K} \equiv 6.023 \cdot 10^{23} \text{ atoms}$

$m_{^{40}\text{K}} = 5.67 \cdot 10^{-7} \cdot m_{\text{body}} \text{[g]} = \frac{6.023 \cdot 10^{23} \cdot 5.67 \cdot 10^{-7} \cdot m_{\text{body}}}{40} \text{[particles]} = N_{^{40}\text{K}}$

$$\frac{N_{^{40}\text{K}}}{m_{\text{body}}} = 8.54 \cdot 10^{15} \text{[particles / g]}$$

to calculate $N_{^{40}\text{K}}$, you need the body mass $m_{\text{body}}$ in gramm.

for 80 kg body: $N_{^{40}\text{K}} = 6.83 \cdot 10^{20} \text{[particles]}$
Calculate the absorbed body dose over an average human lifetime of $t = 70 \text{ y}$ for this source of internal exposure.

* **Dose:**  
  \[ D = \frac{E_{\text{absorbed}}}{m_{\text{body}}} = t \cdot A(^{40}K) \cdot \frac{E_{\text{avg}}}{m_{\text{body}}} \]

* **Activity:**  
  \[ A(^{40}K) = \lambda \cdot N_{^{40}K} = \ln 2 / T_{1/2} \cdot N_{^{40}K} \]

\[ D = 70 \,[y] \cdot \frac{\ln 2}{1.25 \cdot 10^9 \,[y]} \cdot (8.54 \cdot 10^{15} \, [g^{-1}] \cdot m_{\text{body}}) \cdot \frac{0.5 \, [MeV]}{m_{\text{body}}} \]

\[ D = 1.66 \cdot 10^{11} \,[MeV \, / \, kg] = 2.63 \cdot 10^{-2} \,[J \, / \, kg] = 2.63 \cdot 10^{-2} \,[Gy] \]

*with:*  
$1 \,[eV] = 1.602 \cdot 10^{-19} \,[J]$
Exposure to other natural or man-made radioactivity

Tobacco contains a-emitter $^{210}$Po with $T_{1/2} = 138.4\text{ days}$. Through absorption in bronchial system smoking adds 280 mrem/year to the annual dose of US population.
Sources of Natural Radioactivity
Cosmic Ray Bombardment

Cosmic Rays origin from:
- solar flares;
- distant supernovae;

Spectrum of CR

Low energy CR

High energy CR

Fluxes of Cosmic Rays

Knee
(1 particle per m²-year)

Ankle
(1 particle per km²-year)
$^{14}\text{C}$ Enrichment in Environment

$^{14}\text{C}$ is produced by interaction of cosmic rays with $^{14}\text{N}$ in the atmosphere. Fall out of $^{14}\text{C}$ and implementation into bio-material by photosynthesis. Decay counting may start after bio-material is dead.
The Carbon Cycle

Convection establishes a homogeneous distribution of $^{14}$C in the atmosphere!
You find in the ruins of a lost city - in a cold forgotten fireplace - a 25g piece of charcoal. How much carbon is in there?

1 mole of any kind of material contains $N_A = 6.022 \cdot 10^{23}$ particles (Avogadro’s number)

1 mole of material has a mass of A [g]!
- e.g. 1 mole $^{12}\text{C} \equiv 12$ g; 1 mole $^{56}\text{Fe} \equiv 56$ g;
- 1 mole $^{197}\text{Au} \equiv 197$ g; 1 mole of $^{208}\text{Pb} \equiv 208$ g
- 1 mole $\text{AlO}_3 \equiv 27 \text{ g} + 3 \cdot 16 \text{ g} = 75 \text{ g}

The piece of carbon has a mass of $M = 25$ g. This translates into the number of carbon $^{12}\text{C}$ atoms:

$$N(^{12}\text{C}) = \frac{N_A[\text{nuclei/mole}]}{A[\text{g/mole}]} \cdot M =$$

$$= \frac{6.022 \cdot 10^{23}[\text{nuclei/mole}]}{12[\text{g/mole}]} \cdot 25[\text{g}]$$

$$N(^{12}\text{C}) = 1.25 \cdot 10^{24} \text{ atoms}$$
$^{14}$C-Dating, how old is the piece of charcoal?

You analyze some weak activity of $A(t) = dN/dt = 250$ decays/min, this gives you the clue for determining its age.

The atmospheric ratio is: $\frac{^{14}C}{^{12}C} = 1.3 \cdot 10^{-12}$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{5730 \, [y] \cdot 3.15 \cdot 10^7 \, [s/y]} = 3.84 \cdot 10^{-12} \, [s^{-1}]$$

number of $^{12}$C atoms in wood: $N(^{12}C) = 1.25 \cdot 10^{24}$ [atoms];

if $^{14}C/^ {12}C$ has been constant $\Rightarrow N(^{14}C) = 1.25 \cdot 10^{24} \cdot 1.3 \cdot 10^{-12} = 1.63 \cdot 10^{12}$ [atoms]

initial activity: $A_0 = \lambda \cdot N(^ {14}C) = 370$ [decays/min]

$$A(t) = A_0 \cdot e^{-\lambda \cdot t} \Rightarrow t = \frac{\ln \frac{A_0}{A(t)}}{\lambda} = \frac{\ln \frac{370}{250}}{3.84 \cdot 10^{-12} \, [s^{-1}]}; \quad t = 3250 \, [y]$$
Sources of Natural Terrestrial Material

Natural alpha decay chains from long-lived heavy radioisotopes

- Uranium Series: $^{238}\text{U} \rightarrow ^{206}\text{Pb} + 8\alpha$
- Actinium Series: $^{235}\text{U} \rightarrow ^{207}\text{Pb} + 7\alpha$
- Thorium Series: $^{232}\text{Th} \rightarrow ^{208}\text{Pb} + 6\alpha$
- Neptunium Series: $^{241}\text{Pu} \rightarrow ^{209}\text{Pb} + 8\alpha$

There are several long-lived a-emitters in the chain: $^{234}\text{U}$, $^{230}\text{Th}$, $^{226}\text{Ra}$!
Natural Radioactivity in the US

Uranium Concentrations

Source of data: U.S. Geological Survey Digital Data Series DDS-9, 1993

Decay Chain:
- Uranium → Radium → Radon → Lead

half-lives:
- Uranium: 4.5 billion years
- Radium: 3.83 days
- Radon: 1600 years

α decay: from Uranium to Radium
β decay: from Radium to Radon

Radon: 

α: 

Radium: 

Radon: 

Lead: 

Long lived $^{40}$K Radioactivity

$^{40}$K has a half-life of $T_{1/2}=1.28\cdot10^9$ years
its natural abundance is 0.0118 %
Radiation Effects of Nuclear Bomb Tests

Beside shock, blast, and heat a nuclear bomb generates high intensity flux of radiation in form of $\gamma$-rays, x-rays, and neutrons as well as large abundances of short and long-lived radioactive nuclei which contaminate the entire area of the explosion and is distributed by atmospheric winds worldwide.

$T_{1/2} = 5730\text{y}$

Effective half-life $\sim 5-10\text{ y}$

(photosynthesis)
$^{131}I$ Fallout from Nevada Tests

Fig. B.3:CD. Per capita thyroid doses for the population of each county Test Series: Buster-Flange (1957)

Fig. 18.3:CD. Per capita thyroid doses for the population of each county Test Series: Tumbler-Shappell (1959)

Fig. W.K.CD. Per capita thyroid doses for the population of each county Test Series: Nevada (1959)

Fig. XX:3:CD. Per capita thyroid doses for the population of each county Test Series: Trinity (1952)

ATOMIC
Summary

Natural (and also anthropogenic) radioactivity provides a unique tool:

• characteristic decay patterns allow to analyze the content of the material

• characteristic decay times allow to introduce radioactive clocks for dating the material