A Study of Feedback Fundamentals

Panos J. Antsaklis

University of Notre Dame, Notre Dame, IN USA

www.nd.edu/~pantsakl

Symposium to Honor Bill Wolovich

Simposio Tecnico en Honor a Bill Wolovich

Sunday, December 7, 2008

Festschrift – Bill Wolovich

- Polynomial Matrix Description (PMD)
  $Dz = u,\ y = Nz$ Internal Description (ss is a special case)

- Polynomial Fractional Matrix Description (PFMD)
  $y/u = G = ND^{-1}$ External Description
  Linear Multivariable Systems, Springer-Verlag, 1974.

- PMD brings closer together internal and external descriptions
- Fractional matrix description (FMD), extension of N,D to proper and stable-similar properties, excellent at preserving properness in feedback, not clear connection to internal description.

- Successes-parameterization of all stabilizing controllers using dual factorizations, connection to Diophantine and optimal control.

- *Will derive the feedback fundamental result presented below using PMD*

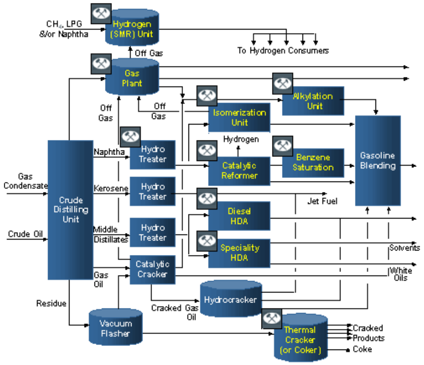
UNIVERSITY OF NOTRE DAME

Feedback-Setting the Stage

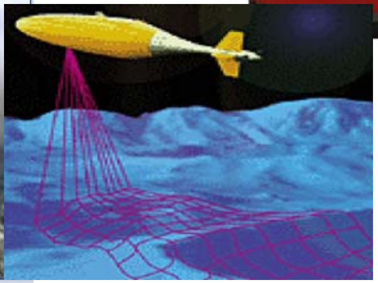
- I have been fascinated by the concept of feedback since I was a student
  -feedback rules!

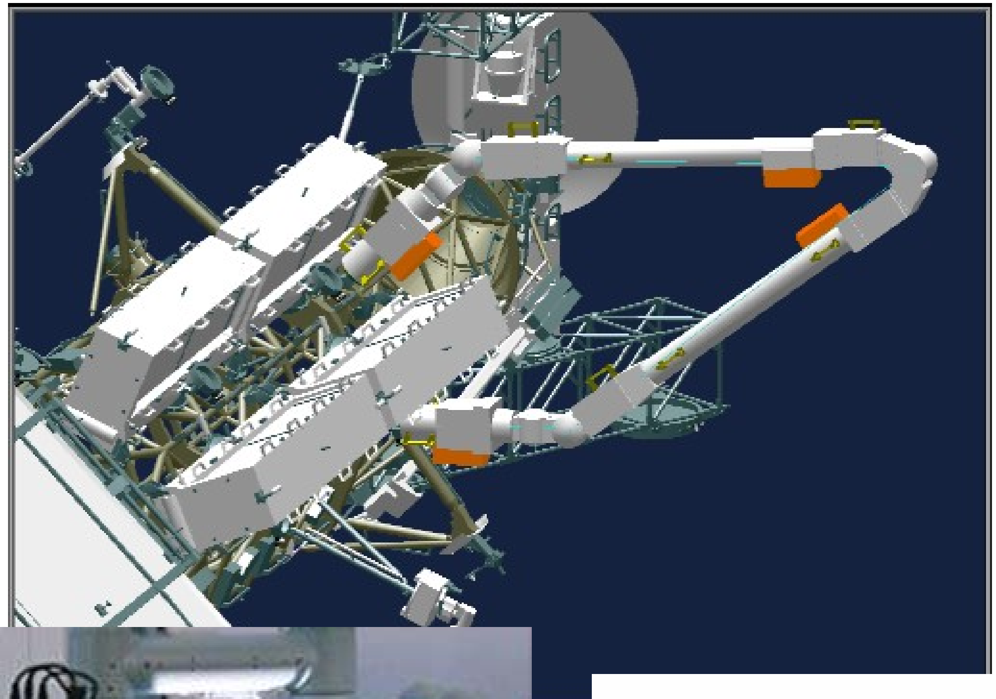
- It seems that feedback is as old as life itself!
  -feedback is associated with sensors-a single cell organism attracted to light adjusts direction.
  -feedback is ubiquitous, it is everywhere, around us and inside us, it is ever-present.

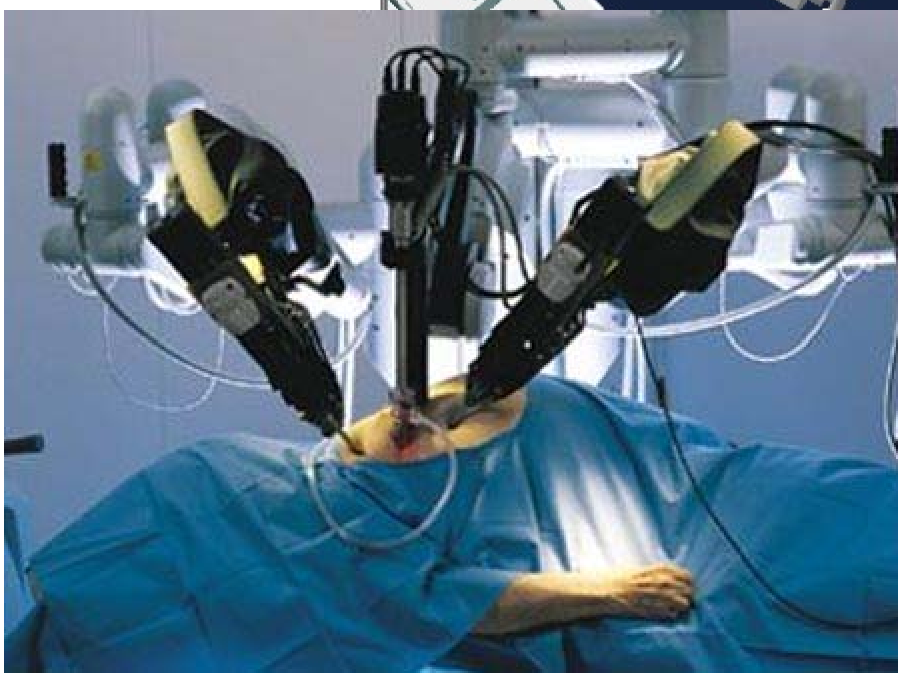
- Feedback is not easy to study because considerations may lead to circular arguments



UAVs operating together







From Computer Desktop Encyclopedia
Reproduced with permission.
© 2004 Intuitive Surgical, Inc.

Feedback-Setting the Stage

- Many powerful methodologies have been introduced in the past half century to design controllers that stabilize the systems and achieve desired performance in a robust way, being tolerant to plant parameter variations and external disturbances.

- Significantly less effort has been spent in the past half century on understanding exactly how and why feedback works so well not only in the control of engineered systems but in biological, social, economic and physical systems as well.

Feedback-Setting the Stage

- Is there a property that transcends models and is omnipresent the moment the loop is closed?

  -A property that does not depend on the choice of feedback gains.

  -That is, what is the most fundamental property of feedback, if one exists?

- A property that is present whenever there is feedback, in Discrete Event Systems and Hybrid Systems, with Supervisory Control and Intelligent Control, in systems with Human Interaction (Man-Machine Interaction), in Economic and Social Systems, and in Biological and Physical systems.

UNIVERSITY OF NOTRE DAME

Feedback-Setting the Stage

- A controls researcher or practitioner would perhaps state that uncertainty reduction is the main feedback property since feedback is necessary for stabilization under uncertainty. And it is needed for reduction of sensitivity to parameter variations, and also for reduction of the effects of external disturbances on performance.

- *But whether a feedback controller will stabilize or destabilize, decrease or increase the effects of parameter variations or of disturbances depends on the particular choices for the gains.*

- For example, any positive gain (in an error feedback configuration) can stabilize 1/s -and it is very robust. But if gain negative, the system is unstable.
- Similar results with sensitivity, where sensitivity my decrease or increase depending on the gain.

- *So what is the property we are looking for? Does it even exist?*

Feedback Fundamental Property: Re-stating the Problem

- What are the fundamental principles, the fundamental mechanisms, which make feedback control so powerfully effective?

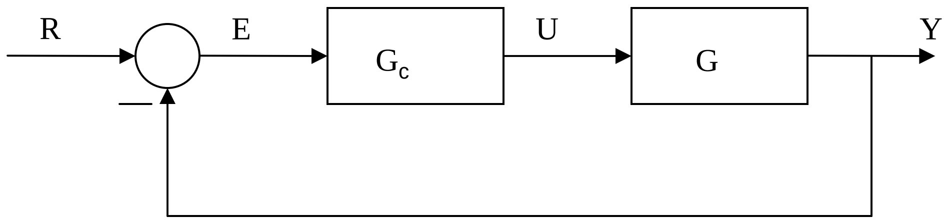
- These fundamental mechanisms should be independent of the particular type of mathematical models used, that is the system may be described by differential equations, by automata, by logic expressions, or by natural language, since we do know that feedback is ubiquitous and works!

- What are these fundamental properties that are present everywhere?

- For the LTI case, analysis best done via polynomial matrix descriptions!

Feedback High Gains

- It is well known that feedback control can be seen as a mechanism that approximately inverts the plant dynamics, producing an “approximate” inverse of the plant at its control input.



$$\frac{Y}{R} = T = \frac{GG_c}{1+GG_c} \quad and \quad \frac{U}{R} = M = \frac{G_c}{1+GG_c}$$

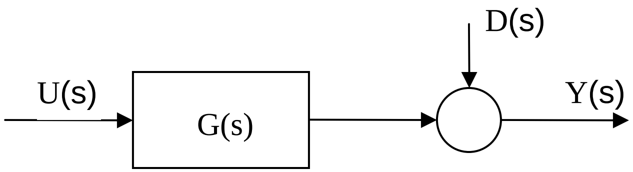
- If then $|GG_c| >> 1$ $\quad \frac{Y}{R} = T \cong 1 \quad and \quad \frac{U}{R} = M \cong \frac{1}{G}$

- That is at the control input U of the plant, the external input R acts through an inverse of the plant so to cancel all plant dynamics and produce an output Y, which is approximately equal to the reference input R.

Open Loop (Feed-forward) Control

- Y(s)/U(s) = $G(s)=\frac{1}{as+1}$

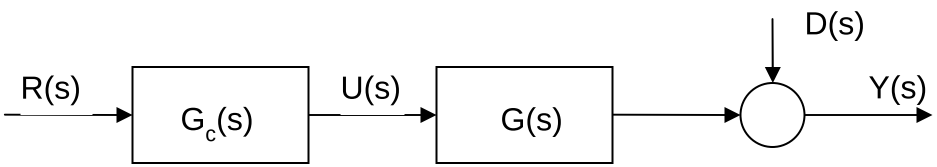
$$a(sY_p(s)-y(0))+Y_p(s)=U(s)$$



$$Y(s)=\frac{a}{as+1}y(0)+\frac{1}{as+1}U(s)+D(s)$$

$$G_c(s)=\frac{bs+1}{cs+1}$$

$$U(s)=\frac{cu(0)-br(0)}{cs+1}+\frac{bs+1}{cs+1}R(s)$$

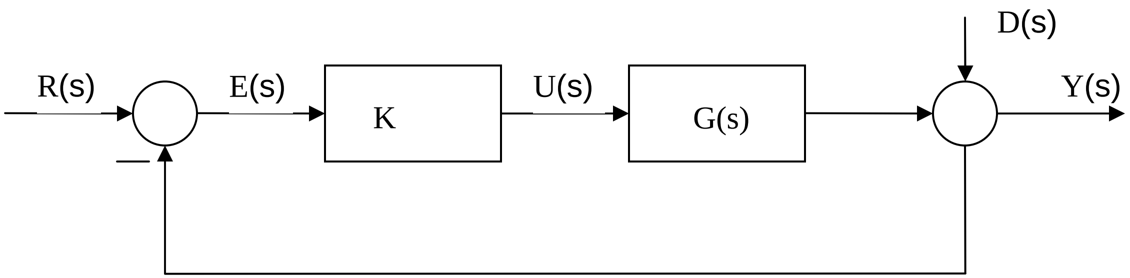


$$Y(s)=\frac{(cs+1)ay(0)+cu(0)-br(0)}{(as+1)(cs+1)}+\frac{bs+1}{(as+1)(cs+1)}R(s)+D(s)$$

$$b=a$$

$$(a-c)y(0)+cu(0)-br(0)=0$$

Closed Loop (Feedback) Control

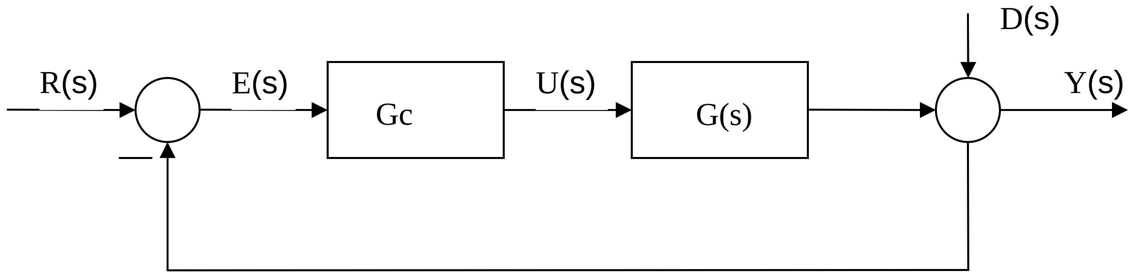


$$Y(s) = \frac{a}{as+1}y(0) + \frac{1}{as+1}U(s) + D(s) \qquad U(s) = -\frac{Kay(0)}{as+1+K} + \frac{K(as+1)}{as+1+K}R(s) - \frac{K(as+1)}{as+1+K}D(s)$$

$$\begin{aligned} Y(s) &= [\frac{a}{as+1}y(0) - \frac{Kay(0)}{(as+1)(as+1+K)}] + \frac{K(as+1)}{(as+1+K)(as+1)}R(s) - \frac{K(as+1)}{(as+1+K)(as+1)}D(s) \\ &= \frac{ay(0)}{as+1+K} + \frac{K}{as+1+K}R(s) - \frac{K}{as+1+K}D(s) \end{aligned}$$

- When a > 0 for K > -1 the closed loop system is stable
- When a < 0 for K < -1 the closed loop system is stable

Feedback Control-A More General Case (1)



$G(s)=n/d \quad G_c(s)=n_c/d_c$

$$Y(s)=\frac{n_o}{d}+\frac{n}{d}U(s)+D(s)$$

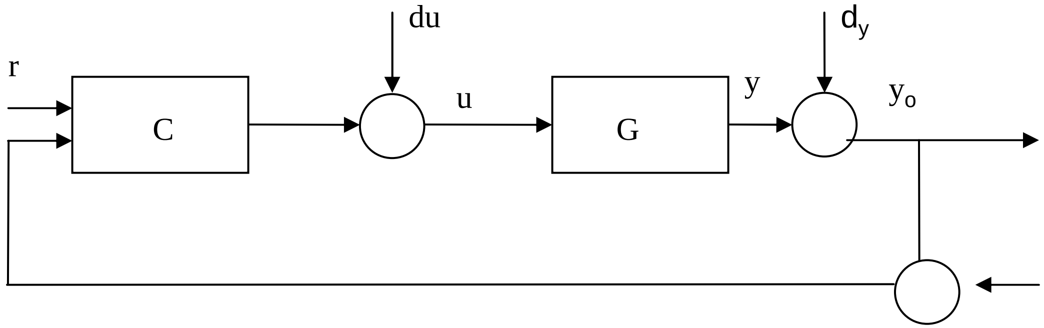
$$U(s)=\frac{n_{co}}{d_c}+\frac{n_c}{d_c}E(s)$$

$$U(s)=\frac{n_{co}d-n_c n_o}{dd_c+nn_c}+\frac{n_c d}{dd_c+nn_c}R(s)-\frac{n_c d}{dd_c+nn_c}D(s)$$

$$Y(s)=\frac{n_o d_c+nn_{co}}{dd_c+nn_c}+\frac{n_c n}{dd_c+nn_c}R(s)+\frac{d_c d}{dd_c+nn_c}D(s)$$

UNIVERSITY OF NOTRE DAME

Feedback Control-A More General Case (2)



$$u = [C_y, C_r]\begin{bmatrix} y + d_y + \eta \\ r \end{bmatrix} + d_u$$

$$y = G(I - C_y G)^{-1}[C_r r + C_y d_y + C_y \eta + d_u]$$
$$u = (I - C_y G)^{-1}[C_r r + C_y d_y + C_y \eta + d_u]$$

$$G = ND^{-1}$$

$$u = D[Xr + Ld_y + L\eta + (I + LN)D^{-1} d_u]$$
$$y = N[Xr + Ld_y + L\eta + (I + LN)D^{-1} d_u]$$

$$y = Tr + (S_o - I)d_y + GQ\eta + GS_i d_u$$
$$u = Mr + Qd_y + Q\eta + S_i d_u$$

$$T = G(I - C_y G)^{-1} C_r = GM = NX$$
$$M = (I - C_y G)^{-1} C_r = DX$$
$$Q = (I - C_y G)^{-1} C_y = DL$$
$$S_o = (I - GC_y)^{-1} = I + GQ$$
$$S_i = (I - C_y G)^{-1} = I + QG$$

$$y_o = y + d_y = Tr + S_o d_y + GQ\eta + GS_i d_u.$$

Related Result-Realizing Stable Transfer Functions with Internal Stability

- Given $y/u = G = ND^{-1}$ the stable rational function matrices (T, M) in y = Tr and u = Mr are realizable with internal stability by means of a two degrees of freedom control configuration if and only if there exists stable rational function matrix X so that

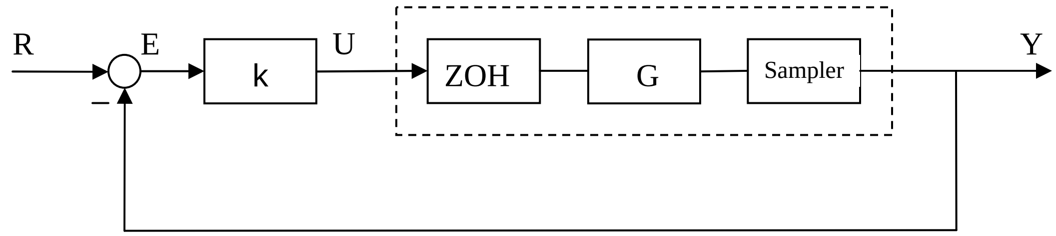
$$\begin{bmatrix} T \\ M \end{bmatrix} = \begin{bmatrix} N \\ D \end{bmatrix} X.$$

- D appears always in M, that is u = Mr = DXr

- Because of uncertainty, feedback must be used to realize T and M and D cancels at the input of the plant – poles of the plant (in D) always cancel with zeros of the map generated between the input to the plant u and the external input r (in D), same as in the simpler cases discussed before

UNIVERSITY OF NOTRE DAME

Feedback Control-Sampled Data



- G(s) = 1/s

$$
\begin{aligned}
H(z) &= (1-z^{-1})Z\{[L^{-1}(G(s)/s)]_{t=kT}\} \\
&= (1-z^{-1})Z\{[L^{-1}(1/s^2)]_{t=kT}\} \\
&= (1-z^{-1})Z\{(kT)\} \\
&= (1-z^{-1})\frac{Tz}{(z-1)^2} = \frac{T}{z-1}
\end{aligned}
$$

- Closed loop transfer function $\frac{kT}{z-1+kT}$ Stability when $0 < k < 2/T$

- which is more restrictive than before (k >0). As T becomes larger, the range for k becomes smaller.

$$U(z) = \frac{k}{1+kH(z)}R(z) = \frac{k(z-1)}{z-1+kT}R(z)$$

- Notice that U(z), as it acts on the plant H(z), cancels the plant dynamics by the pole/zero cancellation of z-1. This cancellation happens independently of the sampling period T.

Feedback Fundamental Mechanism-1

- *The feedback mechanism always generates a signal u the behavior of which is modified by D, the inverse of the map D^{-1} which is the transfer function between z and u in the plant Dz=u. y=Nz.*

$$z = D^{-1} u$$

- *D appears in the numerator of the transfer function between u and r*

$$u = M r = DX r$$

and it has the effect that for such u the behavior of the plant state

$$z = D^{-1} u = D^{-1} DX r = X r$$

is completely freed from behavior determined by the plant modes.

- **So feedback does not really invert the plant y = G u.**

- ***It inverts the map between z and u, always! Namely it inverts the map $z = D^{-1} u$ to generate u = D(.)r = DX r.***

Feedback Fundamental Mechanism-2

- **Arbitrary feedback will result, almost always, to a closed loop system with dynamics (poles) different from the open loop dynamics (poles).**
  Examples: Root-Locus; A-BK; A-BHC; $D(s)D_c(s) + N(s)N_c(s)$.
  Comments: K that preserves open loop locations; Assumption of controllability and observability;

- **Sampled data; nonlinear systems;**

- **Human-in-the loop. Example car and driver.**

- ***For almost any feedback gains the dynamics are completely reassigned, that is the plant behavior drastically changes.***

- ***In feedback control, gains are chosen to assign the new dynamics and not to cancel the old ones.***

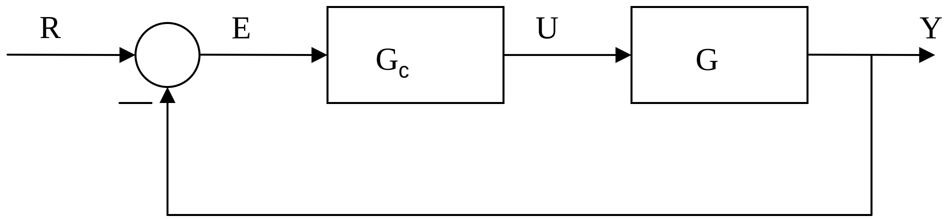
UNIVERSITY OF NOTRE DAME

Open vs Closed Loop

- **Complete change of plant dynamics takes place:**
  a. almost always in closed loop feedback control.
  b. almost never in open loop feed-forward control.
- **Complete assignment of desirable dynamics to the compensated system is:**
  a. straigtforward in the open loop feed-forward control case.
  b. harder in the closed loop feedback case, although typically there is a large range of controller choices that satisfy the requirements for desirable dynamics (control specifications) as one must consider the trade-offs.

- *In the open loop control* after the current plant dynamics have been cancelled out (which is the difficult part) one can simply choose the compensated system dynamics by assigning them to the controller. So the open loop controller should contain all the desirable dynamics.
- *In the closed loop control* the choice of the appropriate controller is a non trivial matter and the field of Control Theory has been studying this problem intensively for at least the past 50 years. Certainly a topic that requires deep understanding. About 25 years ago all stabilizing controllers were conveniently parameterized a result that also showed the large number of choices one has.

Connection to Return Difference (1+GGc)



- At the output th

$$Y = (-GG_cY) \ \text{ or } (1+GG_c)Y = 0$$

$$\text{or } Y = (-GG_cY) + GW + (n_o/dd_cy_o)$$
$$Y = (1+GG_c)^{-1} GW + (1+GG_c)^{-1} (n_o/dd_cy_o)$$

- **For the Return Difference equality to be satisfied D needs to cancel out.**

- **Cancellation of D is due to the creation of the "Return Difference" or to "Forcing the Interconnection" or "Closing the Loop"**

CONCLUDING REMARKS

- **Closing the loop with arbitrary feedback gains will result almost always to a closed loop system with dynamics (poles) different from the open loop dynamics (poles). In feedback all plant poles cancel out automatically. Fundamental property.**

- **Open vs Closed Loop.   The two steps control action: *Canceling old dynamics and inserting new.***

- **We saw the connection to return difference**
- **There are also connections to internal models**
- **There are connections to recursive relations**
- **There are connections to internal feedback-FIR and IIR**

- **Best seen as pole/zero cancellation via D and PMD.**
  The control input generates the exact inverse of input to state map $z = D^{-1}u$***, always.***

- ***Applicable to general feedback systems*: While driving the sensor input and the subsequent driver action makes the car behave very differently in response to a side wind gust from the case where no sensor feedback is received and no action is taken.**

- ***It is conjectured* that the reason for robustness in feedback systems is the fact that feedback cancels the plant dynamics always, independently of the fact they are not known exactly. If feedback were to cancel all poles, zeros and gain of the pant then the resulting overall gain would be independent of the plant. This is the case in fact when the loop gain is very large and the plant is (approximately) inverted. In general the feedback control does not invert the whole plant but, as it was shown, it inverts part of the plant namely the input to state map. Only the denominator of the plant cancels, but not the numerator, and the effect of the parameter uncertainty is reduced, but not eliminated.**