1. Calculate the x-ray K absorption edge for lead (Z=82) and mercury (Z=80) and compare it with the absorption edge for carbon (Z=6) and aluminum (Z=13).

\[
E_K = (Z - 1)^2 \cdot 13.6 \quad [eV] \\
E_K(Pb) = 89.23keV; \quad E_K(Hg) = 84.88keV; \\
E_K(C) = 0.34keV; \quad E_K(Al) = 1.96keV;
\]

15pts

2. You want to perform a radiography analysis of a Byzantine icon, which has a gold layer of \(d=0.3\) mm thickness. The attenuation coefficient of X-rays in gold is \(\mu_{Au}=19.5\text{cm}^{-1}\). Determine the fraction of the transmitted x-ray intensity.

\[
I(d) = I_o \cdot e^{-\mu d}; \quad \frac{I(d)}{I_o} = e^{-\mu d} = e^{-19.5\text{cm}^{-1} \cdot 0.03\text{cm}} = 0.56 \equiv 56\% \\
\]

10pts

3. In a radiograph of an oil painting with a \(d_1=0.2\) mm layer of oil paint (carbon), a \(d_2=0.05\text{mm}\) underground of white lead paint and a wood backing of \(d_3=2\) cm thickness the initial x-ray intensity is reduced by a significant fraction. Calculate the total absorption and the absorbed (not transmitted) fraction of X-ray intensity for each layer.

\[
I(d) = I_o \cdot e^{-\mu_{C}d_{C}} \cdot e^{-\mu_{Pb}d_{Pb}} \cdot e^{-\mu_{W}d_{W}} = I_o \cdot e^{-\mu_{C}d_{C}-\mu_{Pb}d_{Pb}-\mu_{W}d_{W}} = \\
= I_o \cdot e^{-0.21\text{cm}^{-1} \cdot 0.02\text{cm}-22.6\text{cm}^{-1} \cdot 0.005\text{cm}-0.21\text{cm}^{-1} \cdot 2\text{cm}} = 0.584; \quad \text{with } \mu_{C} = \mu_{W} \\
\frac{I_{abs}}{I_o} = 1 - \frac{I(d)}{I_o} = 1 - 0.584 = 0.416 \equiv 41.6\% \\
\]

20pts

4. Calculate the wavelength and the energy of the characteristic L (\(n_l=3\Rightarrow n=2\)) and K (\(n_l=2,3,\Rightarrow n=1\)) X-rays transitions for iron Fe (Z=26), copper Cu Z=29, silver Ag (Z=47), gold Au (Z=79), mercury Hg (Z=80), and lead Pb (Z=82). Assume an effective charge reduction term \(\sigma=1\) for K-transitions and \(\sigma=9\) for L-transitions. Compare the results with the observed characteristic x-ray energies that can be found at the web: http://www.csrri.iit.edu/mucal.html!
\[ E_K = (Z - 1)^2 \cdot 13.6 \left(1 - \frac{1}{n_f^2}\right) \text{[eV]} \]
\[ E_L = (Z - 9)^2 \cdot 13.6 \left(\frac{1}{4} - \frac{1}{n_i^2}\right) \text{[eV]} \]

**transition energies in [keV]**

<table>
<thead>
<tr>
<th>Element</th>
<th>(Z)</th>
<th>(E_{K(2-&gt;1)})</th>
<th>web</th>
<th>(E_{K(3-&gt;1)})</th>
<th>web</th>
<th>(E_{L(3-&gt;2)})</th>
<th>web</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe</td>
<td>26</td>
<td>6.38</td>
<td>6.40</td>
<td>7.56</td>
<td>7.06</td>
<td>0.55</td>
<td>0.00</td>
</tr>
<tr>
<td>Cu</td>
<td>29</td>
<td>8.00</td>
<td>8.05</td>
<td>9.48</td>
<td>8.90</td>
<td>0.76</td>
<td>0.00</td>
</tr>
<tr>
<td>Ag</td>
<td>47</td>
<td>21.58</td>
<td>22.16</td>
<td>25.58</td>
<td>24.94</td>
<td>2.73</td>
<td>2.98</td>
</tr>
<tr>
<td>Au</td>
<td>79</td>
<td>62.06</td>
<td>68.78</td>
<td>73.55</td>
<td>77.97</td>
<td>9.26</td>
<td>9.71</td>
</tr>
<tr>
<td>Hg</td>
<td>80</td>
<td>63.66</td>
<td>70.82</td>
<td>75.45</td>
<td>80.26</td>
<td>9.52</td>
<td>9.98</td>
</tr>
<tr>
<td>Pb</td>
<td>82</td>
<td>66.92</td>
<td>74.96</td>
<td>79.32</td>
<td>84.92</td>
<td>10.07</td>
<td>10.55</td>
</tr>
</tbody>
</table>

5. What distinguishes the red color vermilion from minium? What characteristic x-ray lines would you expect for these pigments in x-ray fluorescence analysis?

Vermilion is HgS would expect Hg L-line at \(~10\) keV; minium is Pb\(_2\)O\(_4\) you expect Pb L-lines at \(~10.5\) keV

6. Calculate the range of 3 MeV protons for a PIXE analysis of a gold layered iron cup. Your energy loss of the protons in gold is about 290MeV/mm. If the gold layer is 0.005 mm thin, how deep will the protons penetrate into the iron.

\[
\frac{\Delta E}{\Delta x} = \frac{290\text{MeV}}{1\text{mm}} \Rightarrow \Delta E = 290\text{MeV} \cdot 0.005 = 1.45\text{MeV} \\
R_A = 3.2 \cdot 10^{-4} \cdot \sqrt{A_A} \cdot R_{Air}; \\
\rho_{Fe} = 7.86g / cm^3; \quad A_{Fe} = 56; \quad R_{Air}(1.55\text{MeV}) = 10cm \\
R_{Fe} = 3.05 \cdot 10^{-3} cm = 30.5\mu m = 0.0305mm \\

The protons lose 1.5 MeV of their initial energy in the thin layer of gold and penetrate 0.0305 mm into the iron cup.
7. What characteristic x-ray transition do you expect to show in a PIXE analysis of Gall-ink? What x-ray lines would you see in a PIXE analysis of the color pigments azurite, ultramarine, yellow ochre, and cerulean blue?

- Gall ink has Fe component, Fe line at ~7 keV
- Azurite has Cu component with line at ~8.9 keV
- Ultramarine has only light element components up to S, difficult to see because of low x-ray energies, highest line is the one from S at ~2.4 keV
- Yellow ochre has Fe component with line at ~7 keV
- Cerulean blue has Co and Sn components with lines at ~8.4 keV and 3.4 keV & 25.7 keV respectively

15pts