Under consideration for publication in Network Science

A dyadic reciprocity index for repeated interaction networks∗

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Abstract

A wide variety of networked systems in human societies are composed of repeated communications between actors. A dyadic relationship made up of repeated interactions may be *reciprocal* (both actors have the same probability of directing a communication attempt to the other) or *non-reciprocal* (one actor has a higher probability of initiating a communication attempt than other). In this paper we propose a theoretically motivated index of reciprocity appropriate for networks formed from repeated interactions based on these probabilities. We go on to examine the distribution of reciprocity in a large-scale social network built from trace-logs of over a billion cell-phone communication events across millions of actors in a large industrialized country. We find that while most relationships tend toward reciprocity, a substantial minority of relationships exhibit large levels of non-reciprocity. This is puzzling because behavioral theories in social science predict that persons will selectively terminate non-reciprocal relationships, keeping only those that approach reciprocity. We point to

∗ Research was sponsored by the Army Research Laboratory and was accomplished in part under Cooperative Agreement Number W911NF-09-2-0053, by the Defense Threat Reduction Agency (DTRA) grant HDTRA 1-09-1-0039 (Anthony Strathman and Zoltán Toroczkai), and by the National Science Foundation (NSF) grant BCS-0826958. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U.S. Government. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation hereon. Special acknowledgments go to Albert László Barabási for providing the source data.

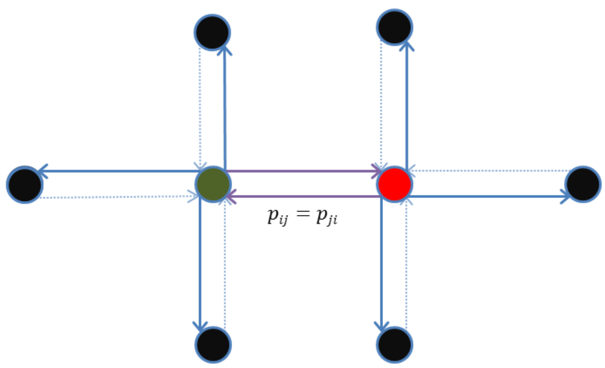
two structural features of human communication behavior and relationship formation—the division of contacts into strong and weak ties and degree-based assortativity—that either help or hinder the ability of persons to obtain communicative balance in their relationships. We examine the extent to which deviations from reciprocity in the observed network are partially traceable to the operation of these countervailing tendencies.

1 Introduction

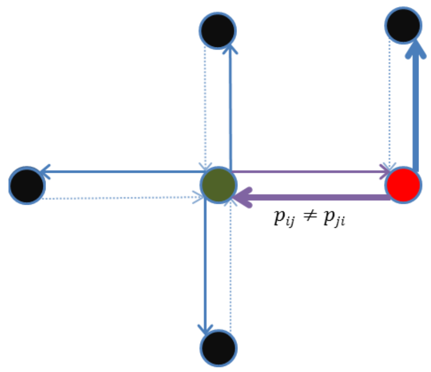
Reciprocity is one of the most important properties of the links connecting entities in networked systems (Garlaschelli & Loffredo, 2004; Boccaletti *et al.*, 2006; Wasserman & Faust, 1994; Zamora-López *et al.*, 2008; Skvoretz & Agneessens, 2007). The study of dyadic reciprocity began in the sociometric and social network analysis tradition as a way to characterize the relative behavioral or cognitive "balance" in social relationships (Hallinan, 1978; Hallinan & Hutchins, 1980; Hammer, 1985; Davis, 1963; Mandel, 2000; Krackhardt, 1987; Newcomb, 1968). These studies defined reciprocity in a very simple—but fundamentally limited—way. A dyad was reciprocal if both partners nominated one another as friends, or—in the tradition of "balance theory" (Heider, 1958; Newcomb, 1961; Newcomb, 1979; Davis, 1979; Doreian, 2002)—if it was found that the relationship had the same valence (positive or negative) for both participants. Dyads were viewed as non-reciprocal either when one partner reported considering the other one a friend or a close associate and the other did not, or if one partner displayed positive sentiments towards a partner who felt negatively towards him or her. The fundamental hypothesis of balance concerned a dynamic prediction: over time ties that were imbalanced were expected either to become balanced or to dissolve (Hallinan, 1978; Newcomb, 1961; Newcomb, 1979; Doreian, 2002).

This definition of "reciprocity" fit very well with the representation of social networks in early graph theory as consisting of binary (1,0) edges connecting two vertices (Wasserman & Faust, 1994). Analysts can then establish the level of reciprocity in the network via the so-called "dyadic census." In the usual representation, we have a directed adjacency matrix A, where $a_{ij} = 1$ if actor i chooses actor j as a neighbor and $a_{ij} = 0$ otherwise. Three types of dyads can then be defined: asymmetrical—sometimes also referred to as "non-reciprocal" ($a_{ij} = 1$ and $a_{ji} = 0$ or $a_{ij} = 0$ and $a_{ji} = 1$), symmetrical ($a_{ij} = a_{ji} = 1$) and null ($a_{ij} = a_{ji} = 0$), otherwise known as the UMAN classification (Carley & Krackhardt, 1996; Wasserman & Faust, 1994). The phenomenon of dyadic reciprocity at the level of the whole network has been studied by comparing the relative distribution of asymmetric and mutual dyads in a graph (Mandel, 2000; Garlaschelli & Loffredo, 2004). Non-reciprocity is high if the proportion of asymmetric dyads is larger than would be obtained by chance in a graph with similar topological properties (for instance a graph with the same number of vertices and edges). This classical definition of reciprocity has been extended and developed for the analysis of reciprocity in complex systems (social, technological, biological, etc.) organized as networks (Garlaschelli & Loffredo, 2004). The information contained in the distribution of reciprocal versus non-reciprocal nominations may be used to extract an underlying status dimension governing the direction of the choices (Ball & Newman, 2012). The idea here is that low status actors direct nominations towards high-status actors with those nominations unlikely to be returned; reciprocal nominations, on the other hand, should be more common among actors of comparable rank (Gould, 2002; Schaefer, 2012).

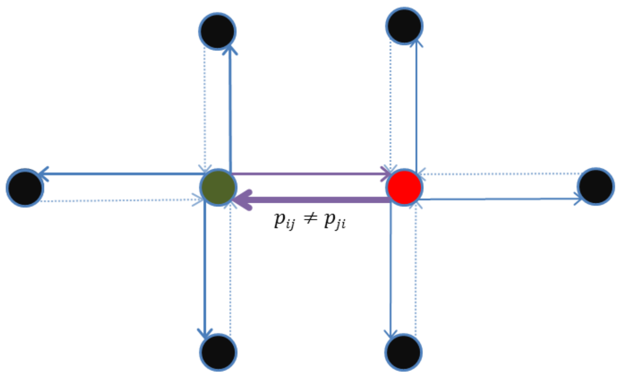
In spite of its utility, the binary classification of dyads into three types misses one of the most important features of a dyadic relationship: the relative *frequency* of contact between the two partners (Hammer, 1985; Eagle *et al.*, 2008; Eagle *et al.*, 2009; Kovanen *et al.*, 2010). This is a dimension of dyadic relationships that has always been considered crucial in previous treatments of the dynamics and static correlates of dyadic ties (Hammer, 1985;



(a) Reciprocity as a result of outdegree matching



(b) Non reciprocity as a result of outdegree-mismatch.



(c) Non-reciprocity as a result of directed weight-mismatch.

Fig. 1: Idealized local-structural scenarios producing different levels of dyadic reciprocity.

Marsden & Campbell, 1984; Feld, 1981; Peay, 1980), but which has not been treated in depth in the existing literature, mainly due to the lack of reliable behavioral data on repeated social interactions among humans in natural environments (Eagle *et al.*, 2008). This is in spite of the fact that our intuitive notions of what reciprocity take into account information about the relative "balance" not of one-shot mutual nominations or sentiments, but of repeated behavioral interactions, exchanges or flows within a dyad (Wellman & Wortley, 1990; Borgatti, 2005; Borgatti *et al.*, 2009; Hammer, 1985).

Accordingly, a more empirically accurate definition of reciprocity can only be obtained in the context of a *weighted graph* (Barrat *et al.*, 2004; Yook *et al.*, 2001; Kossinets & Watts, 2006)—also referred to as "valued graphs" (Peay, 1980; Freeman *et al.*, 1991; Yang & Knoke, 2001). In this representation, instead of a tie being thought of as simply being present or absent, the adjacency matrix is now defined by weights ($a_{ij} = w_{ij}$) which indicate the relative flow strength of the arc (e.g., the count of the number of interactions initiated by i and directed towards j).

In this context, what have traditionally been considered "non-reciprocal" dyads—e.g., one partner in the dyads nominates the other but not vice-versa—(Carley & Krackhardt, 1996; Ball & Newman, 2012) can be better thought of as the one-way, that is, completely unbalanced limit of the interactions between the agents forming the dyad. This is the case in which one partner in the relationship initiates all contact attempts and receives no reciprocation from the other member of the dyad. Intuitively it is doubtful whether we can call this a relationship in the first place (Kovanen *et al.*, 2010). In the very same way, what have been traditionally conceived of as "reciprocal" (e.g. "mutual" or "symmetric") dyads can exhibit high levels of communicative imbalance with most of the interaction being one-way (Hammer, 1985; Karsai *et al.*, 2012). Consider for instance a dyadic relationship in which one partner is five times more likely to direct a communication towards the other person than the reverse. It is not very intuitive to call this dyad "reciprocal," but that is precisely what measurements methods that discard the information encoded in the weights do.

Advancing research on the causes and dynamics of dyadic reciprocity in networked systems requires that we define dyadic reciprocity for weighted graphs. Recent work on human communication networks has begun to address this challenge (Karsai *et al.*, 2012; Wu *et al.*, 2010; Xu *et al.*, 2012; Kovanen *et al.*, 2010). This includes the development of temporally bounded measures of communicative balance for two actors that incorporates the number of communications sent along an edge (Karsai *et al.*, 2012), as well as the development of a measure of "edge bias" that incorporates directional edge-weights between two actors (Kovanen *et al.*, 2010; Xu *et al.*, 2012). The present work is meant to contribute to this emerging effort to develop an edge-level index of reciprocity for weighted communication networks characterized by repeated interactions. Our main difference with extant approaches is that we focus on the incoming and outgoing communication *probabilities* (the normalized edge weight) rather than the raw weight in the construction of our index. We will see below that this allows us to more fruitfully conceptualize what we mean by reciprocity in weighted graphs.

The rest of the paper is organized as follows: In the following section, we present our proposed index of reciprocity and explore some of its mathematical properties and substantive implications. We follow with an empirical account of the distribution and correlates of dyadic reciprocity so defined in a weighted social network built from trace logs of cell phone communications (section 3). Finally, we compare (section 3.3) the observed distribution to that obtained when we vary key dimensions of the networks that we argue are important topological and structural drivers of reciprocity. We close by outlining the implications of our argument and results (section 4).

2 Weighted reciprocity

2.1 Weighted reciprocity metric

We seek to define an index of dyadic reciprocity that captures the degree of communicative imbalance a two-way relationship between two actors but which also incorporates information on the larger system of relationships in which the dyad is embedded (Hammer, 1985). Consistent with the notion of reciprocity as balance, this index should have the following properties: first, it should be at a minimum (indicating reciprocity) when the weight of the directed arc going from vertex i to vertex j approaches the weight of the directed arc going from vertex j to vertex i. Second, it should increase monotonically with the weight difference between the two directed arcs. Third, it should normalize the weight difference to adjust for the fact that some persons are simply more or less communicative than others (they contact all of their partners more or less frequently). Finally, the index should be the same irrespective of directionality ($R_{ij} = R_{ji}$).

One index that satisfies these conditions is:

$$R_{ij} = |ln(p_{ij}) - ln(p_{ji})| \tag{1}$$

With,

$$p_{ij} = \frac{w_{ij}}{w_{i+}} \tag{2}$$

Where w_{ij} is the raw weight corresponding to the directed $i \to j$ arc, and w_{i+} is the strength of the i^{th} vertex as given by (Barrat *et al.*, 2004):

$$w_{i+} = \sum_{j \in N(i)} w_{ij} \tag{3}$$

Where $N(i)$ is the set of vertices that lie in i's neighborhood (i.e. are connected to i via an outgoing directed arc). In the case of a social network where the weights are given by the number of communications directed from one actor to another, the strength of each vertex ($w_{\cdot+}$) can be defined as the actors *communicative propensity*. This is the likelihood that at any given moment a given actor will be active or "on", which in our case means being the initiator of a communication event. We should expect that in human communication networks there should exist substantial heterogeneity across vertices in communicative propensity (Xu *et al.*, 2012; Karsai *et al.*, 2012)—with some persons being constantly active, and others communicating more sparingly—which is a phenomenon that is characteristic of other physical and biological systems (Barrat *et al.*, 2004; Barthelemy *et al.*, 2003; Serrano *et al.*, 2009).

Note that the "normalized weight" p_{ij} (Serrano *et al.*, 2009) is the instantaneous probability that if i makes a communication attempt it will be directed towards j (and vice-versa for p_{ji}); as a probability their sum across $j \in N(i)$ is constrained to be 1. A substantive interpretation of a reciprocity index based on the ratio of normalized weights is that a dyad is reciprocal when two persons have the same probability of communicating with one another, and a dyad is non-reciprocal when the probability of one person directing a communication towards another differs substantially from the probability of that person returning that communication (In the following we will just simply call R_{ij} our "reciprocity index" with the caveat that it really stands for the amount of imbalance or non-reciprocity characterizing the dyad). Factors that affect this probability, such as the number of neighbors connected to each vertex, the relative communicative propensities of each vertex or the dispersion of edge-weights across neighbors for each vertex, should thus be implicated in moving each dyad closer or farther away from the ideal of full reciprocity. Observe that in the limit if one actor initiates all directed communication attempts while the other actors initiates none, then reciprocity is not defined ($R_{ij} = \infty$), which is consistent with the intuition that there can be no definition of reciprocity when there is no actual *two-way* relationship to speak of.

2.2 Some special cases

The characterization of reciprocity given above allows us to outline some idealized conditions under which we should expect full reciprocity and under which we should expect systematic deviations from the reciprocity ideal. To build some intuition it helps to rewrite equation 1 as:

$$R_{ij} = \left| ln \left[\frac{w_{ij}}{w_{ji}} \frac{w_{j+}}{w_{i+}} \right] \right| \tag{4}$$

The first idealized condition that we can consider is an *equidispersion regime* (Serrano *et al.*, 2009; Barthélemy *et al.*, 2005). Under this condition, persons distribute their communicative activity equally across partners, with the only constraint being the number of partners (k_i^{out}) and their communicative propensity (w_{i+}). It is easy to appreciate that under this regime the expected directed weights are given by:

$$\widehat{w}_{ij} = \frac{w_{i+}}{k_i^{out}} \tag{5}$$

Substituting 5 into 2 we find that the expected p_{ij} under this regime is simply:

$$\widehat{p}_{ij} = \frac{1}{k_i^{out}} \tag{6}$$

Indicating a strong trade-off between the normalized outflow and the range of contacts for each vertex (Aral & Van Alstyne, 2009). Finally, substituting 6 into 1 shows that in this case the reciprocity equation simplifies to:

$$\widehat{R}_{ij} = |ln(k_j^{out}) - ln(k_i^{out})| \tag{7}$$

Because vertex strength ($w_{.+}$) drops out of the picture under the equidispersion constraint, if persons disperse their directed communications equally across neighbors, and have the same number of outgoing arcs ($k_i^{out} = k_j^{out}$), then reciprocity is assured. This is the situation depicted in Figure 1a. Thus, when equidispersion holds, deviations from the reciprocity ideal are solely traceable to the magnitude of the degree-differences across the two vertices in a dyad and are *independent of vertex strength differences*.

A case of non-reciprocity produced by non-assortative mixing by degree is shown in Figure 1b. Here the two vertices match in strength but differ in outdegree. In this case, even if the two actors were to distribute their communicative activity equally across neighbors they would not be able to reach reciprocity. The reason for this is that the more sociable green vertex is forced to divide her energy over a larger number of neighbors than the red vertex, reducing the outgoing *probability* of communication in relation to the incoming probability corresponding to her less sociable neighbor (Aral & Van Alstyne, 2009). *This implies that, holding all else equal, degree-assortativity in social networks (the existence of more same-degree dyads than we would expect by chance) should drive the average* *reciprocity of a random dyad towards the maximum reciprocity point* ($R_{ij} = 0$). Non-assortativity (or negative assortativity) should move dyads towards less reciprocal relations.

As shown in Figure 1c, deviations from the ideal of reciprocity can be produced even when persons share the same number of neighbors and have the same communicative propensities but they do not distribute their communicative activity equally across contacts (Hammer, 1985). In the example shown above, the green vertex follows the equidispersion rule but the red vertex does not. Instead the red vertex concentrates her communicative activity on the green vertex at the expense of her other neighbors. Setting $w_{i+} = w_{j+}$ in 4, gives us the expected reciprocity for this case:

$$R_{ij} = |ln(w_{ij}) - ln(w_{ji})| \tag{8}$$

In other words, when vertices have the same strength and have the same number of neighbors, but $R_{ij} \neq 0$, we can be sure that either: (1) at least one of the vertices is investing *more* in that relationship than in his or her other relationships; or (2) at least one of the vertices is investing in that relationship *less* than he or she does in his other relationships. Naturally, both things could be happening at the same time (one partner under-invests while the other one over-invests).

A fourth case that would produce systematic non-reciprocity according to 4 would be one in which the directed weights for each arc in the dyad match ($w_{ij} = w_{ji}$), but the vertex strength of the partners is different. In this case, the level of non-reciprocity for that dyad is given by:

$$R_{ij} = |ln(w_{j+}) - ln(w_{i+})| \tag{9}$$

Note that the case of equal weight but non-equal vertex strength is redundant since it is implies that either one partner is under-investing or another partner is over-investing in the relationship; this is therefore another version of the non-equidispersion story shown in 1c. This is intuitive since, as we saw above, when both vertices disperse their communicative activity equally across neighbors R_{ij}, is independent of vertex strength differences. Thus, any dependence of the expected value of R_{ij} on either w_i+ or w_{j+} when $w_{ij} = w_{ji}$ can only be produced by deviations from equidispersion.

In a real communication network, we should expect the values of w_{ij} to vary across neighbors for each vertex: equidispersion is an ideal that will usually fail to be met in real social networks (Almaas *et al.*, 2004; Barthelemy *et al.*, 2003; Wu *et al.*, 2010; Karsai *et al.*, 2012; Xu *et al.*, 2012). Empirical evidence indicates that persons typically divide their neighborhood into core and peripheral members, directing strong (large weight) ties toward core members and keeping only weak (small weight) ties with peripheral members (Granovetter, 1973; Marsden, 1987). Arcs that are considered strong ties in ego's neighborhood should have much larger weights than those that are considered weak ties. Non-reciprocity results when they are mismatches in the directional tie strength between two vertices: one member of the dyad considers a strong tie what from the point of the view of the other member is a weak tie. *Thus, holding all else equal, deviations from the equidispersion ideal should move the average dyad away from the reciprocity point* ($R_{ij} = 0$).

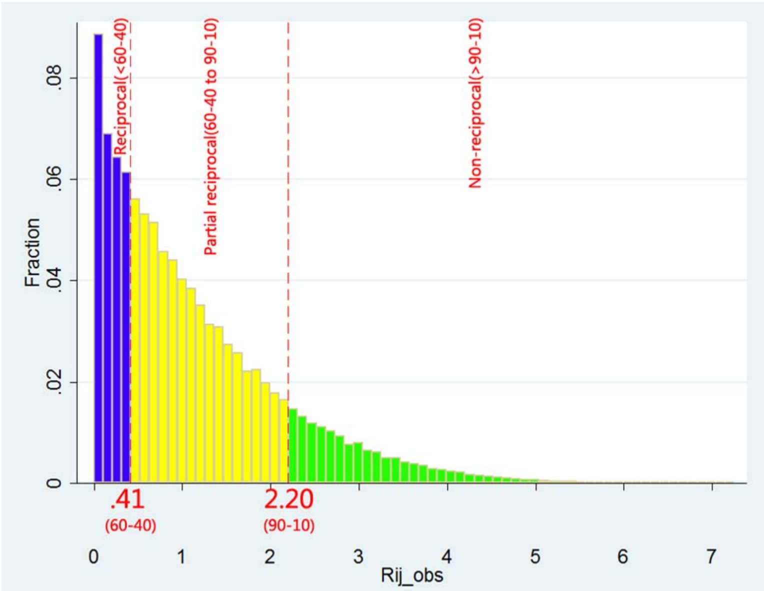


Fig. 2: Distribution of reciprocity scores across dyads in the cell phone network. For each edge connecting two vertices, the reciprocity score (R) is given by the absolute value of the logged-ratio of the normalized weights ($p_{ij} = w_{ij}/w_{i+}$) corresponding to each directional arc: $R_{ij} = R_{ji} = |ln(p_{ij}/p_{ji)}|$. $R = 0$ indicates full reciprocity.

One additional special case deserves mention: that of a vertex with only one outgoing edge. Consider a pair of vertices i, j for which $k_i^{out} > 1$ and $k_j^{out} = 1$. Because $w_{ji} = w_{j+}$, then equation 4 reduces to:

$$R_{ij} = |ln(p_{ij})| \tag{10}$$

Indicating that when one vertex has another vertex as its sole partner, then reciprocity is purely a function of the behavior of the vertex with more than one outgoing edge and independent of the behavior of the vertex with only one outgoing edge.

A trivial case emerges when $k_i^{out} = k_j^{out} = 1$. Here $R_{ij} = 0$ irrespective of the magnitude of the difference between m_{ij} and m_{ji}. This is another case of the principle—illustrated in equation 7 above—that when the corresponding communication probabilities match (in this case, $p_{ij} = p_{ji} = 1$), then reciprocity is independent of vertex strength differences. This is consistent with the proposal that perceived deviations from the reciprocity ideal only come into play when dyads are embedded in a system of other relationships (Hammer, 1985).

3 The empirical distribution of weighted reciprocity

The data that we will consider in what follows consist of a weighted graph of a human communication network constructed from trace-logs of over 1 billion cellular telephone voice calls made by 8 million subscribers of a single cellular telephone provided in a European country over a two-month period in 2008. Among these 8 million subscribers

Table 1: Four variants of weighted reciprocity: (1) quantity computed in a network with assortative mixing by degree and equal flow dispersion ($\widehat{R}_{ij}^{obs}$); (2) quantity computed in a network with a neutral mixing pattern and equal flow dispersion ($\widehat{R}_{ij}^{rw}$; (3) quantity computed in a network with assortative mixing by degree and unequal flow dispersion (R_{ij}^{obs});(4)) quantity computed in a network with a neutral mixing pattern and equal flow dispersion (R_{ij}^{rw}).

	Assortativity	Non-assortativity
Equidispersion	$\widehat{R}_{ij}^{obs}$	$\widehat{R}_{ij}^{rw}$
Non-equidispersion	R_{ij}^{obs}	R_{ij}^{rw}

there are over 34 million directed arcs, that is instances in which a subscriber made at least one call to another subscriber. Of these 34 million arcs, about 16.8 million (49%) are asymmetric dyads, meaning that the directed arc is not reciprocated. The remaining 17.2 million symmetric arcs are in 8.6 million mutual dyads consisting of two arcs, indicating that each person in the dyad made at least to the other person during this time period. The focus below is on these 8.6 million mutual dyads given that reciprocity, as characterized above, is only defined for these types of dyads. We define the weight (w_{ij}) of the incoming and outgoing arcs for each vertex as the *number of calls* either received from or made to each neighbor (respectively) during the time period in question.

The mean level of reciprocity among these 8.6 million dyads is .634 with a standard deviation of .523. The median is .511 indicating a skewed distribution titled towards zero. This is clearly seen in Figure 2 which depicts the observed distribution of reciprocity computed according to equation 1. We divide the observed dyads into three classes: *reciprocal dyads* are those in which the communication probability ratio (taking the largest probability as the numerator) ranges from 1.0 to 1.5 ((0 to .41 when taking the natural log of the probability ratio). *Partially reciprocal dyads* are those in which the communication probability ratio is larger than 1.5 but smaller than 9.0 (.41 to 2.20 on the logged scale). Finally, *non-reciprocal* dyads are those with a probability ratio exceeding 9.0 (2.20 on the logged scale). We find that a substantial minority (28%) of dyads belong to the reciprocal class, about 58% of dyads can be considered partially reciprocal, and a non-trivial minority of dyads (14%) exhibit extreme non-reciprocity, with one partner being more than nine times more likely to contact the other than being contacted by that partner.

It is clear that a substantial proportion of dyads in the observed social network feature relatively large degrees of weighted non-reciprocity. Had we confined ourselves to the purely binary definition of reciprocity as mutuality or symmetry, we would have missed the large levels of communicative imbalance encoded in the directed weights. This result suggests that there are systematic features of human communicative behavior that drive dyads towards non-reciprocity in spite of often noted psychological preferences and normative expectations for balance in human social relationships (Gouldner, 1960; Newcomb, 1979; Hammer, 1985). We investigate some of the topological and structural factors that push social networks either towards and away from reciprocity in what follows.

3.1 Comparing the observed distribution to alternative regimes

To what extent are the patterns of reciprocity observed in this social network deviations from what we would expect by chance? To answer this question we compare the observed reciprocity distribution to that obtained from three-alternative regimes, corresponding to three out of the four different configurations in a two-dimensional space defined by the presence or absence of degree assortativity (Park & Newman, 2003; Newman & Park, 2003; Newman, 2003; Newman, 2002; Catanzaro *et al.*, 2004), versus the presence or absence of a tendency toward equidispersion in the weight distribution of the arcs emanating from each vertex (Barthélemy *et al.*, 2005; Barthelemy *et al.*, 2003; Serrano *et al.*, 2009). This is shown in shown in Table 1.

As we have already noted, the observed social network is located in the lower-left corner of the table (R_{ij}^{obs}). This is a network displaying positive degree assortativity and a tendency for non-negligible proportion of actors to distribute their communicative activity inequitably across neighbors. The Pearson correlation coefficient (r) between the (excess) degree sequences of each of the two vertices across linked dyads in the observed network is positive: $r_{k_i k_j}^{obs} = 0.33$, which is a value typical for human social networks (Newman & Park, 2003).

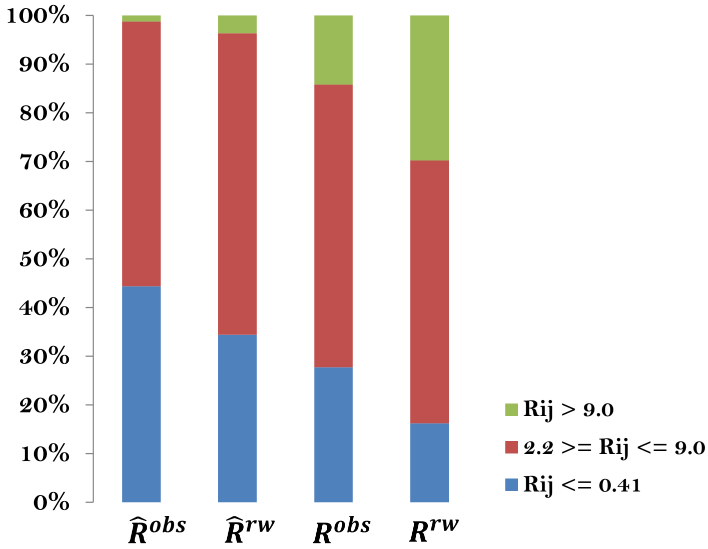
The tendency for a substantial number of vertices (among those whose $k_i^{out} \geq 2$) to concentrate their communicative outflow on a minority of their contacts can be quantified by can quantified by calculating $H = \sum_j p_{ij}^2$ (Serrano *et al.*, 2009), which is equivalent to Herfindahl's concentration (H) index (Herfindahl, 1950) . We normalize the index to remove any dependence between the expected minimum and vertex degree by computing:

$$H^* = \frac{H - 1/k_i^{out}}{1 - 1/k_i^{out}} \tag{11}$$

This calculation reveals that about 10% of vertices have an non-equidispersion score of 0.66 or above (with 1.0 indicating the extreme case of concentrating all communicative activity on a single partner) and 25% have a score of .43 or higher. Only about 25% of vertices come close to the equidispersion ideal ($H^* \leq 0.10$). This concentration of communicative activity on a small subset of the total possible number of partners has been observed in other large scale communication networks (Karsai *et al.*, 2012; Wu *et al.*, 2010; Xu *et al.*, 2012). The systematic prevalence of substantial inequalities in directed weights across vertices is a property that this social system shares with other physical and biological networked structures (Almaas *et al.*, 2004; Barthelemy *et al.*, 2003; Csermely, 2004; Csermely, 2006).

3.2 Procedure

We proceed to generate three alternative comparison networks, all of which preserve the most relevant topological and statistical features of the original network (the number of vertices, the number of edges, and the degree-distribution), but which either remove assortativity, impose equidispersion in the distribution of directed weights across neighbors for all vertices, or do both. We remove assortativity in the original network using the Maslov-Sneppen local rewiring algorithm (Maslov & Sneppen, 2002). This procedure



	$\widehat{R}^{obs}$	$\widehat{R}^{rw}$	R^{obs}	R^{rw}
Dispersion	=	=	≠	≠
Assortativity	Yes	No	Yes	No
$\mathbf{R}^{\mu}$	0.63	0.81	1.12	1.63
$\mathbf{R}^{\sigma}$	0.52	0.62	0.98	1.19

Fig. 3: Distribution of dyadic reciprocity in the observed cell phone network and three artificial variations. Blue bars are *reciprocal dyads*, red bars are *partially reciprocal dyads* and green bars are *non-reciprocal dyads* as defined in section 3 above.

preserves the number of edges attached to each vertex, but makes the vertex-to-vertex connections independent of degree. We can verify that the algorithm is successful by computing the degree-correlation after reshuffling. The resulting network is indeed non-assortative ($r^{rw}_{k_ik_j} = 0$) indicative of a "neutral" mixing pattern.

The first alternative network depicted in the upper-left corner of Table 1 is an assortative-equidispersed network. This is just like the originally observed network, except that the number of calls across partners are redistributed and forced to be same ($p_{ij} = 1/k^{out}_i$ for all arcs and $R_{ij} = R_{ji} = |ln(k^{out}_j/k^{out}_i)|$ for all dyads); here reciprocity is given by $\widehat{R}^{obs}_{ij}$). The second comparison system is a non-assortative, equidispersed network (upper-right corner of table 1). This is just like the first network, except that now the links are reshuffled to remove degree-assortativity according to the procedure described above; here reciprocity is given by $\widehat{R}^{rw}_{ij}$). Finally, the non-assortative non-equidispersed network (lower-right hand corner) is just like this last network, except that the distribution of calls across neighbors matches that of the observed network; here reciprocity is given by R^{rw}_{ij}.

Because assortativity and non-equidispersion pull in opposite directions with respect to reciprocity, we should observe that $\widehat{R}^{obs}_{ij} < R^{obs}_{ij}$ due to the non-equidispersion effect; that is reciprocity in the observed network (where there is non-equidispersion) is farther away from zero than in a network with similar characteristics but where persons distributed calls equally across partners. We should also observe that $R^{obs}_{ij} < R^{rw}_{ij}$ due to the assortativity

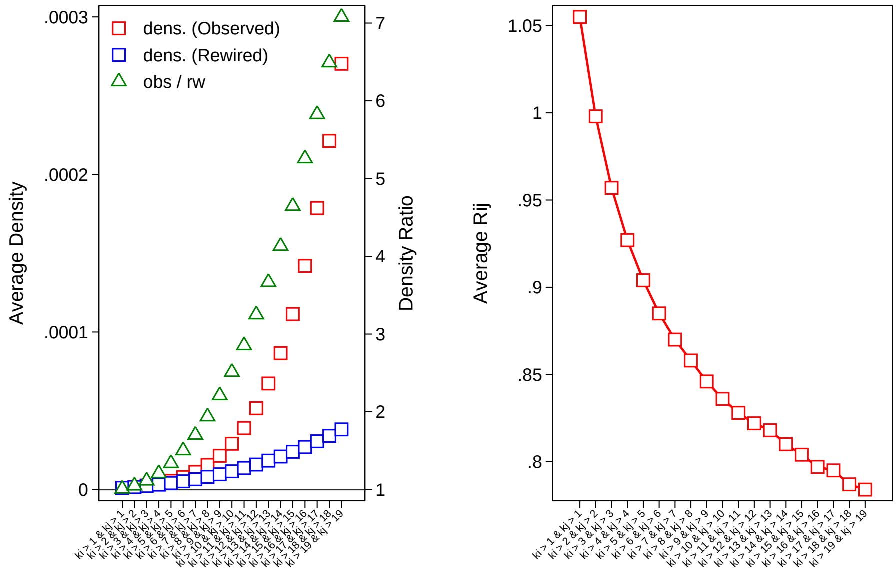


Fig. 4: **Left-hand panel**: *rich club structure in the cell phone network*. The ratio of the density in the original network (red squares, left-axis) and in the re-wired network (blue squares, left-axis) is shown by the green triangles and the right-axis. This ratio increases exponentially as we restrict the calculation to subsets of vertices of increasingly high degree, indicating the existence of a relatively well-connected rich-club. **Right-hand panel**: *Non-reciprocity within the rich-club*. Average reciprocity (R_{ij}) comes closer to zero as we gradually restrict the calculation to the rich club subset of the graph in both the observed (red squares) and in the re-wired (blue squares) network, suggesting that edges that connect high-degree vertices are more reciprocal.

effect; that is reciprocity in the observed network (which is degree-assortative) is closer to zero than in a topologically equivalent network with a neutral mixing pattern because, as noted above, reciprocity is more likely among dyads with degree similar vertices. Finally, due to the non-equidispersion effect, we should expect that $\widehat{R}^{rw}_{ij} < R^{rw}_{ij}$. That is even in a network without assortativity, one in which persons distribute calls equally across neighbors should have reciprocity values closer to zero than one where this condition does not obtain. If these three inequalities hold, then we should find the following partial ordering of expected (average) non-reciprocity across the four networks:

$$\widehat{R}^{obs}_{ij} < min\left(\widehat{R}^{rw}_{ij}, R^{obs}_{ij}\right) \leq max\left(\widehat{R}^{rw}_{ij}, R^{obs}_{ij}\right) < R^{rw}_{ij} \tag{12}$$

The most reciprocal network should be the one which has both assortativity and equidispersion, and the least reciprocal network should be one without assortativity and without equidispersion. Note that the ordering of the expected values of $\widehat{R}^{rw}_{ij}$ and R^{obs}_{ij} cannot be predicted a priori, since the question of which force is greater, (1) the ability of assortativity to drive reciprocity towards zero or (2) the ability of non-equidispersion to move the same quantity away from zero, is an empirical issue. We can however expect that reciprocity in these two networks should fall in between the two extremes described above, since they are positive in a factor that lowers reciprocity and negative on a factor that increases it. If assortativity is a stronger factor in driving non-reciprocity towards zero than non-equidispersion is in driving it away from zero, then we should find that $\widehat{R}_{ij}^{rw} > R_{ij}^{obs}$. If the opposite is the case, then we should find that $\widehat{R}_{ij}^{rw} < R_{ij}^{obs}$.

3.3 Results

3.3.1 Reciprocity, Assortativity, and Equidispersion

Figure 3 summarizes the differences in the relative distribution of reciprocity across all four networks, and gives the means and standard deviations of our measure of reciprocity in each network. The results largely agree with expectations regarding the two regimes that should fall at the lower and higher extremes of weighted reciprocity. The network without assortativity and without equidispersion (R^{rw}) displays proportionally more dyads with extreme levels of non-reciprocity, and has the highest mean Reciprocity score (R^{μ} = 1.63). While only about 14% of dyads in the observed network (R^{obs}) exhibit extreme non-reciprocity (e.g. one partner being nine times more likely to initiate a communication attempt than than the other), this proportion more than doubles once we remove the assortativity bias but keep everything the same (30%). Meanwhile while about 28% of dyads enjoy some level of reciprocity in the observed network, this number drops to 16% in the non-assortative version of the same network. Also as expected, the network displaying reciprocity values closest to the zero (full reciprocity) level is the one that has both assortativity and equidispersion $\hat{R}^{obs}$). Here the mean Reciprocity score is 0.63, and the proportion of reciprocal dyads is 44% (in comparison to 28% in the original data), and the proportion of extremely non-reciprocal dyads is only 1%.

The results shown in figure 3 provide an answer to the question of which of the two tendencies observed in human communication networks—assortativity or non-equidispersion—contributes more to system level reciprocity. The answer is clear: adding equidispersion to the least reciprocal network results in a much more dramatic move towards reciprocity than does adding assortativity to the same network (compare the difference between R^{rw} and $\widehat{R}^{obs}$ to the difference between R^{rw} and R^{obs}). In this respect, while assortativity keeps human communication networks from resembling the least reciprocal of our baseline networks, the tendency to disperse communication activity inequitably across contacts is responsible for the bulk of the observed non-reciprocity.

Accordingly, the final ordering of expected reciprocity (with smaller values indicating more reciprocity) for all the four networks is as follows:

$$\widehat{R}_{ij}^{obs} < \widehat{R}_{ij}^{rw} < R_{ij}^{obs} < R_{ij}^{rw}$$

3.3.2 Reciprocity and Rich-Club Structure

The presence of “rich club” structure is a property of complex networks that is analytically and empirically independent from the presence of degree-assortativity. Networks exhibit rich-club structure whenever we observe higher-levels of connectivity among well-connected vertices (e.g. those with lots of outgoing edges) than we would expect from topological constraints alone (Colizza *et al.*, 2006). Many networked social systems exhibit such a rich-club pattern, and our cell-phone communication is no exception. The left-hand panel of Figure 4 shows that the observed density within the well-connected subset of the network is substantially larger than we would expect by chance, as given by the observed density in the randomized network. The right-hand panel of Figure 4 shows that the average R_{ij} is smaller (closer to zero) within the rich-club than in the network as a whole, suggesting that vertices within the rich-club tend to maintain relatively reciprocal relations with one another. This is consistent with sociological intuitions connecting status-equality and reciprocity (Gould, 2002; Ball & Newman, 2012).

4 Discussion

In this paper we have defined a metric for reciprocity applicable to weighted networks. Under this conceptualization, reciprocity is defined as balance in the number of communications flowing from one partner to another, normalized by the communicative activity of each person. This yields a notion of reciprocity that is interpretable as a *matching* of the *probabilities* that the two vertices in a dyad will initiate directed contact attempts towards each other. When persons match in overall communicative propensity ($w_{\cdot +}$), reciprocity reduces to the (absolute value of the logged) ratio of the weights of the incoming and outgoing arcs. When the weights (w_{ij}, w_{ji}) of the arcs are the same, reciprocity simplifies to the (absolute value of the logged) ratio of the strength of the vertices. The most revealing special case results when vertices disperse their communication attempts equally across neighbors. In this case reciprocity simplifies to the (absolute value of the logged) ratio of the *number of neighbors* (outdegree) of each vertex.

We examined the distribution of reciprocity as defined here in a social network built from trace logs of cell-phone communications between individuals during a two month period. We found that these relationships exhibit varying levels of balance, with the majority of relationships exhibiting moderate to very large imbalances. In this respect, while reciprocity might certainly be a communicative preference across persons, there are systematic features of human communication behavior and network topology that prevent it from becoming a statistical "norm" as would be predicted by cognitive balance and normative theories of reciprocity (Heider, 1958; Gouldner, 1960; Newcomb, 1979; Hallinan, 1978). One such feature consists precisely of the propensity to divide contacts into strong and weak ties, thus concentrating communicative activity on a few partners at the expense of others (Wu *et al.*, 2010; Karsai *et al.*, 2012). Eliminating this tendency—by imposing equidispersion of weights on the observed network—moves it closer to the ideal of full-reciprocity. In addition, we demonstrate that one systematic feature that differentiates social networks from other types of networks—namely, the tendency of like to associate with like as manifested in the degree-assortativity property (Park & Newman, 2003)—makes the observed communication network more reciprocal. Assortative mixing creates reciprocity by facilitating the matching of probabilities across incoming and outgoing arcs (see Figure 1). When we remove assortativity from the observed network, we observe a substantial overall decrease in the proportion of reciprocal dyads. It is worth noting, however, that the effect of non-equidispersion in moving reciprocity away from the ideal of zero is stronger than the effect of assortativity in moving this quantity closer to zero. Finally, we show that our communication network contains a core of well-connected nodes that are also more likely to be connected to another. Within this "elite" core, edges tend to be more reciprocal than outside of it.

Of course, it is not our claim that the two structural properties that we have considered here (degree assortativity and non-equidispersion) are the only factors contributing to observed patterns of reciprocity (and non-reciprocity). Various dynamics endogenous to human relations (such as the tendency of parents to call their children more than the reverse, or cultural rules that prescribe unilateral communication flows across persons of different status), and some unique to cell-phone communications (e.g. calling plans that make it more financially feasible for one person to call others rather than to receive calls) undoubtedly contribute to the overall distributional pattern of reciprocity in the observed network. Our only claim is that these endogenous and idiosyncratic sources of "non-reciprocity" operate alongside *systematic* relational processes connected to the local topology of networked systems and general behavioral propensities (such as heterogeneous concentration of communicative activity on a single neighbor). The focus of this paper has been on isolating these systematic processes using a new measure of weighted reciprocity that allows us to disentangle the effects of degree assortativity and equal dispersion on reciprocity.

Our findings have important implications for how we think about the phenomenon of dyadic reciprocity in social and other networked systems. The most obvious implication is that non-reciprocity should be a recurrent feature of all networked systems that feature a high degree of vertex-level heterogeneity in the distribution of communications across neighbors. Thus, it appears that in the case of human social networks, the preference to concentrate communicative activity on a small subset of the total possible number of contacts, systematically generates non-reciprocity for the rest of ego's neighbors. This may generate endogenous dynamics of relationship formation and termination as persons attempt to satisfy preferences for reciprocity in personal relationships while at the same time inequitably distributing their communicative activity across partners.

In addition, our results point to some ideal conditions under which reciprocity should be more likely to be observed. For instance, smaller groups or dense communities that impose homogeneity in most topological characteristics (including the number of neighbors as in fully-connected cliques) should exhibit more weighted reciprocity than social systems that induce large inequalities in connectivity across partners (e.g., systems characterized by "popularity tournament" dynamics) (Martin, 2009; Waller, 1937; Gould, 2002; Barabasi & Albert, 1999). Networked systems that induce anti-correlation in the number of neighbors of each vertex in a dyad should—all else being equal—be characterized by high-levels of non-reciprocity. In the same way, positive correlations across vertices on other relevant characteristics (e.g. average outgoing arc weight or vertex strength) should move social relationships towards the reciprocity ideal, while mismatches in these vertex-level traits should increase non-reciprocity. In this respect, observed tendencies for persons to match in these traits may be the indirect result of an underlying tendency to preserve more reciprocal relationships available in the network and terminate the least reciprocal—biased selection into reciprocal relationships—than a direct preference to be concordant on these surface features. Preferences for reciprocity operating at a local level are likely to be an important mechanism that generates some distinctive global properties in human social networks.

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