Interactions of heavy charged particles with matter

Linear stopping power: \( S = -\frac{dE}{dx} \)

Bethe formula:

\[
\frac{dE}{dx} = \frac{4\pi e^4 z^2}{m_0 v^2} N Z \left[ \ln\frac{2m_0c^2\beta^2}{I} - \ln(1 - \beta^2) - \beta^2 \right]
\]

- \( N \): number density \((A/\rho)\) of absorber
- \( Z \): Atomic number of absorber
- \( m_0 \): electron rest mass
- \( v \): velocity of primary particle
- \( Z \): charge of primary particle
- \( I \): average ionization potential

\( \Rightarrow \) Range can be calculating by integrating \( dE/dx \)
How to calculate $dE/dx$: SRIM 2008

SRIM allows you to not only calculate stopping power and range tables but it also provides Monte Carlo simulations of the interaction of a beam of particles with various layers of matter (the interface can be seen here).

SRIM can be downloaded from: [http://www.srim.org/](http://www.srim.org/)
Interaction of fast electrons with matter

Electrons interact through Coulomb scattering from atomic electrons

1) $e^-$ travel at rel. speed
2) $e^-$ suffer large deflections

Energy loss due to collisions:

$$-\left(\frac{dE}{dx}\right)_c = \frac{2\pi e^2}{m_0 v^2} NZ \left[ \ln \frac{m_0 v^2 E}{2E} - \ln 2(2\sqrt{1-\beta^2} - 1 + \beta^2) + (1 - \beta^2) + \frac{1}{8}(1 - \sqrt{1-\beta^2})^2 \right]$$

Energy loss due to radiation (Bremsstrahlung)

$$-\left(\frac{dE}{dx}\right)_r = NEZ(Z+1)e^4 \left[ 4 \ln \frac{2E}{m_0 c^2} - \frac{4}{3} \right]$$

$$\frac{dE}{dx} = \left(\frac{dE}{dx}\right)_c + \left(\frac{dE}{dx}\right)_r$$

$$\left(\frac{dE}{dx}\right)_r \approx \frac{T + mc^2}{mc^2} \frac{Z}{1600}$$
Simulation of 20 keV electrons in Gold

Gold
0.2 microns
20 keV electrons
Interactions of photons with matter

Photons (\(\gamma\), x-rays,…) interact with matter through 3 processes:

1) Photoelectric absorption

The gamma ray is absorbed by an atom and an energetic photoelectron is ejected (generally from the k-shell)

\[ E_{e^-} = h\nu - E_b \]

2) Compton scattering

3) Pair production

If \( E_\gamma > 2 \times \text{rest-mass energy of an electron (1.02 MeV)} \)

Then \( \gamma \rightarrow e^+ + e^- \) is possible
Compton scattering

By conservation of momentum and energy we can write:

\[ \frac{h \nu}{c} = \frac{h \nu'}{c} \cos \theta + \frac{mc \beta \cos \phi}{\sqrt{1 - \beta^2}} \]
\[ 0 = \frac{h \nu'}{c} \sin \theta + \frac{mc \beta \sin \phi}{\sqrt{1 - \beta^2}} \]

Eliminating \( \beta \) and \( \phi \) (unobserved) we can write:

\[ h \nu' = \frac{h \nu}{1 + \frac{h \nu}{mc^2 (1 - \cos \theta)}} \]

The angular distribution of scattered \( \gamma \)-rays is predicted by the *Klein-Nishina formula*:

\[ \frac{d\sigma}{d\Omega} = Z \rho \left[ \frac{1}{1 + \alpha (1 - \cos \theta)} \right]^2 \left[ \frac{1 + \cos^2 \theta}{2} \right] \left[ 1 + \frac{\alpha^2 (1 - \cos \theta)^2}{(1 + \cos^2 \theta) [1 + \alpha (1 - \cos \theta)]} \right] \]

*Figure 2-10* A polar plot of the number of photons (incident from the left) Compton scattered into a unit solid angle at the scattering angle \( \theta \). The curves are shown for the indicated initial energies.
The 3 $\gamma$-ray interactions

Figure 7.8 The three $\gamma$-ray interaction processes and their regions of dominance.