# Recent Advances in Positive Systems:
# The Servomechanism Problem

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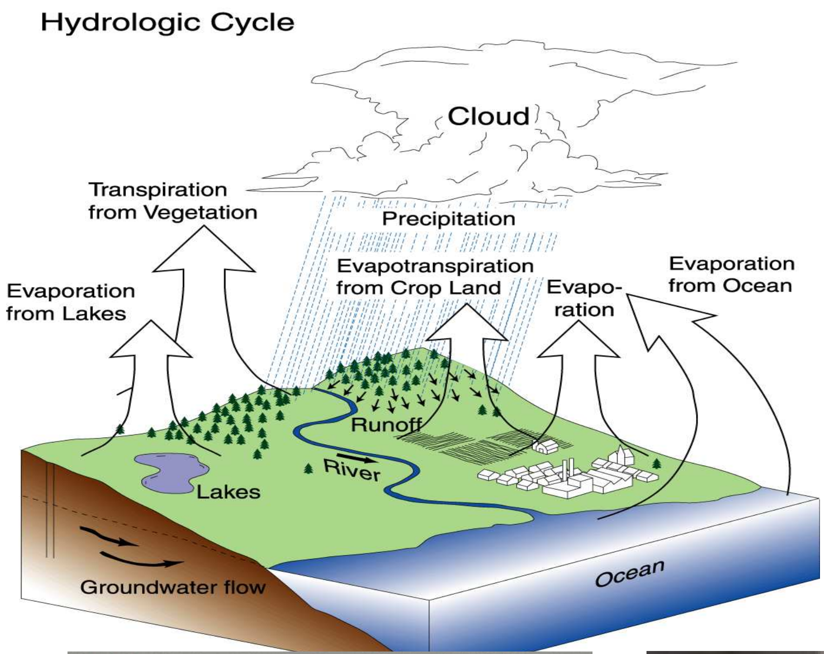
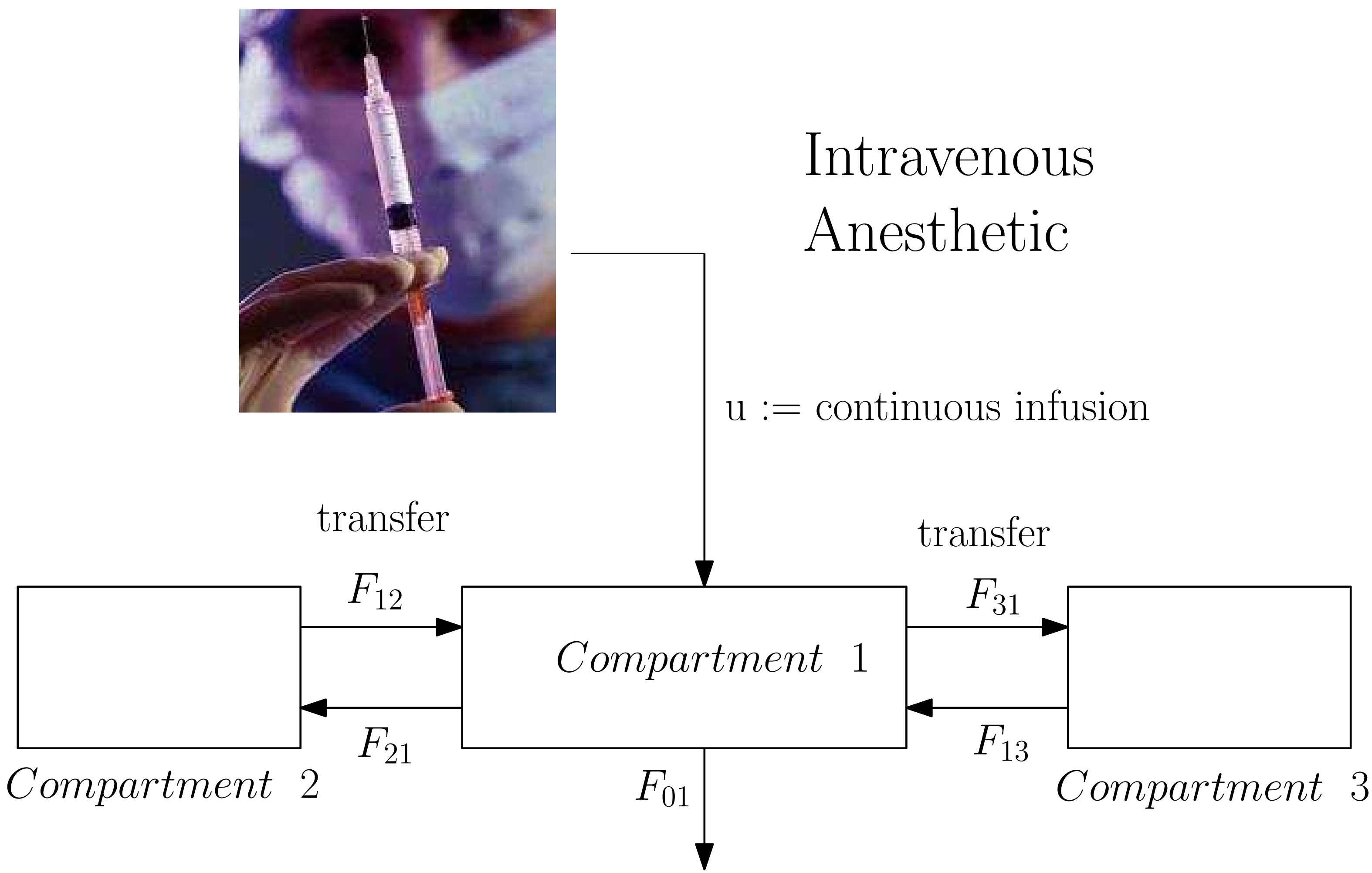
# *Outline:*

- Motivation - why positive systems?
- Introduction and background to positive LTI systems
- SISO results and examples
- MIMO results and examples
- Extensions and future development
- References

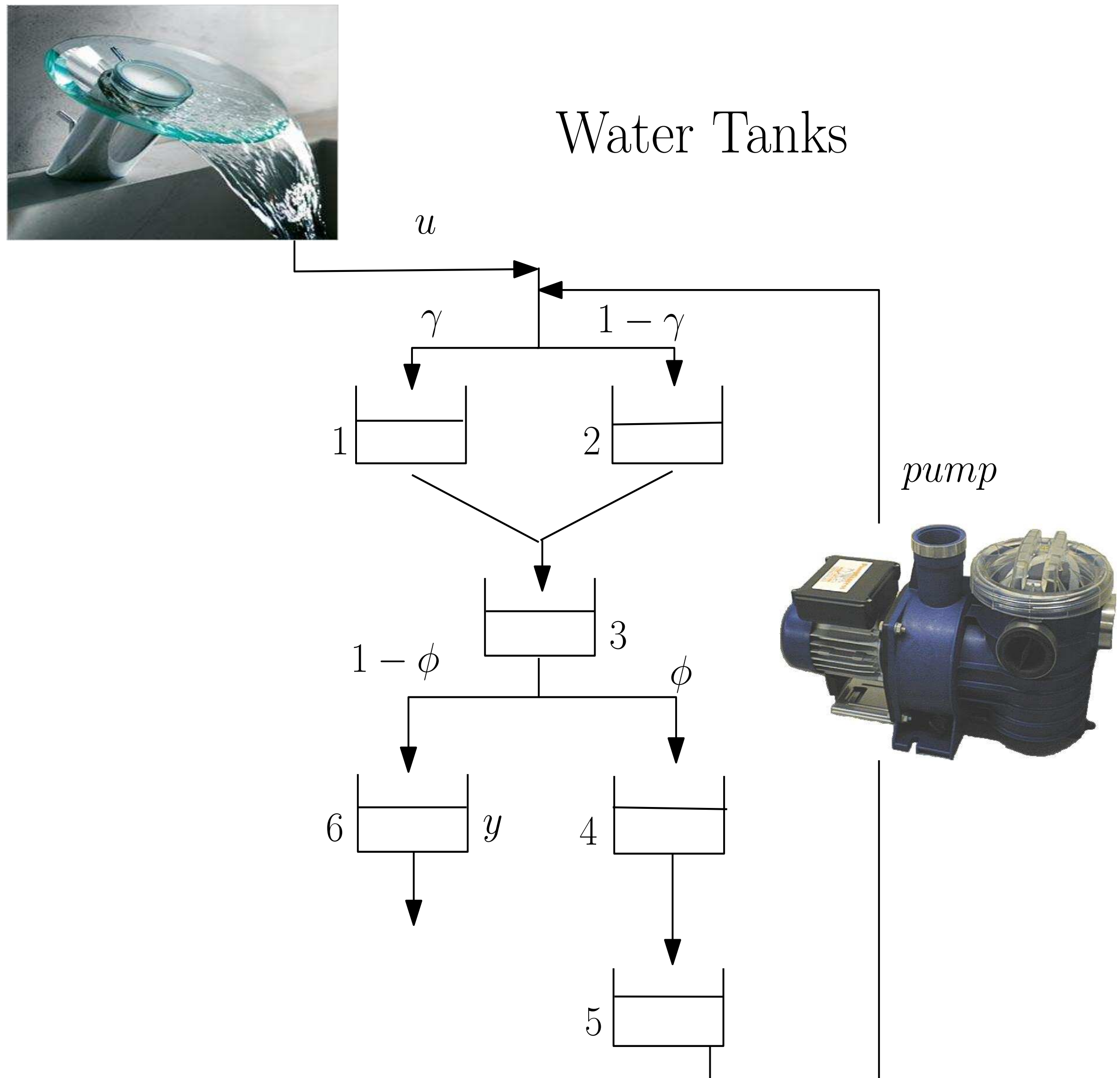
# *Motivation - Why Positive Systems:*




# Motivation - Intravenous Maintenance of Anesthesia:



# ***Motivation - Water Tanks:***



# *Introduction to positive systems:*

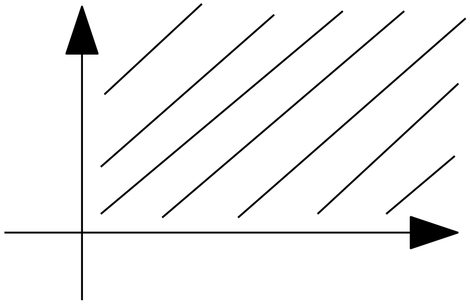
- Motivation - why positive systems? ✓
- Introduction and background to positive LTI systems
  - Positive LTI System Definition
  - "Almost" Positive LTI Systems
  - System of Interest

## *Positive LTI System Definition:*

**Definition 1** *A linear system*

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \quad (1)$$

*where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{r\times n}$, and $D \in \mathbb{R}^{r\times m}$ is considered to be a* positive linear system *if for every nonnegative initial state and for every nonnegative input the state of the system and the output remain nonnegative.*



# ***Positive LTI System Definition:***

It turns out that Definition 1 has a very nice interpretation in terms of the matrix quadruple $(A, B, C, D)$.

**Theorem 1** *A linear system* (1) *is positive if and only if the matrix $A$ is a Metzler matrix, and $B$, $C$, and $D$ are nonnegative matrices.*

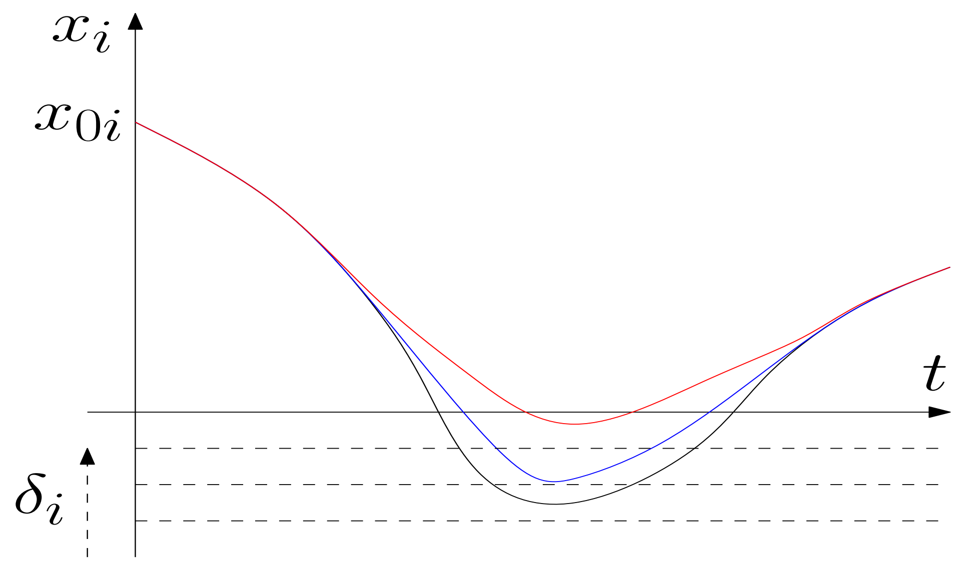
A matrix $A$ is Metzler if all the off-diagonal terms are nonnegative.

$$\begin{bmatrix} \ddots & \geq 0 \\ \geq 0 & \ddots \end{bmatrix}$$

# Almost *Positive LTI Systems:*

An arbitrary linear system is considered to be an *almost-state (output) positive linear system* with respect to $x_0$ if for any given $\delta = (\delta_1, \delta_2, ..., \delta_{n(r)}) \in \mathbb{R}_+^{n(r)} \setminus \{0\}$ there exists a $u_\delta$ such that the state $x$ (output $y$) of the system satisfies

$$x_i(t)(y_i(t)) \geq -\delta_i, \quad \forall i = 1, 2, ..., n(r), \ \ \forall t \in [0, \infty).$$



# *Problem statement:*

- Motivation - why positive systems? ✓
- Introduction and background to positive LTI systems ✓
- SISO results and examples
  - Servomechanism problem for SISO positive LTI systems

# System of Interest:

SISO system

$$\begin{aligned} \dot{x} &= Ax + bu + e_\omega \omega \\ y &= cx + du + f\omega \\ e &:= y - y_{ref} \end{aligned} \quad (2)$$

$u \in \mathbb{R}^m$ is the input, $x \in \mathbb{R}^n_+$ is the state, $y \in \mathbb{R}_+$ is the output to be regulated, $\omega \in \mathbb{R}^\Omega$ are the disturbances, $y_{ref} \in \mathbb{R}_+$ is the tracking signal and $e \in \mathbb{R}^r$ is the error in the system. Matrix $A$ is stable Metzler, and matrices $b$, $c$, $e_\omega\omega$, $f\omega$, and $d$ are nonnegative.

# Key Assumptions:

**Assumption 1** *Given* (2) *assume that*

$$rank(d - cA^{-1}b) = 1$$

*and for all tracking and disturbance signals in question, it's assumed that the steady-state of the system* (2) *given by*

$$\begin{bmatrix} x_{ss} \\ u_{ss} \end{bmatrix} = -\begin{bmatrix} A & b \\ c & d \end{bmatrix}^{-1} \begin{bmatrix} e_\omega & 0 \\ f & -1 \end{bmatrix} \begin{bmatrix} \omega \\ y_{ref} \end{bmatrix} \quad (3)$$

*and has the property that* $u_{ss} \in \mathbb{R}_+$.

# Servomechanism problem for SISO positive LTI systems:

**Problem:** Consider the plant (2), with initial condition $x_0 \in \mathbb{R}^n_+$, under Assumption 1. Find a nonnegative controller $u$ that

(a) guarantees closed loop stability;

(b) ensures the plant (2) is nonnegative, i.e. the states $x$ and the output $y$ are nonnegative for all time; and

(c) ensures tracking of the reference signals, i.e. $e = y - y_{ref} \to 0$, as $t \to \infty$, $\forall y_{ref} \in Y_{ref}$ and $\forall \omega \in \Omega$. In addition,

(d) assume that a controller has been found so that conditions (a), (b), (c) are satisfied; then for all perturbations of the nominal plant modal which maintain properties (a) and (b), it is desired that the controller can still achieve asymptotic tracking and regulation, i.e. property (c) still holds.

# Servomechanism problem for SISO positive LTI systems:

**Problem:** Consider the plant (2), with initial condition $x_0 \in \mathbb{R}^n_+$, under Assumption 1. Find a nonnegative controller $u$ that

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(c) ensures tracking of the reference signals, i.e. $e = y - y_{ref} \to 0$, as $t \to \infty$, $\forall y_{ref} \in Y_{ref}$ and $\forall \omega \in \Omega$. In addition,

(d) the controller is robust.

# Servomechanism problem for "almost" positive LTI systems:

**Remark 1**

In the sequel when almost-state and almost-output positivity will be considered, then in the previous problem the words state and output should be replaced by almost-state and almost-output, respectively. Additionally, the constraint of nonnegativity on the input will be lifted, i.e. the input can be bidirectional.

We call this problem the servomechanism problem for "almost" positive LTI systems.

# Clamping Tuning Regulator:

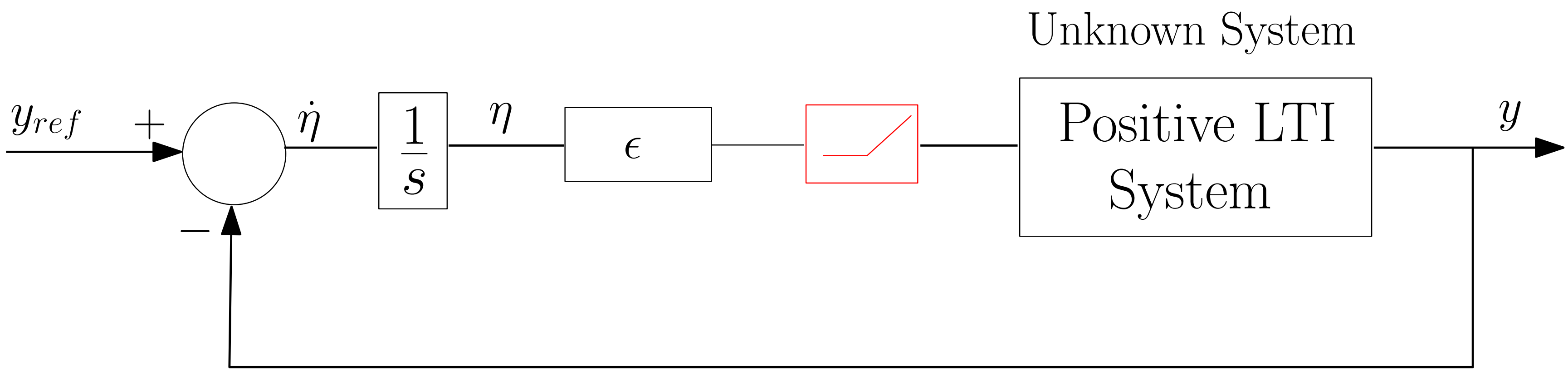
$$\begin{array}{rcl} \dot{\eta} & = & \epsilon(y_{ref} - y), \quad \eta_0 = 0 \\ u & = & k\eta \end{array}, \qquad (4)$$

where

$$k = \begin{cases} 0 & \eta \leq 0 \\ 1 & \eta > 0 \end{cases}$$

and $\eta_0 = 0$ and $\epsilon \in (0, \epsilon^*]$, $\epsilon^* \in \mathbb{R}_+ \setminus \{0\}$.

# *Clamping Tuning Regulator:*




$$\begin{aligned} \dot{\eta} &= \epsilon(y_{ref} - y), \quad \eta_0 = 0 \\ u &= k\eta \end{aligned}$$

# Key Assumptions:
# Finding $rank(d - cA^{-1}b) = 1$

**Algorithm 1** *It is assumed that the output of the system is measurable and the input is excitable the disturbance set to zero, i.e.* $\omega = 0$*.*

1. *Apply an input* $u = \overline{u}$ *to* (2)*, with* $\overline{u}$ *having a non-zero steady-state value.*
2. *Measure the corresponding steady-state value of the output* $y = \overline{y}$*.*
3. *If* $\overline{y} \neq 0$*, then the existence condition holds true.*

$$\dot{x} = 0 = Ax_{ss} + bu_{ss} + e_\omega \omega$$

$$\dot{\eta} = \epsilon(y_{ref} - y) = 0 = cx_{ss} + du_{ss} + f\omega - y_{ref}$$

Isolating for $x_{ss}$ we get:

$$x_{ss} = -A^{-1}bu_{ss} - A^{-1}e_\omega \omega.$$

By substituting $x_{ss}$ and isolating for $u_{ss}$ we obtain:

$$u_{ss} = \frac{cA^{-1}e_\omega \omega - f\omega + y_{ref}}{d - cA^{-1}b}.$$

$$\dot{x} = 0 = Ax_{ss} + bu_{ss} + e_\omega \omega$$

$$\dot{\eta} = \epsilon(y_{ref} - y) = 0 = cx_{ss} + du_{ss} + f\omega - y_{ref}$$

Isolating for $x_{ss}$ we get:

$$x_{ss} = -A^{-1}bu_{ss} - A^{-1}e_\omega \omega.$$

Therefore:

$$cA^{-1}e_\omega \omega - f\omega + y_{ref} \geq 0.$$

# *Solution:*

**Theorem 2** *Consider system* (2) *under the clamping tuning regulator. Further assume that*

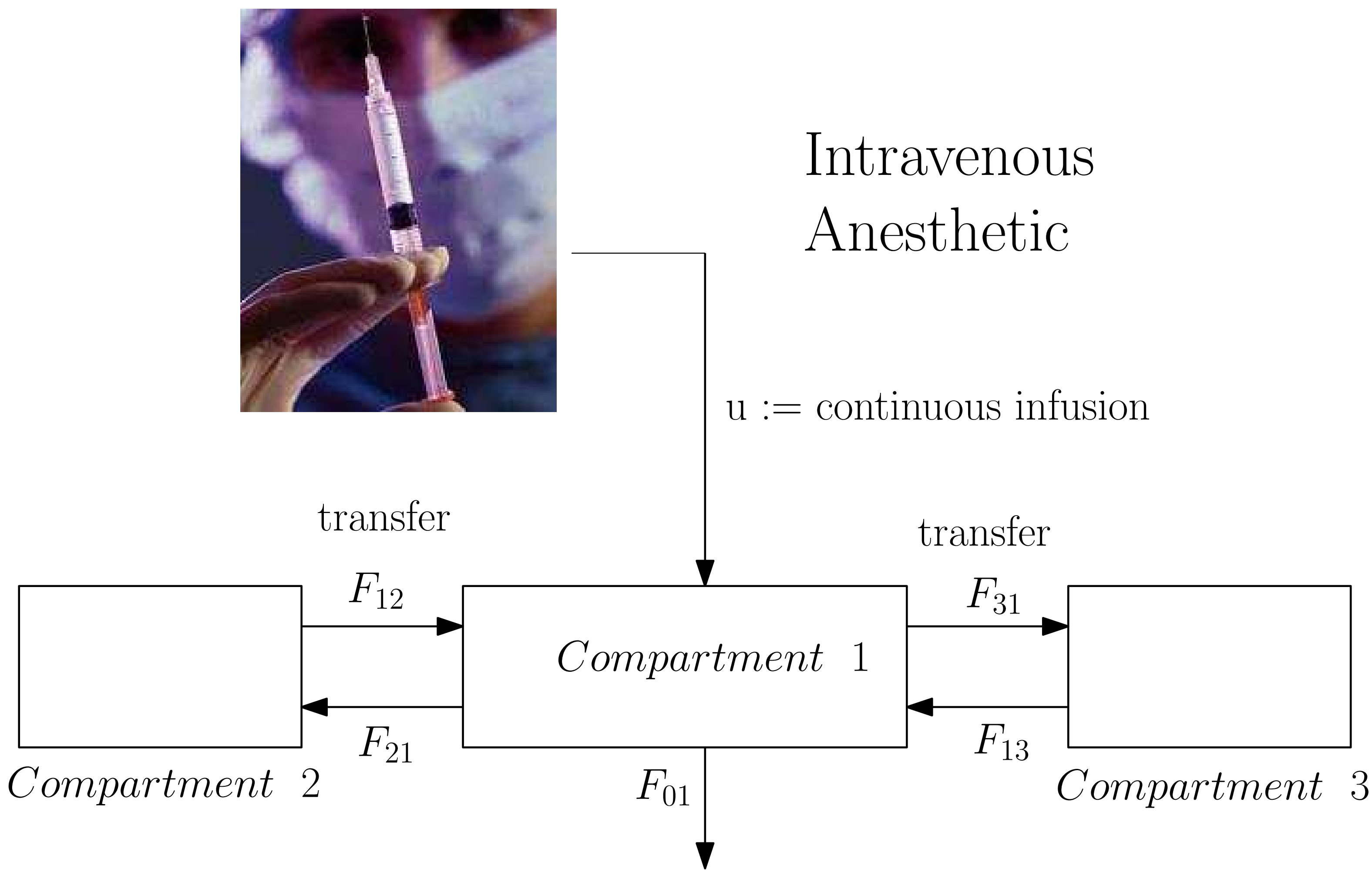
- $rank(d - cA^{-1}b) = 1$
- $x_0 \in \mathbb{R}^n_+$
- $u_{ss} > 0$ .

*Then there exists an* $\epsilon^*$ *such that for all* $\epsilon \in (0, \epsilon^*]$ *the clamping tuning regulator solves the servomechanism problem.*

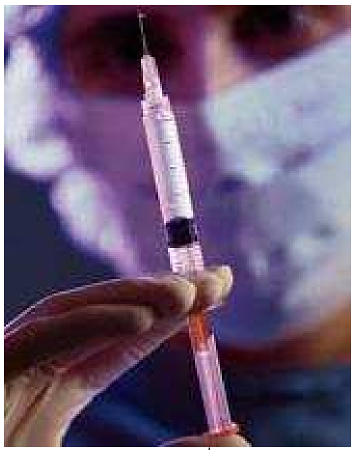
**Algorithm 2**

1. *Check the existence condition* $rank(d - cA^{-1}b) = 1$ *by Algorithm 1.*
   (a) *If Algorithm 1 returns* $\overline{y} = 0$*, then there does not exist a solution to the servomechanism problem.*
   (b) *Otherwise, go to Step 2.*
2. *Apply the clamping regulator to the unknown plant.*
   (a) *If the clamping controller remains at zero for* $t \in [t_+, \infty)$*, where* $t_+ \geq 0$*, and no tracking/regulation occurs, then the servomechanism problem is not solvable under any control law.*
   (b) *Otherwise, the clamping regulator solves the servomechanism problem.*

# Intravenous Anesthetic



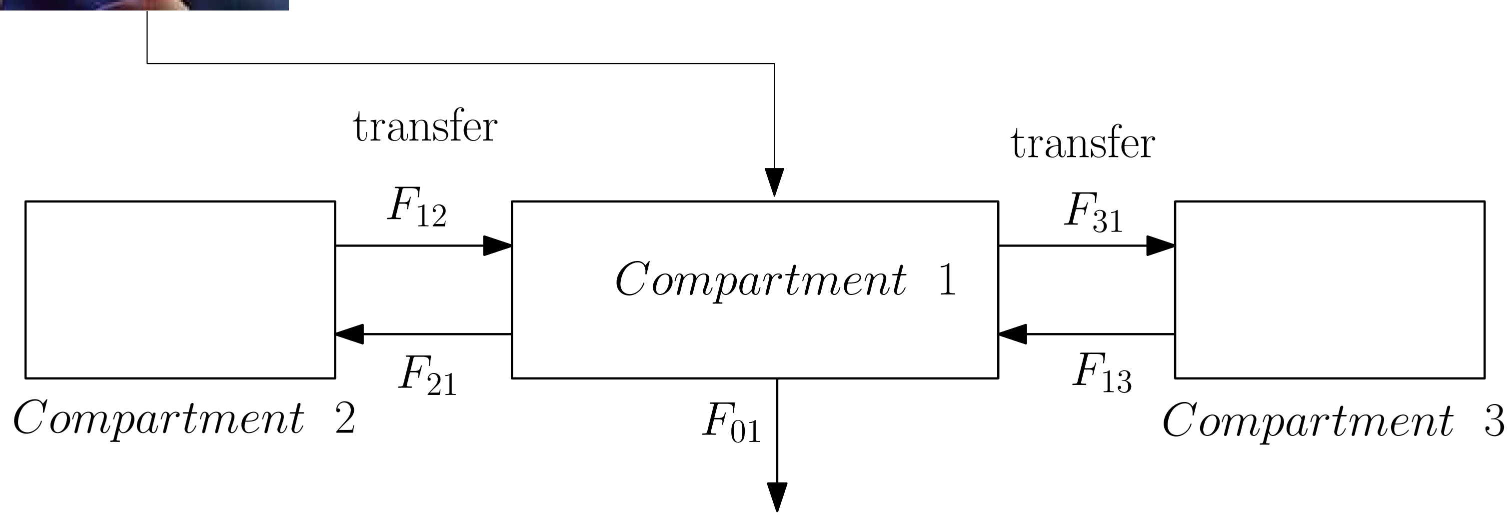
# Intravenous Anesthetic



$$*stable, \ rank(d - cA^{-1}b) = 1$$

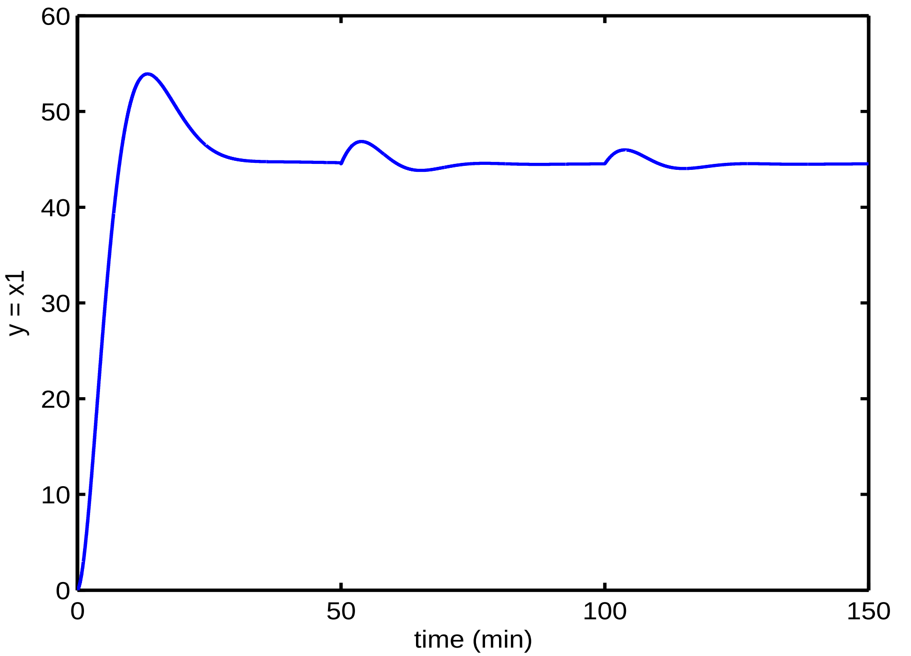
$$\dot{x} = \begin{bmatrix} -(f_{01}+f_{21}+f_{31}) & f_{12} & f_{31} \\ f_{21} & -f_{12} & 0 \\ f_{31} & 0 & -f_{13} \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} e_\omega \\ 0 \\ 0 \end{bmatrix} \omega$$

$$y = [1 \ \ 0 \ \ 0]x$$



# Intravenous Anesthetic

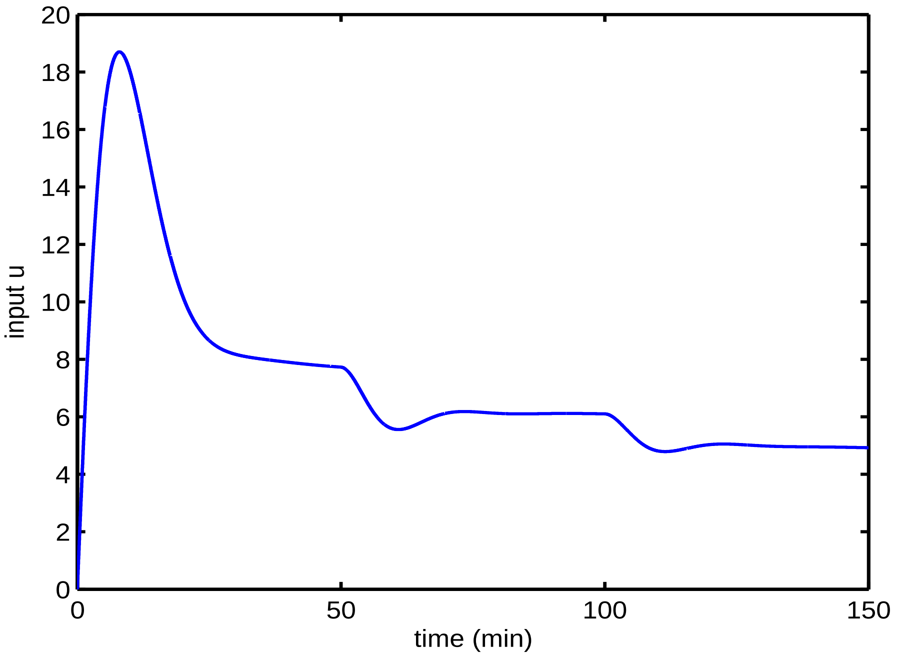
## Output




| Time | $f_{01}$ | $f_{21}$ | $f_{12}$ | $f_{31}$ | $f_{13}$ | $e_\omega \omega$ |
|---|---|---|---|---|---|---|
| $[0, 50)$ | 0.152 | 0.207 | 0.092 | 0.040 | 0.0048 | 0.5 |
| $[50, 100)$ | 0.119 | 0.114 | 0.055 | 0.041 | 0.0033 | 0.5 |
| $[100, 150]$ | 0.119 | 0.114 | 0.055 | 0.041 | 0.0033 | 1.5 |

# *Intravenous Anesthetic*

Input




| Time | $f_{01}$ | $f_{21}$ | $f_{12}$ | $f_{31}$ | $f_{13}$ | $e_\omega \omega$ |
|---|---|---|---|---|---|---|
| $[0, 50)$ | 0.152 | 0.207 | 0.092 | 0.040 | 0.0048 | 0.5 |
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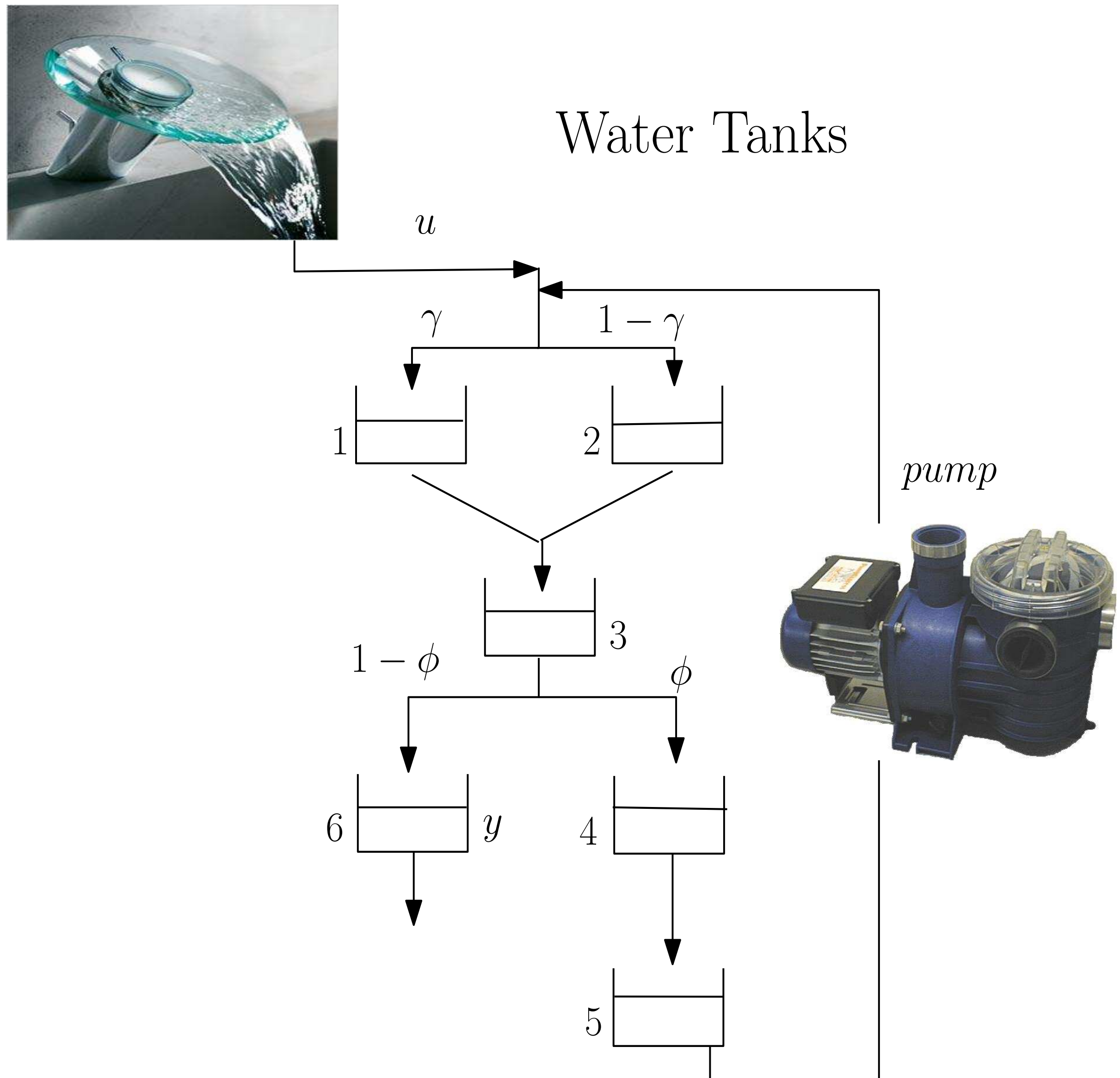
# Water Tanks:



# Water Tanks:

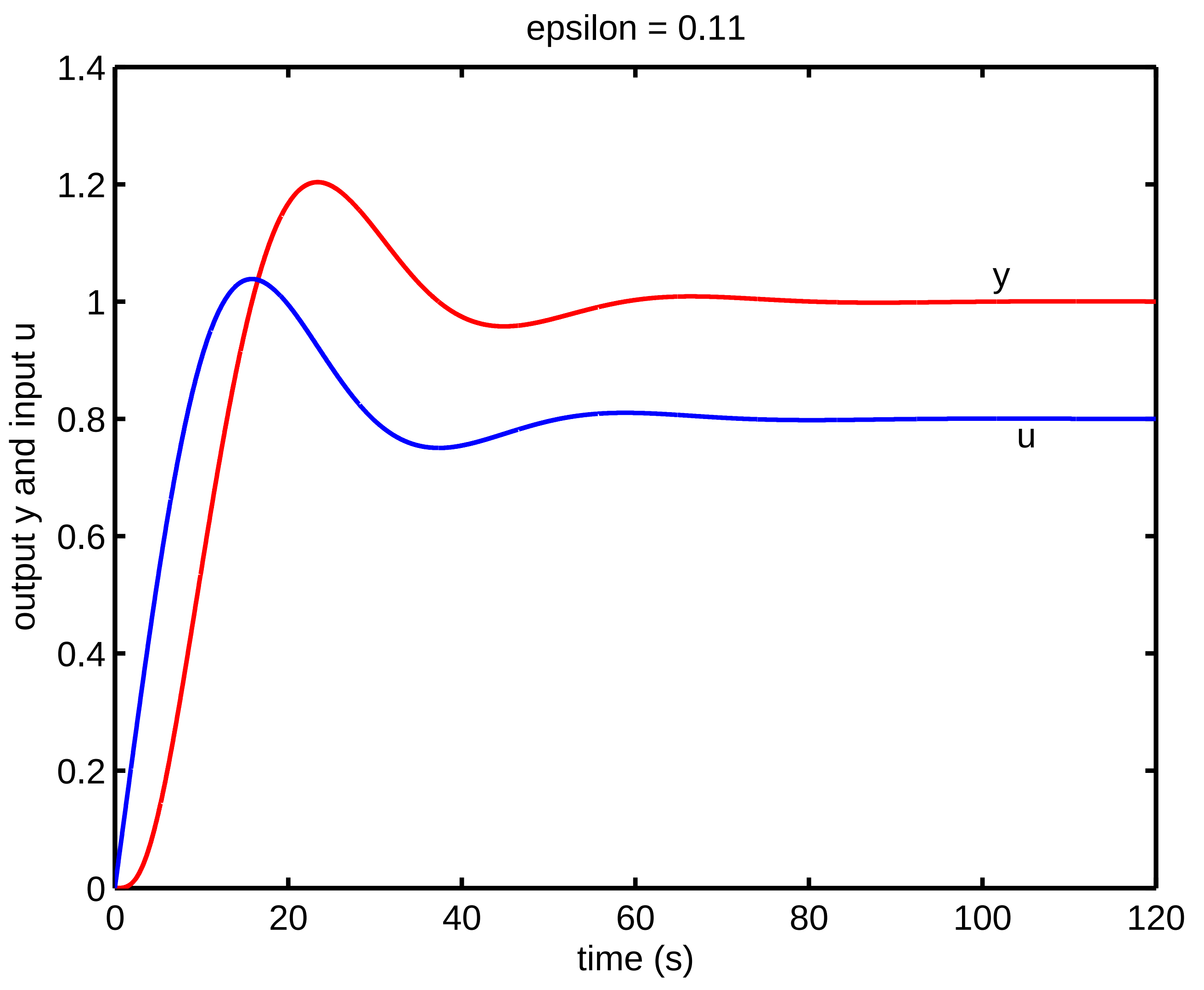
$$\dot{x} = \begin{bmatrix} -0.8 & 0 & 0 & 0 & 2 & 0 \\ 0 & -0.7 & 0 & 0 & 0 & 0 \\ 0.8 & 0.7 & -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.15 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0.35 & 0 & 0 & -0.8 \end{bmatrix} x + \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \omega$$

Also, assume the output $y$ is of the form

$$y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} x$$

* $\epsilon = 0.5$.

# Simulation:

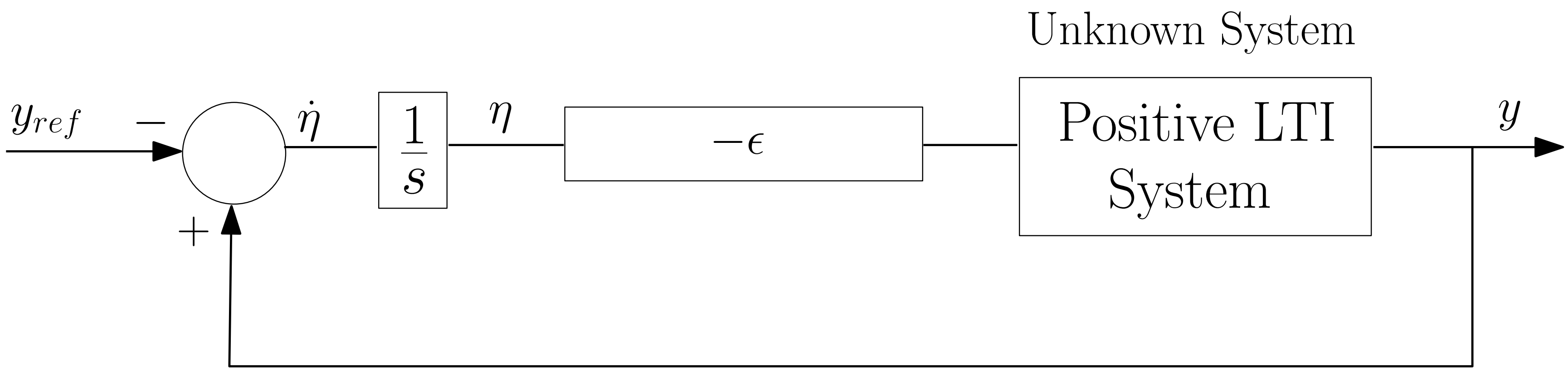


epsilon = 0.11
output y and input u
time (s)
y
u
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
20
40
60
80
100
120

What about “almost” positivity?

$$\begin{aligned} \dot{\eta} &= y - y_{ref} \\ u &= -\epsilon\eta \end{aligned} \qquad (5)$$

where $\eta_0 = 0$ and $\epsilon \in (0, \epsilon^*]$, $\epsilon^* \in \mathbb{R}_+ \setminus \{0\}$.



The controller:

$$\begin{aligned} \dot{\eta} &= y - y_{ref} \\ u &= -\epsilon\eta \end{aligned} \qquad (6)$$

where $\eta_0 = 0$ and $\epsilon \in (0, \epsilon^*]$, $\epsilon^* \in \mathbb{R}_+ \setminus \{0\}$, under the assumption that

$$rank(d - cA^{-1}b) = 1,\ x_{ss} \in \mathbb{R}^n_+ \text{ and } x_0 \in \mathbb{R}^n_+$$

solves Problem 1 under Remark 1, i.e. the servomechanism problem for "almost" positivity can be attained under (5).

# Optimal approach: LQcR control

Consider the same problem under the controller:

$$\begin{aligned} \dot{\eta} &= y - y_{ref}, \quad \eta_0 = 0 \\ u &= max\{[K_x \;\; K_\eta] \begin{bmatrix} x \\ \eta \end{bmatrix}, 0\} \end{aligned}$$

where $K_x \in \mathbb{R}^{1 \times n}$ and $K_\eta \in \mathbb{R}$ are found by solving the cheap control problem:

$$\int_0^\infty \epsilon^2 e^T e + \dot{u}^T \dot{u} d\tau \qquad (7)$$

where $\epsilon > 0$

# Optimal approach: LQcR control

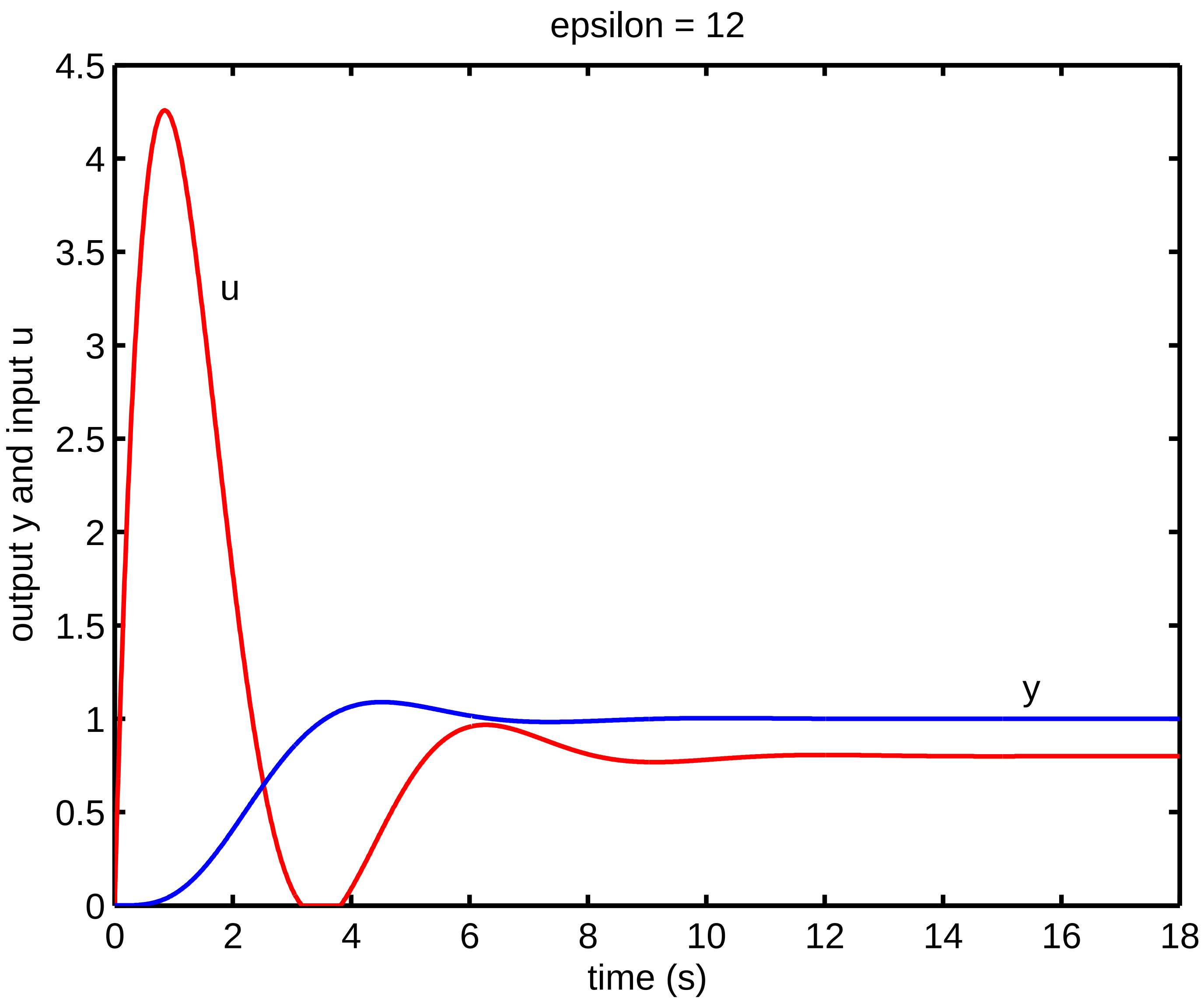
Consider the same problem under the controller:

$$\begin{aligned} \dot{\eta} &= y - y_{ref}, \quad \eta_0 = 0 \\ u &= max\{[K_x \;\; K_\eta] \begin{bmatrix} x \\ \eta \end{bmatrix}, 0\} \end{aligned}$$

for the system:

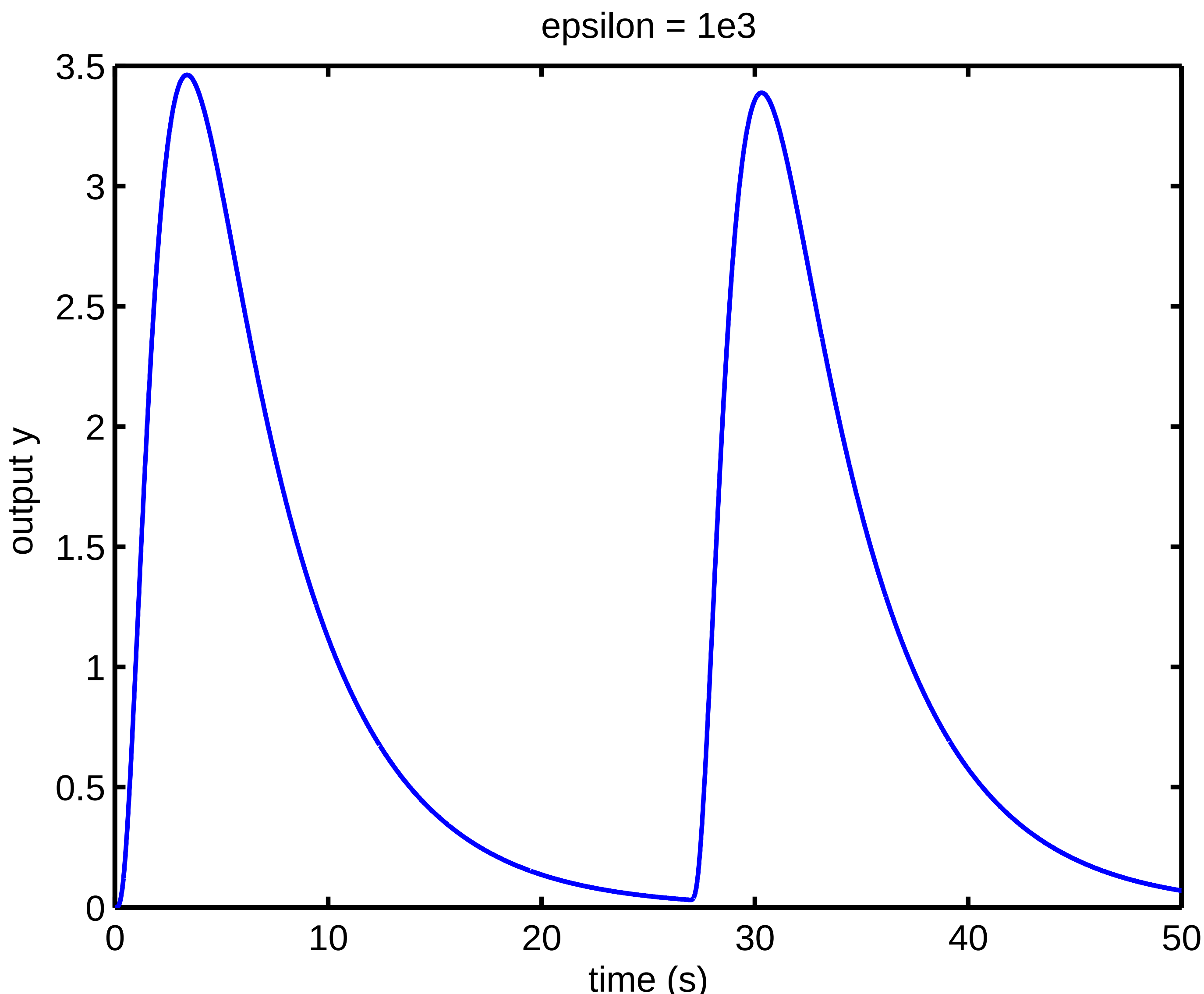
$$\begin{aligned} \begin{bmatrix} \ddot{x} \\ \dot{e} \end{bmatrix} &= \begin{bmatrix} A & 0 \\ c & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ e \end{bmatrix} + \begin{bmatrix} b \\ d \end{bmatrix} \dot{u} \\ e &= [0 \;\; 1] \begin{bmatrix} x \\ e \end{bmatrix} \end{aligned}$$

# *Optimal Approach: LQcR control*

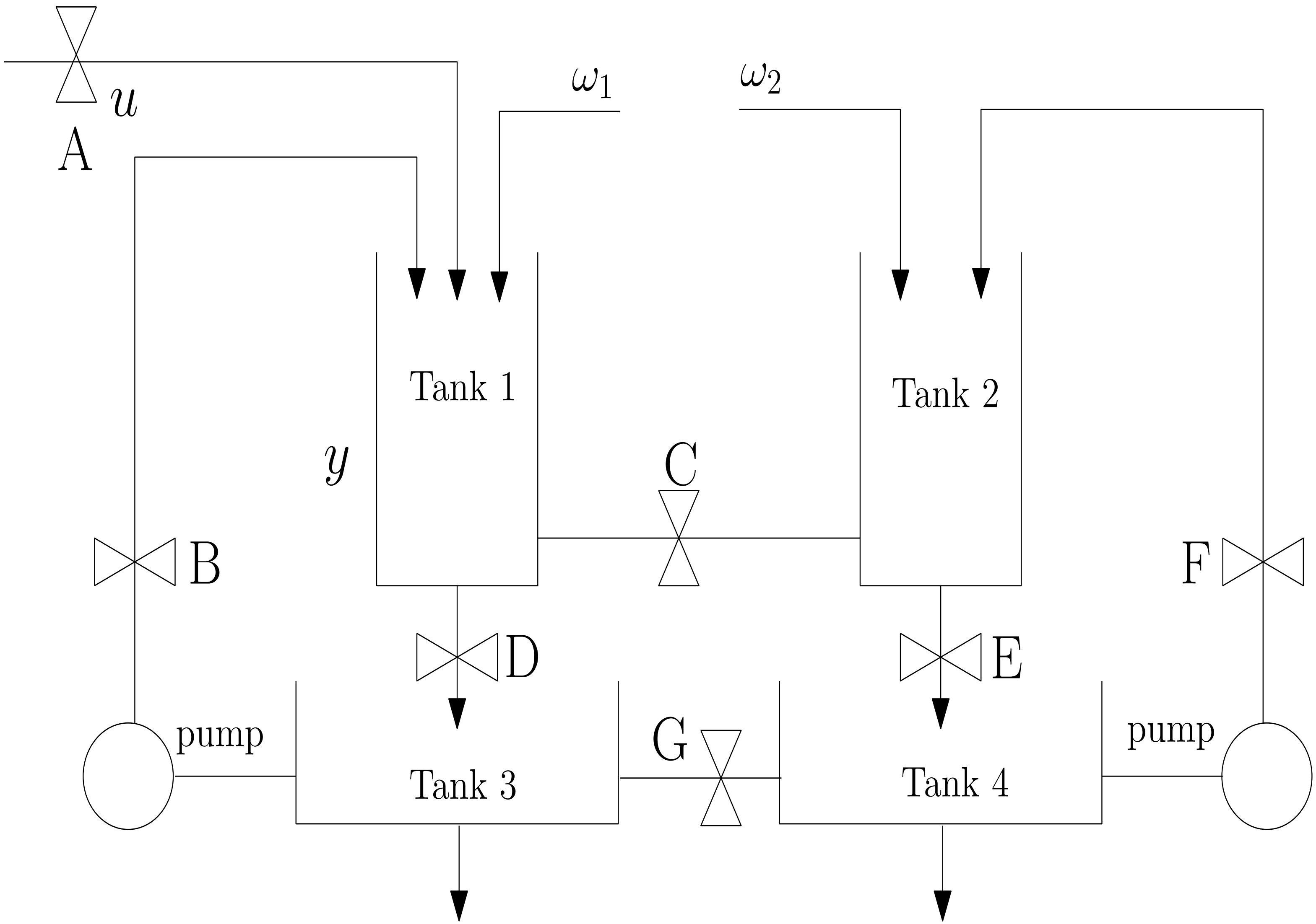


# *Optimal Approach: LQcR control*

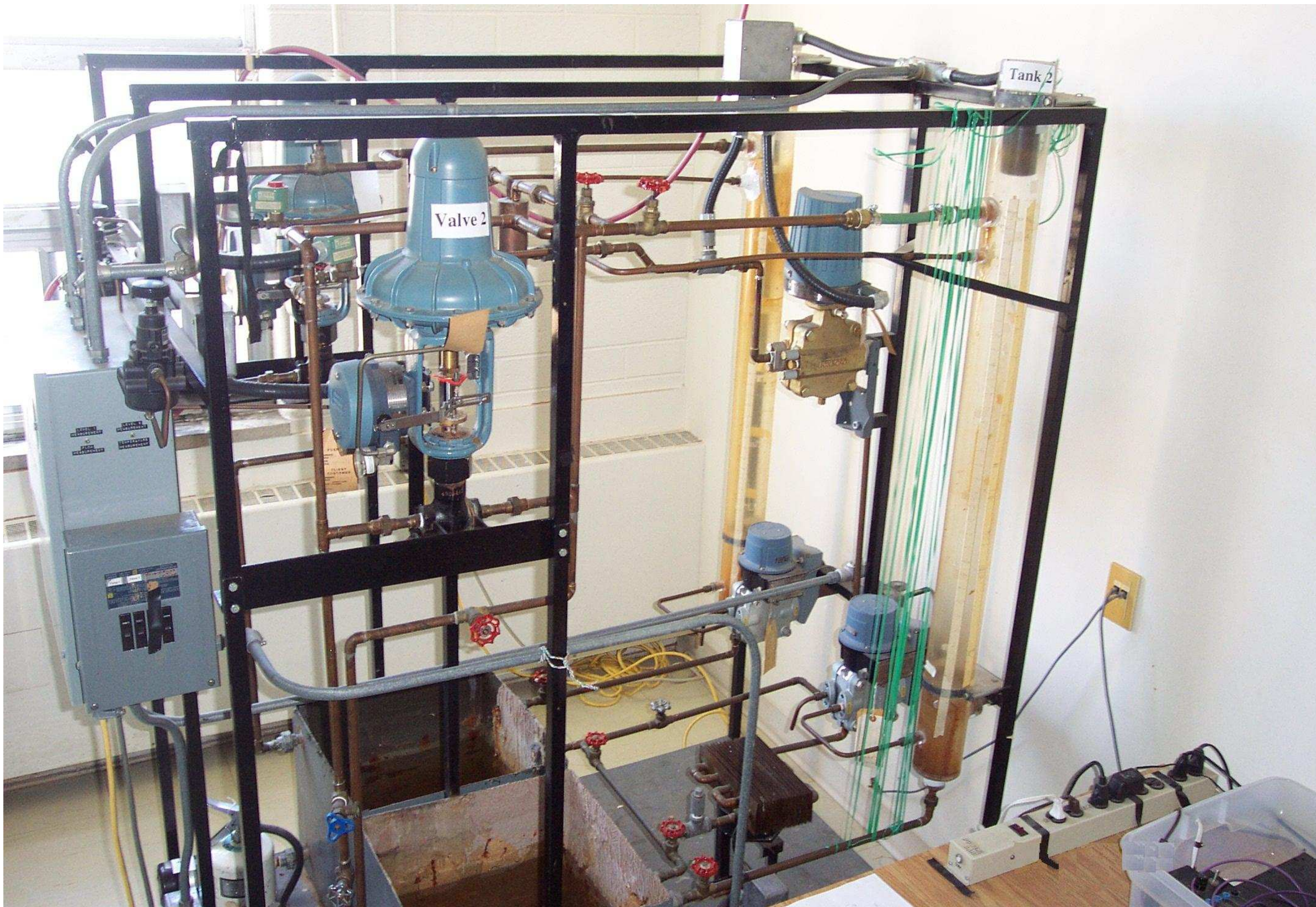
We cannot blindly use the standard LTI approach! E.g.



# Experimental results: LQcR control



# *Experimental results: LQcR control*



## *Results and examples:*

- Motivation - why positive systems? ✓
- Introduction and background to positive LTI systems ✓
- SISO results and examples ✓
- MIMO results and examples

# System of Interest:

MIMO case

$$
\begin{aligned}
\dot{x} &= Ax + Bu + E\omega \\
y &= Cx + Du + F\omega \\
e &:= y_{ref} - y
\end{aligned} \tag{8}
$$

$u \in \mathbb{R}^m$ is the input, $x \in \mathbb{R}^n_+$ is the state, $y \in \mathbb{R}_+$ is the output to be regulated, $\omega \in \mathbb{R}^{\Omega}_+$ are the disturbances, $y_{ref} \in \mathbb{R}_+$ is the tracking signal and $e \in \mathbb{R}^r$ is the error in the system. Matrix $A$ is stable Metzler, and matrices $B$, $C$, $D$, $E$, $F$ are nonnegative with $m = r$.

## *Problem of Interest:*

Find a controller $u \in \mathbb{R}^m_+$ for all reference tracking signals $y_{ref} \in \mathbb{R}^r_+$ and for all disturbance signals $\omega \in \mathbb{R}^{\overline{\Omega}}_+$ such that

(a) closed loop stability is maintained;

(b) nonnegativity of states $x$ and outputs $y$ occurs for all time;

(c) tracking of reference signals occurs, i.e. $e = y_{ref} - y \to 0$, as $t \to \infty$, $\forall y_{ref} \in \mathbb{R}^r_+$ and $\forall \omega \in \mathbb{R}^{\overline{\Omega}}_+$.

(d) assume that an LTI controller has been found so that conditions (a), (b), (c) are satisfied; then for all perturbations of the nominal plant modal which maintain properties (a) and (b), it is desired that the controller can still achieve asymptotic tracking and regulation, i.e. the controller is robust.

# In General No Solution:

**Theorem:** There does not exist a solution to the problem of interest for almost all positive systems (8).

Reason:

$$\begin{aligned} u_{ss} &= K_r y_{ref} + K_d \omega \\ &= (D - CA^{-1}B)^{-1} y_{ref} \\ &\qquad - (D - CA^{-1}B)^{-1}(F - CA^{-1}E)\omega \\ &\geq 0 \end{aligned}$$

# *Key Assumption:*

**Assumption 2** *Given* (8) *assume that*

$$rank(D - CA^{-1}B) = r$$

*and for all tracking and disturbance signals in question, it's assumed that the steady-state of the system* (8) *is given by*

$$\begin{bmatrix} x_{ss} \\ u_{ss} \end{bmatrix} = - \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} E & 0 \\ F & -I \end{bmatrix} \begin{bmatrix} \omega \\ y_{ref} \end{bmatrix}$$

*and has the property that* $u_{ss} = K_r y_{ref} + K_d \omega \in \mathbb{R}^m_+$.

1. Apply an input vector $u = [0 \ldots 0 \; \overline{u}_i \; 0 \ldots 0]^T$ to (8), $\forall i = 1, ..., m$.
2. Measure the corresponding steady-state value of the output vectors $y = \overline{y}_i \in \mathbb{R}^r$, $\forall i = 1, ..., m$.
3. Solve the equation:

$$K_1 \begin{bmatrix} \overline{u}_1 & 0 & \ldots & 0 \\ 0 & \overline{u}_2 & \ldots & 0 \\ & & \ddots & \\ 0 & 0 & \ldots & \overline{u}_m \end{bmatrix} = \begin{bmatrix} \overline{y}_1^1 & \overline{y}_2^1 & \ldots & \overline{y}_m^1 \\ \overline{y}_1^2 & \overline{y}_2^2 & \ldots & \overline{y}_m^2 \\ & & \ddots & \\ \overline{y}_1^r & \overline{y}_2^r & \ldots & \overline{y}_m^r \end{bmatrix}$$

for $K_1 = (D - CA^{-1}B)$. Note $K_r = K_1^{-1}$.

1. Apply a disturbance vector $\omega = [0\ \ldots\ 0\ \overline{\omega}_i\ 0\ \ldots\ 0]^T$ to (8), $\forall i = 1, \ldots, \tilde{\Omega}$.
2. Measure the corresponding steady-state value of the output vectors $y = \overline{y}_i \in \mathbb{R}^r$, $\forall i = 1, \ldots, \tilde{\Omega}$.
3. Solve the equation:

$$K_2 \begin{bmatrix} \overline{\omega}_1 & 0 & \ldots & 0 \\ 0 & \overline{\omega}_2 & \ldots & 0 \\ & & \ddots & \\ 0 & 0 & \ldots & \overline{\omega}_{\tilde{\Omega}} \end{bmatrix} = \begin{bmatrix} \overline{y}_1^1 & \overline{y}_2^1 & \ldots & \overline{y}_{\tilde{\Omega}}^1 \\ \overline{y}_1^2 & \overline{y}_2^2 & \ldots & \overline{y}_{\tilde{\Omega}}^2 \\ & & \ddots & \\ \overline{y}_1^r & \overline{y}_2^r & \ldots & \overline{y}_{\tilde{\Omega}}^r \end{bmatrix}$$

for $K_2 = (F - CA^{-1}E)$. Note: $K_d = -K_r K_2$.

# Tuning Regulators and Feedforward control:

Tuning Regulator:

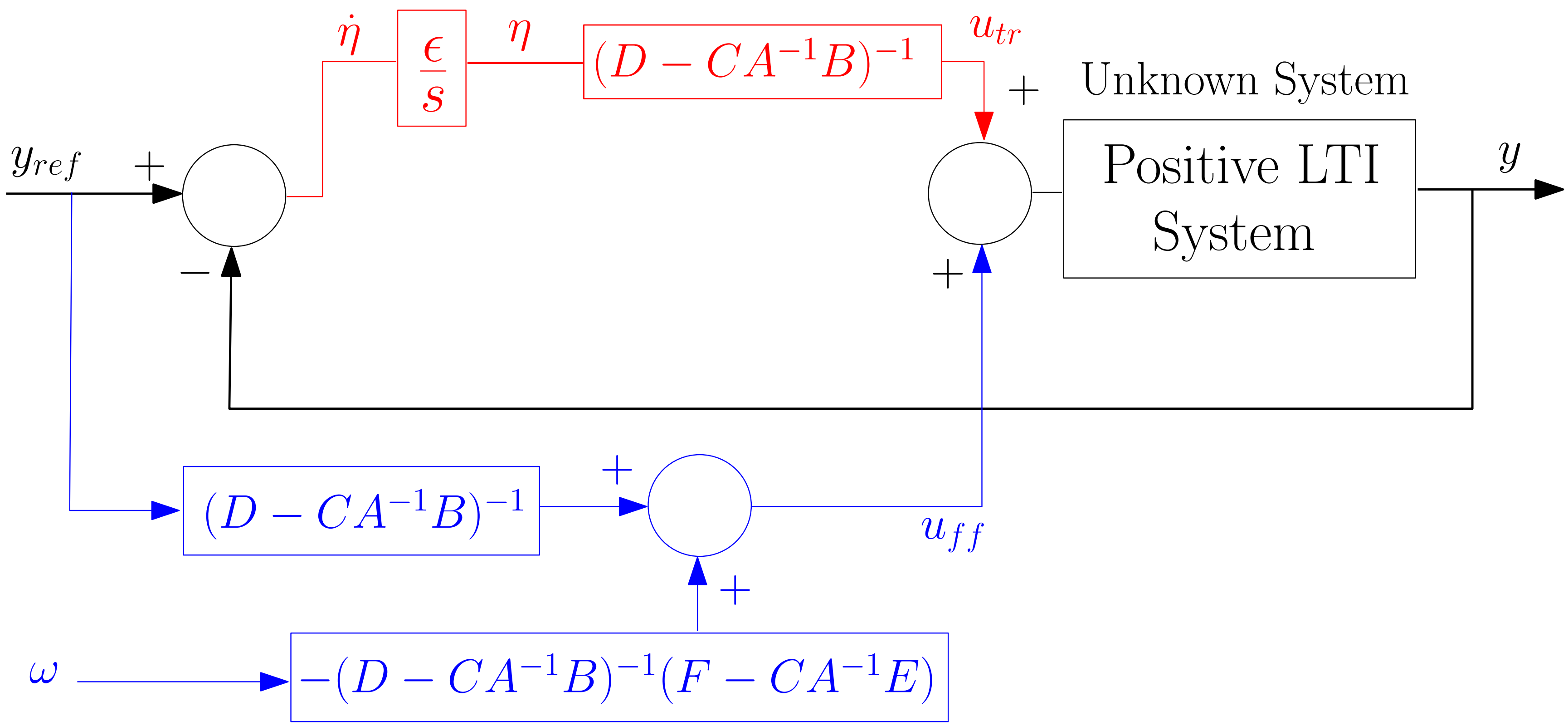
$$\begin{aligned} \dot{\eta} &= \epsilon(y_{ref} - y) \\ u_{tr} &= (D - CA^{-1}B)^{-1}\eta \end{aligned} \qquad (9)$$

where $\epsilon \in (0, \epsilon^*]$, $\epsilon^* \in \mathbb{R}_+ \setminus \{0\}$.

Feedforward Control:

$$u = (D - CA^{-1}B)^{-1} y_{ref} - (D - CA^{-1}B)^{-1}(F - CA^{-1}E)\omega$$

# Tuning Regulators and Feedforward control:



**New Problem:** Obtain the largest subclass of tracking signals $y_{ref} \in Y_{ref} \subset \mathbb{R}^r_+$ and disturbance signals $\omega \in \Omega \subset \mathbb{R}^{\overline{\Omega}}_+$ such that the original Problem of Interest is satisfied.

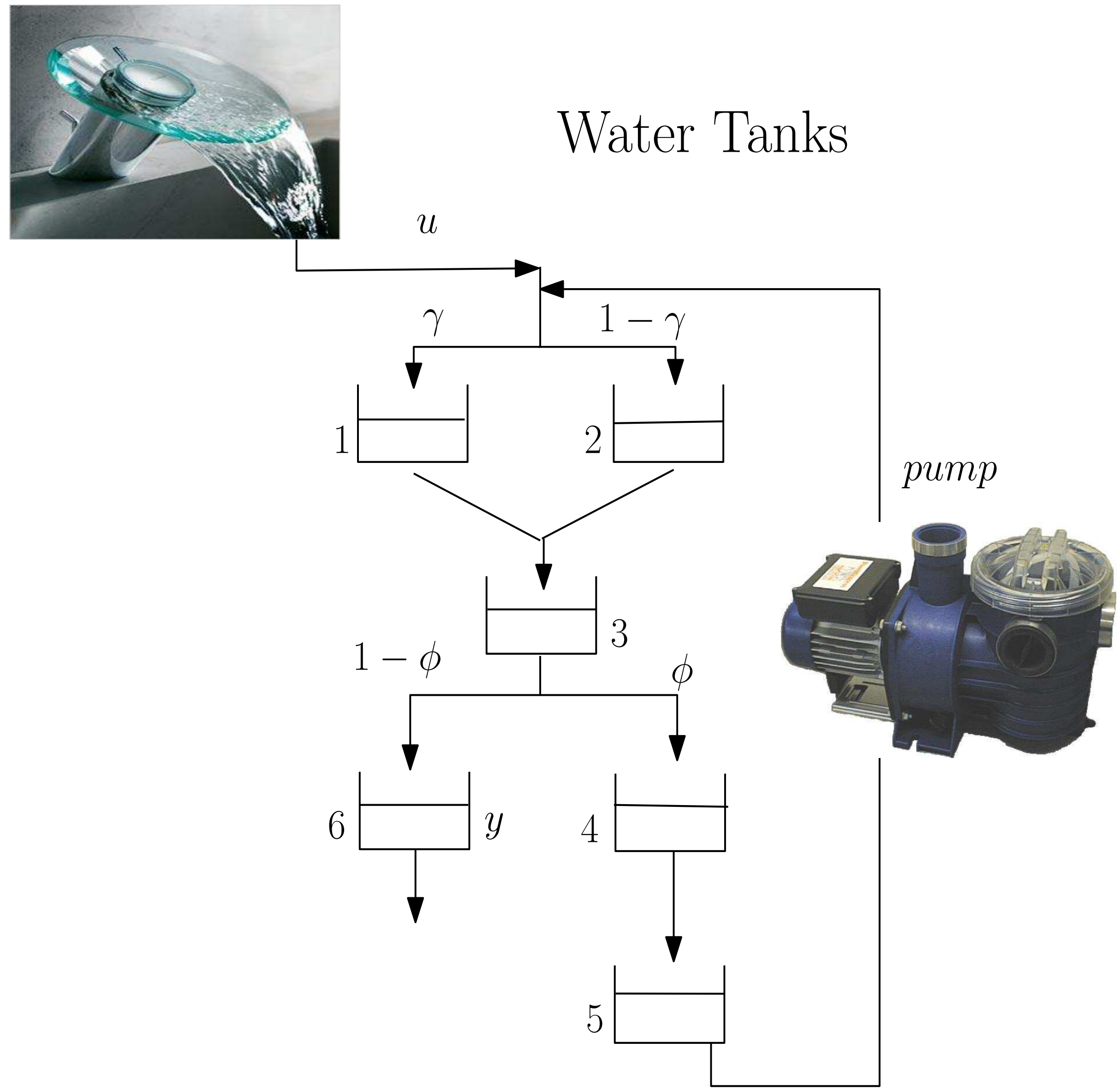
**Theorem:** The original problem is solvable if and only if

$$(y_{ref}, \omega) \in Y_{ref} \times \Omega := \{(\overline{y}_{ref}, \overline{\omega}) \in \mathbb{R}^r_+ \times \mathbb{R}^{\overline{\Omega}}_+ \mid K_r \overline{y}_{ref} > -K_d \overline{\omega} \text{ component-wise}\}. \quad (10)$$

Moreover, it suffices to use the feedforward compensator and the tuning regulator control as the control input $u$, i.e.

$$u = u_{ff} + u_{tr}.$$

# Water Tanks:



# Water Tanks:

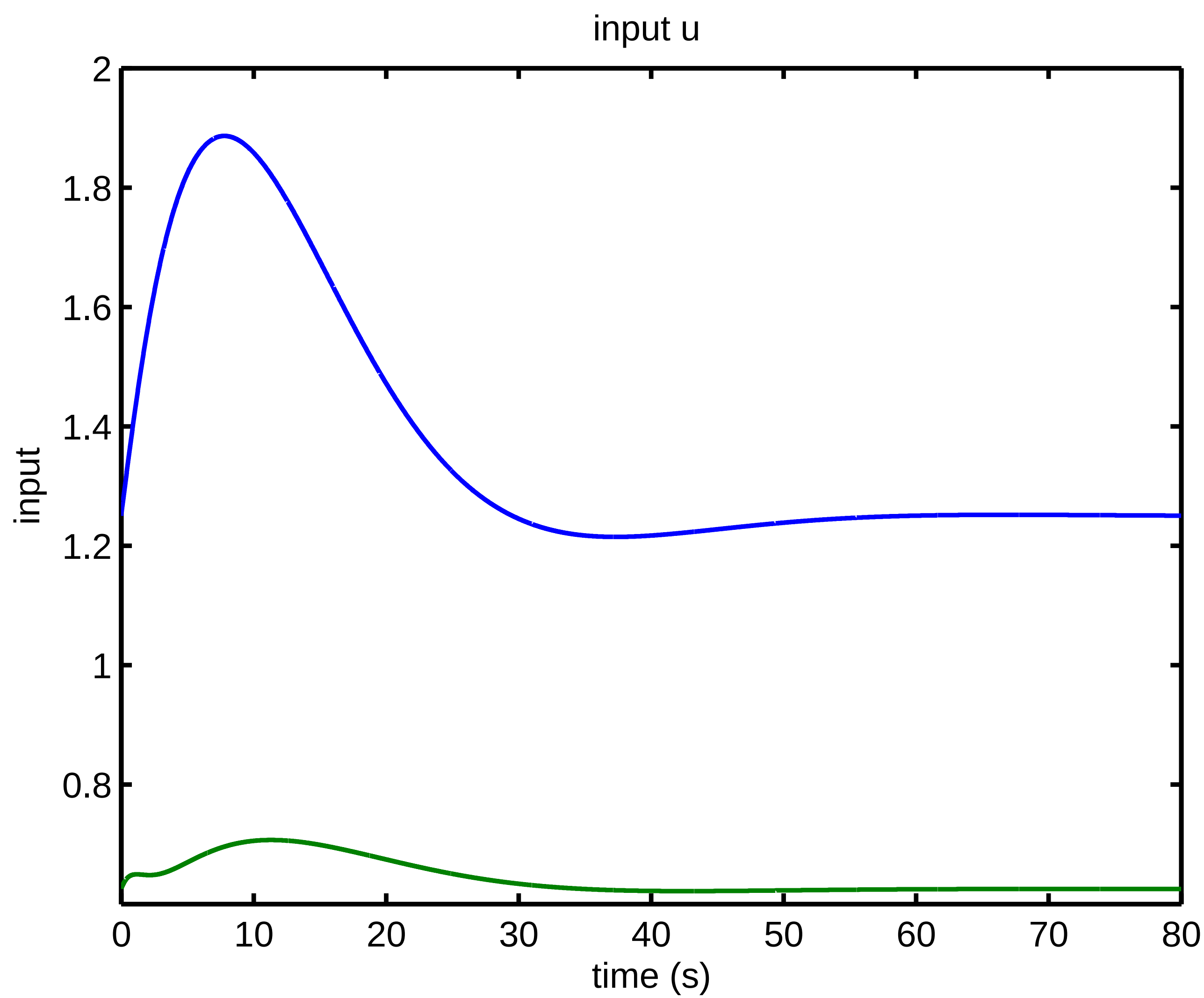
$$\dot{x} = \begin{bmatrix} -0.8 & 0 & 0 & 0 & 2 & 0 \\ 0 & -0.7 & 0 & 0 & 0 & 0 \\ 0.8 & 0.7 & -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.15 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0.35 & 0 & 0 & -0.8 \end{bmatrix} x + \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u + \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \omega$$

Also, assume the output $y$ is of the form

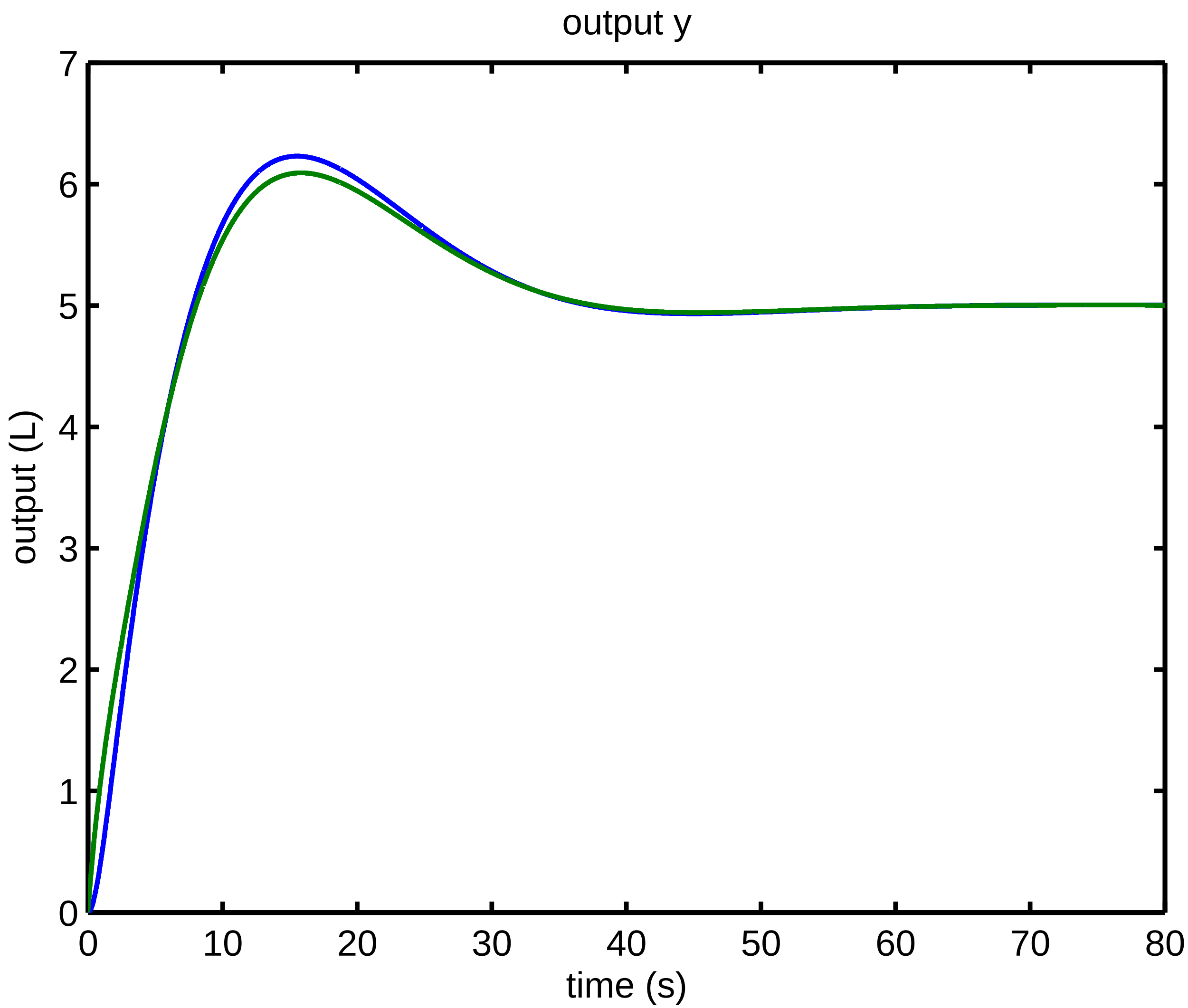
$$y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} x$$

* $\epsilon = 0.1$.

# Simulation:



# Simulation:



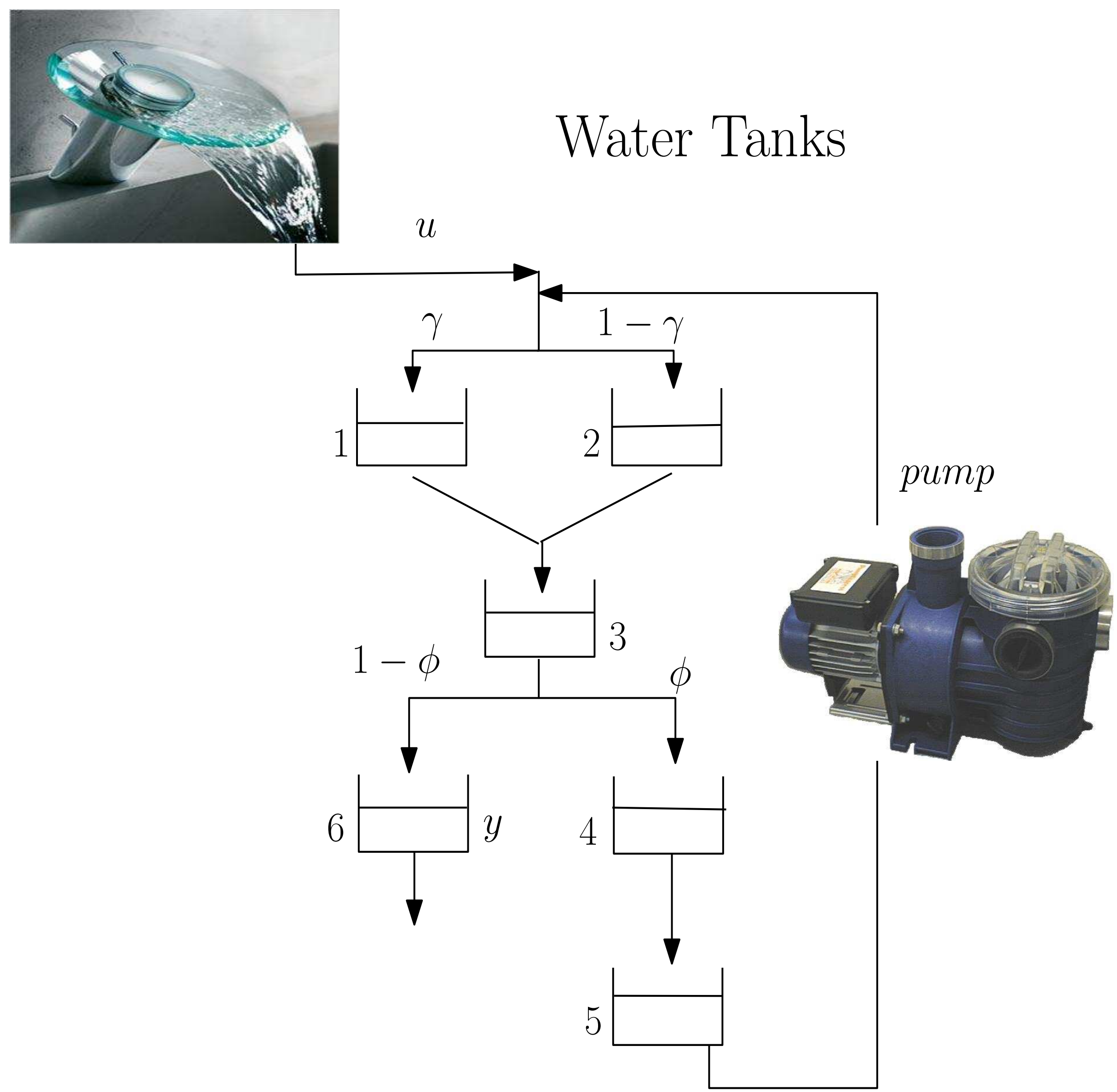
# *Extensions and future development:*

- Motivation - why positive systems? ✓
- Introduction and background to positive LTI systems ✓
- SISO results and examples ✓
- MIMO results and examples ✓
- Extensions and future development

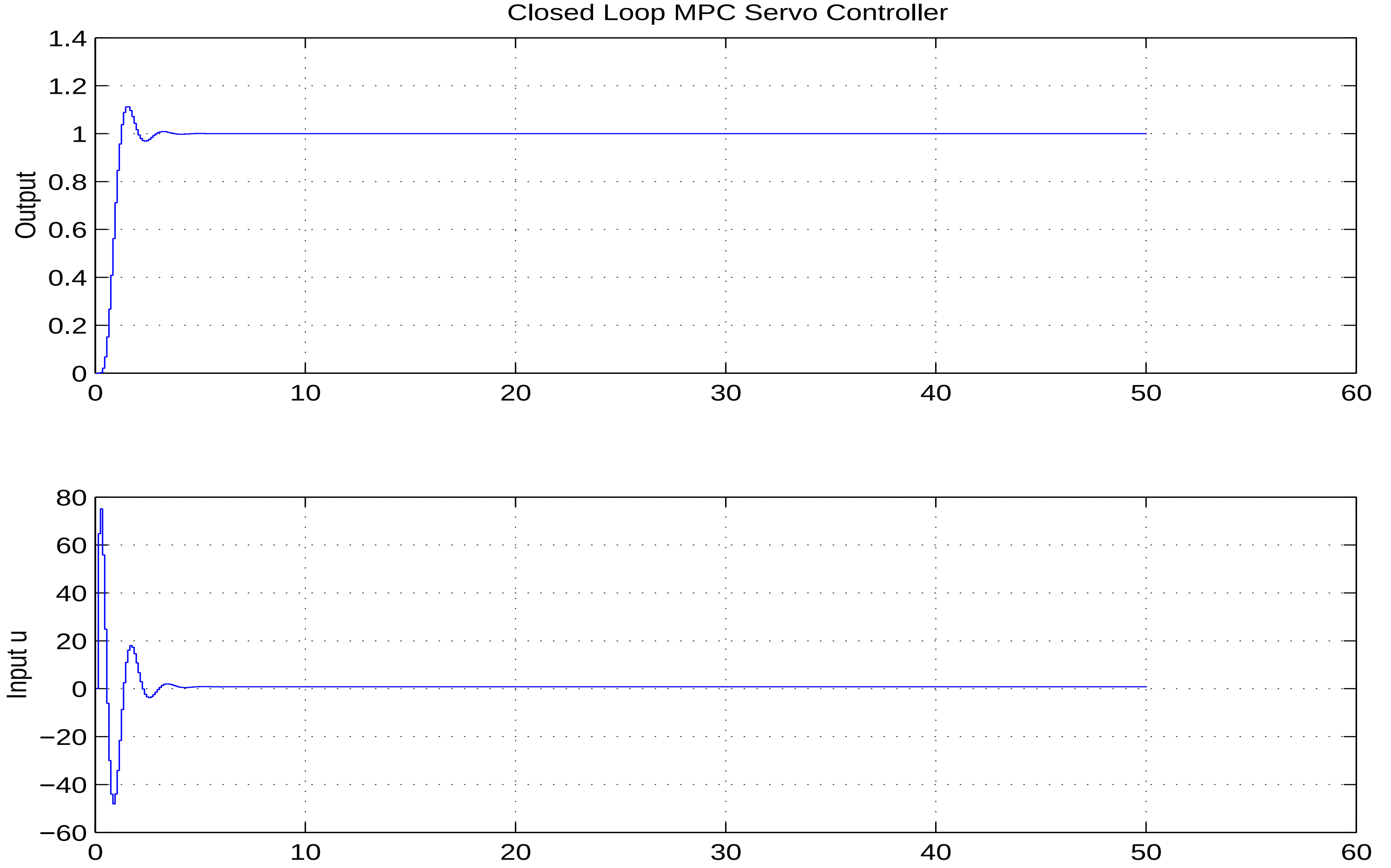
# *Extensions and future development:*

- Assume model is known - want to solve the same type of problem using optimal control techniques
- Previous study, aside from “almost” positivity, has been for positive systems constraint by nonnegative control - want to solve the same type of problem for nonpositive and bidirectional control - presently ongoing!

# Water Tanks Example with nonnegative input

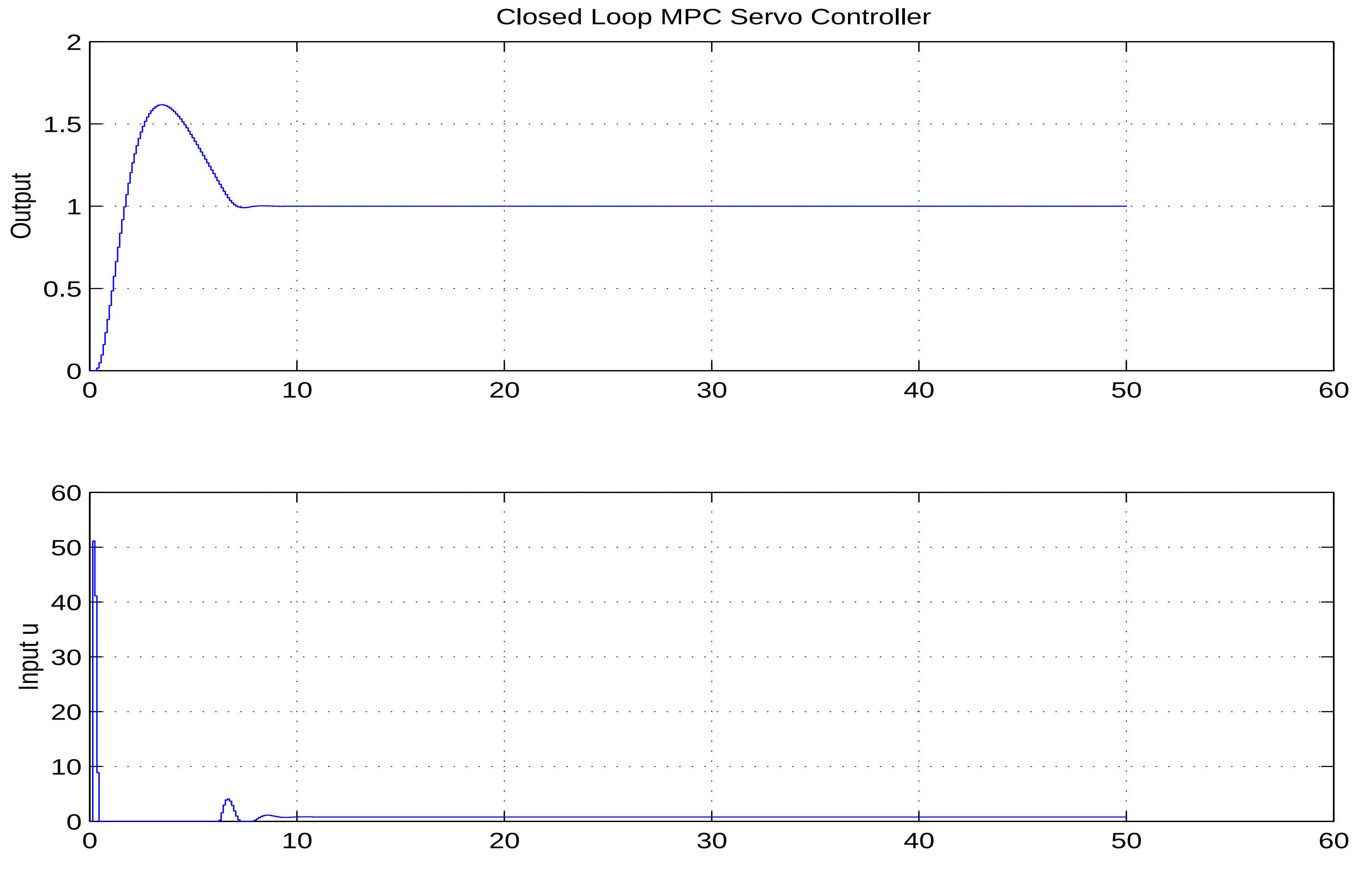


# Results for Optimal Control: Water Tanks Example (Figure 1)



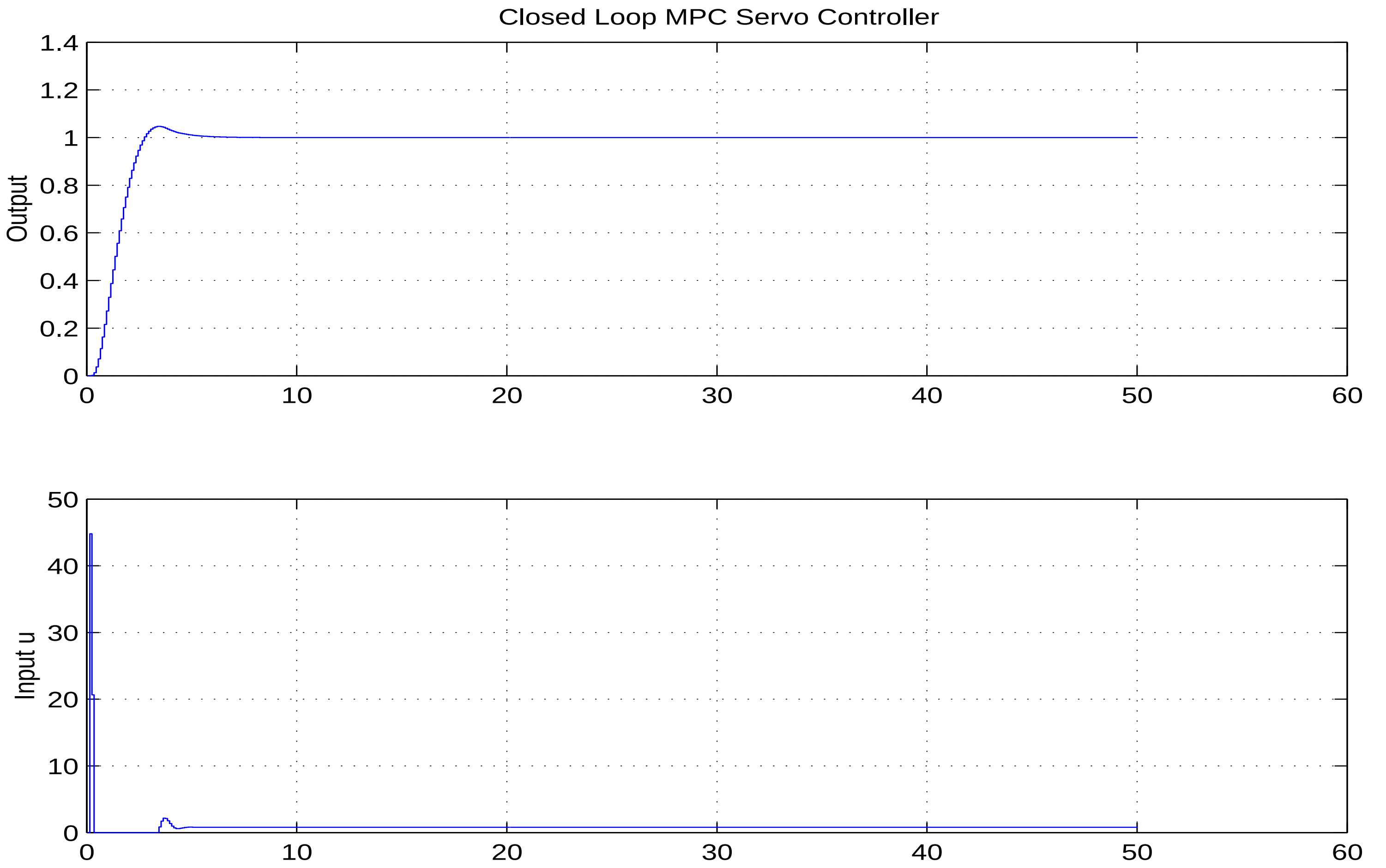

* no constraints, $y_{ref} = 1$, $\omega = 0$, violation of $u \geq 0$

# Results for Optimal Control: Water Tanks Example (Figure 2)



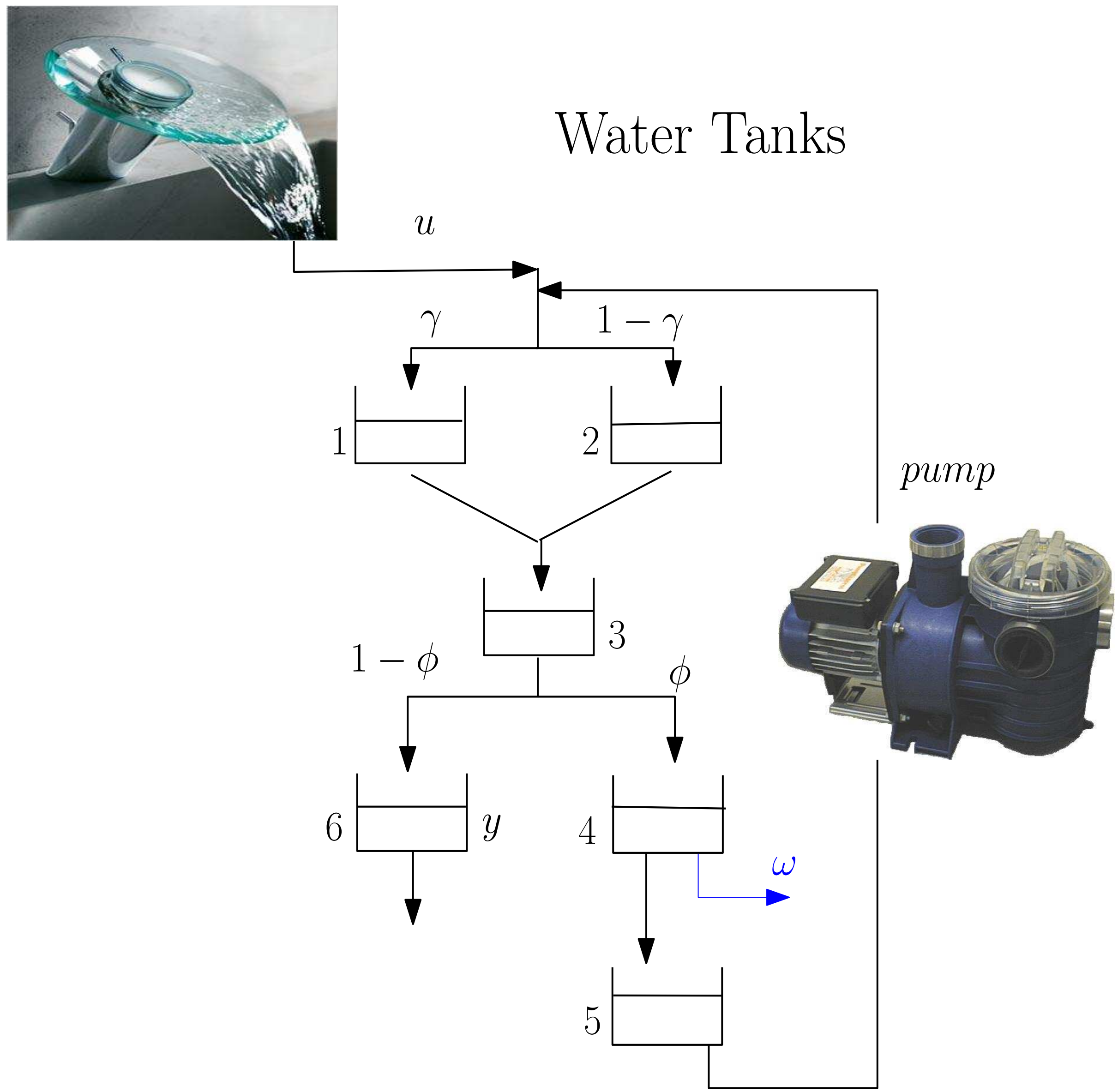

* same as Figure 1, but $u \geq 0$ is satisfied

# Results for Optimal Control: Water Tanks Example (Figure 3)

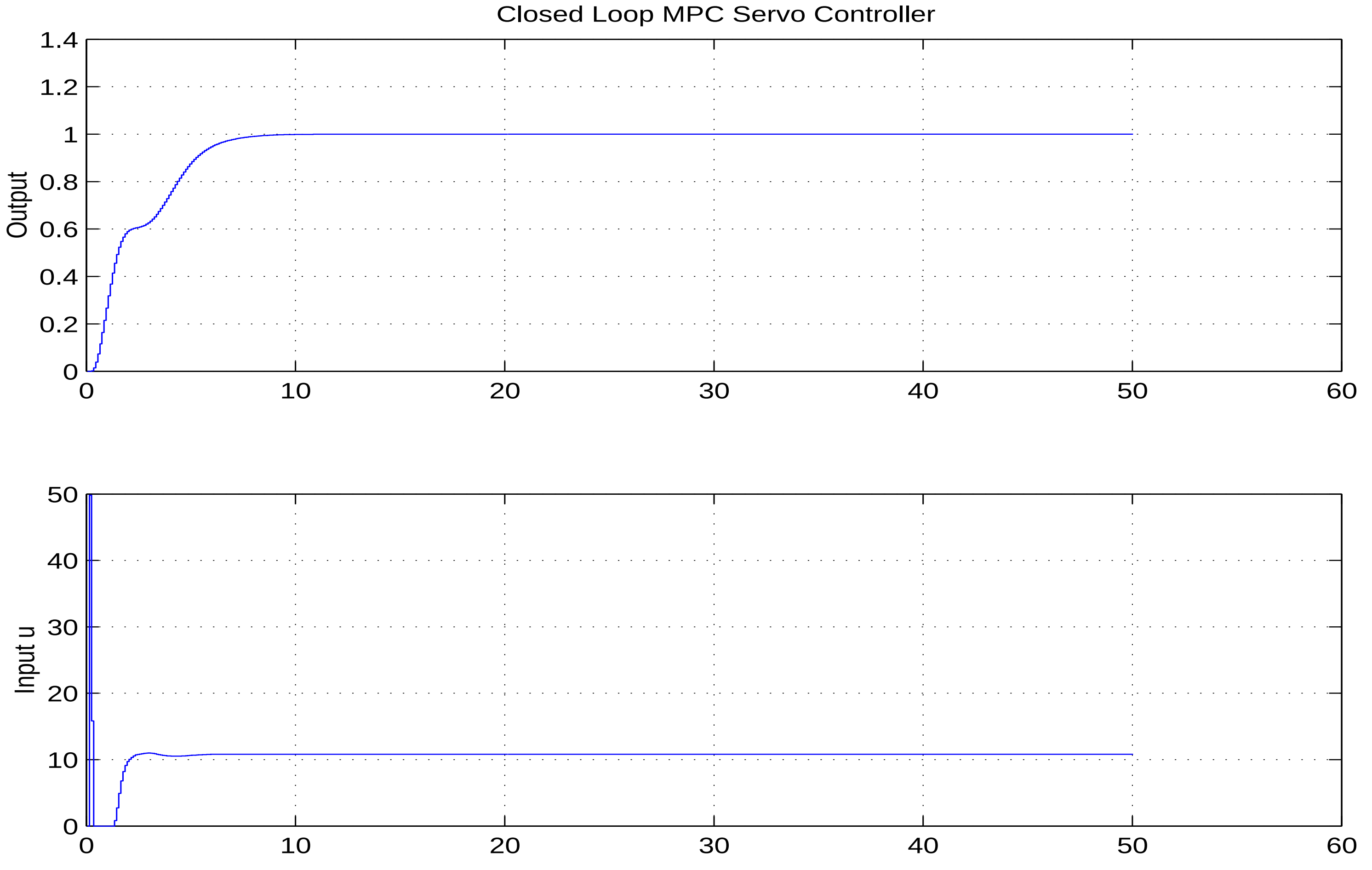



* same as Figure 2, but an improved controller

# Water Tanks Example with nonnegative input

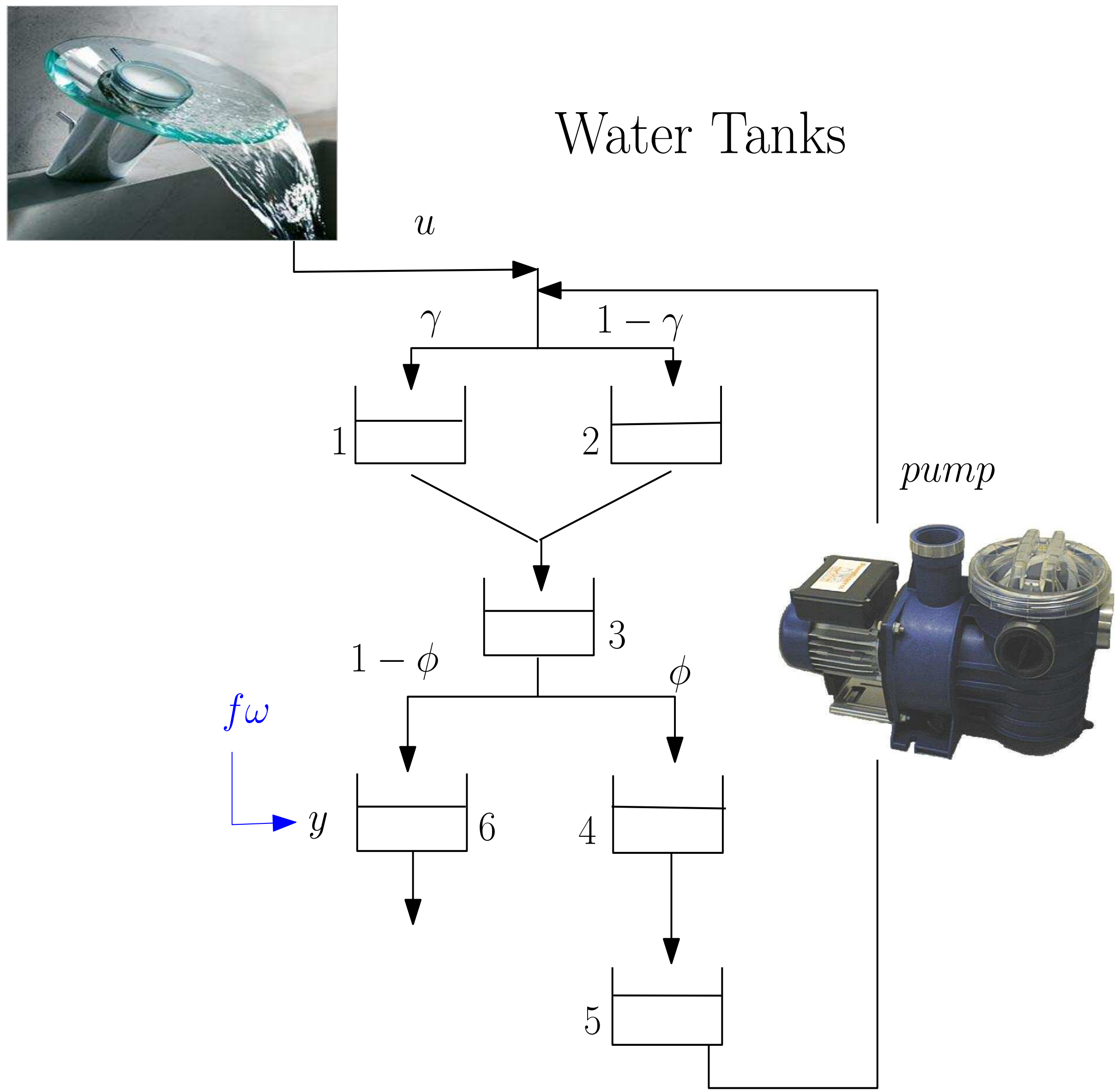


# Results for Optimal Control: Water Tanks Example (Figure 4)

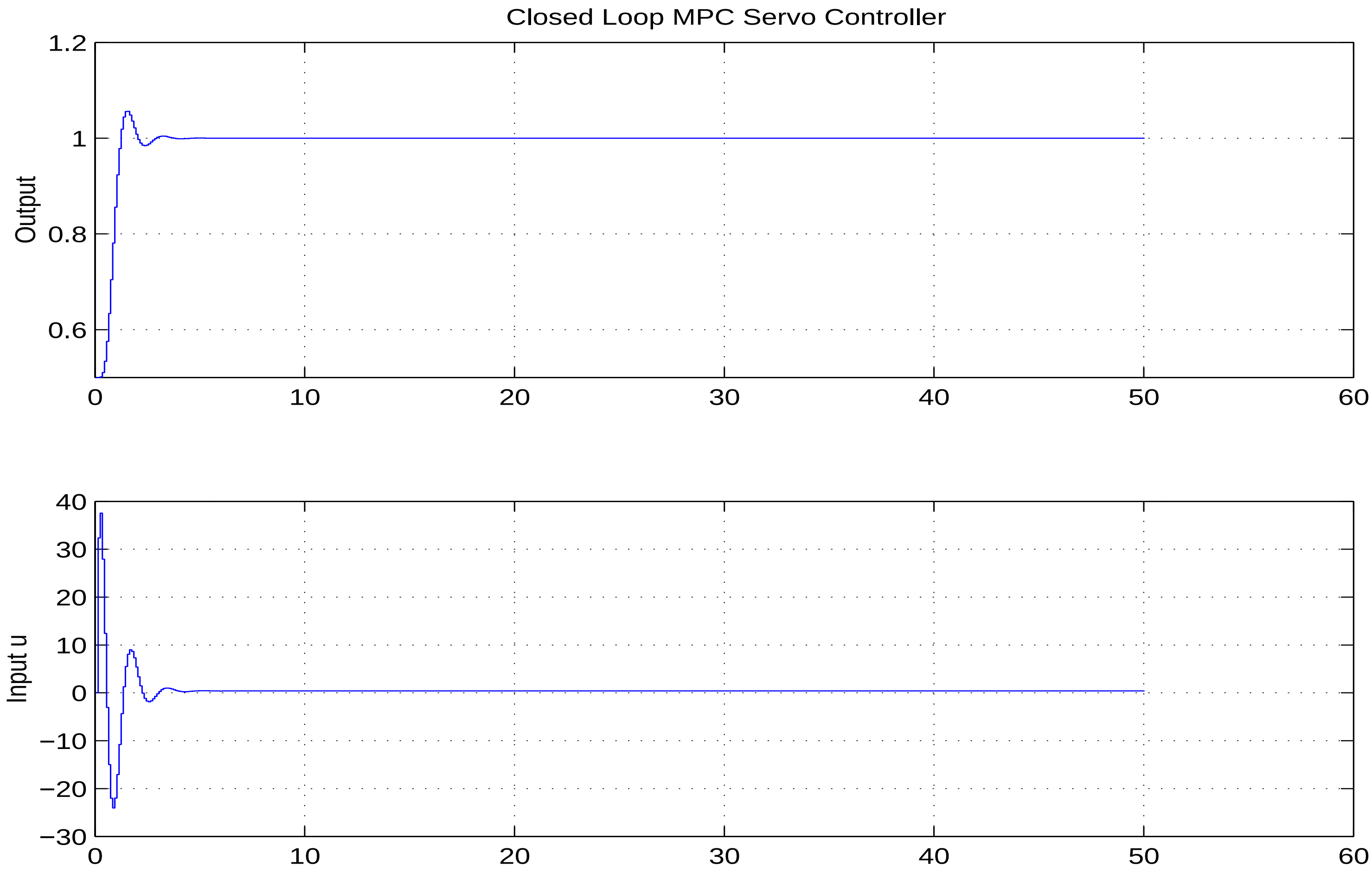



* same as Figure 3, but $e_\omega = [0\ 0\ 0\ -1\ 0\ 0]^T$

# Water Tanks Example with nonnegative input

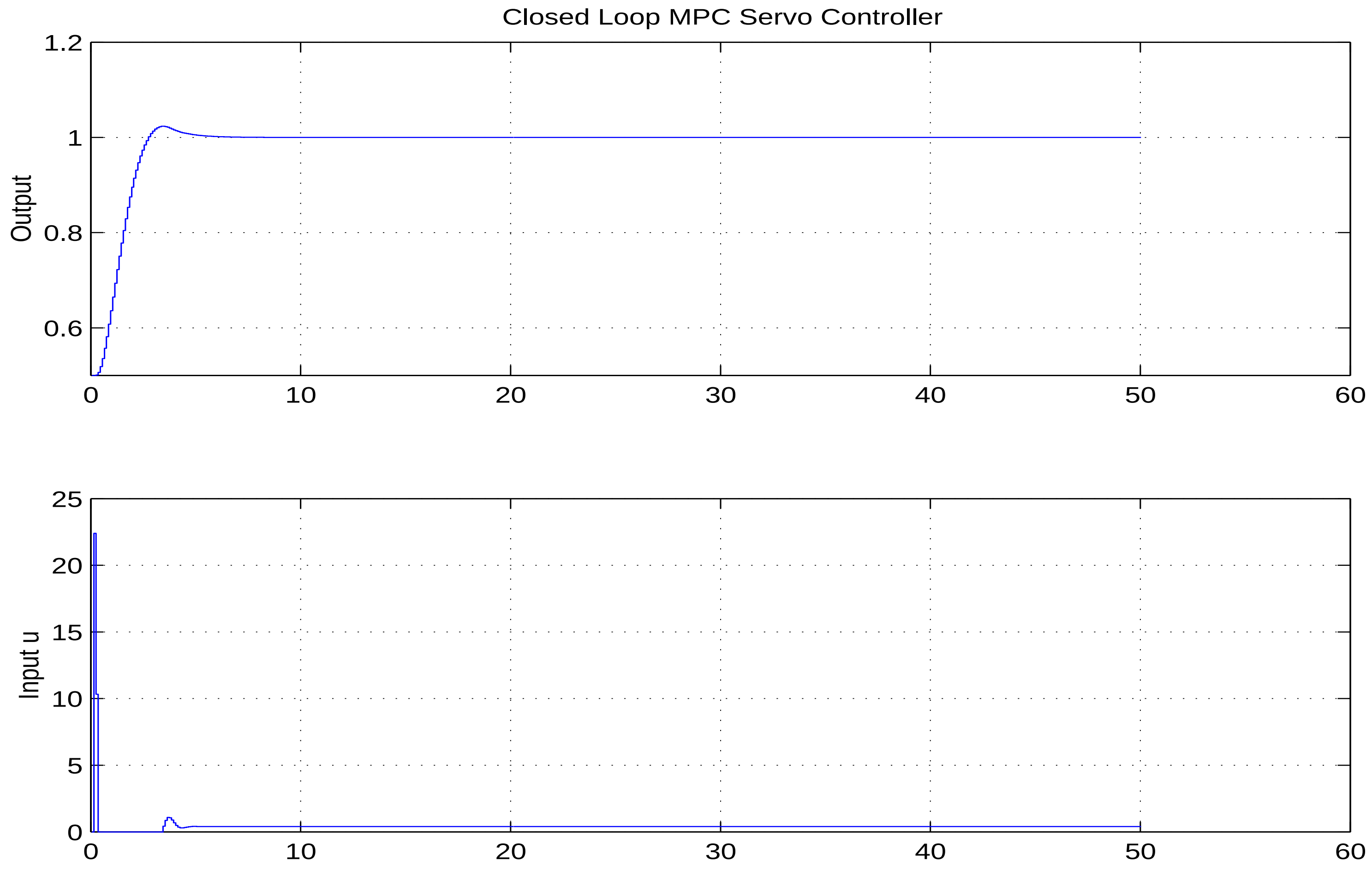


# Results for Optimal Control: Water Tanks Example (Figure 5)



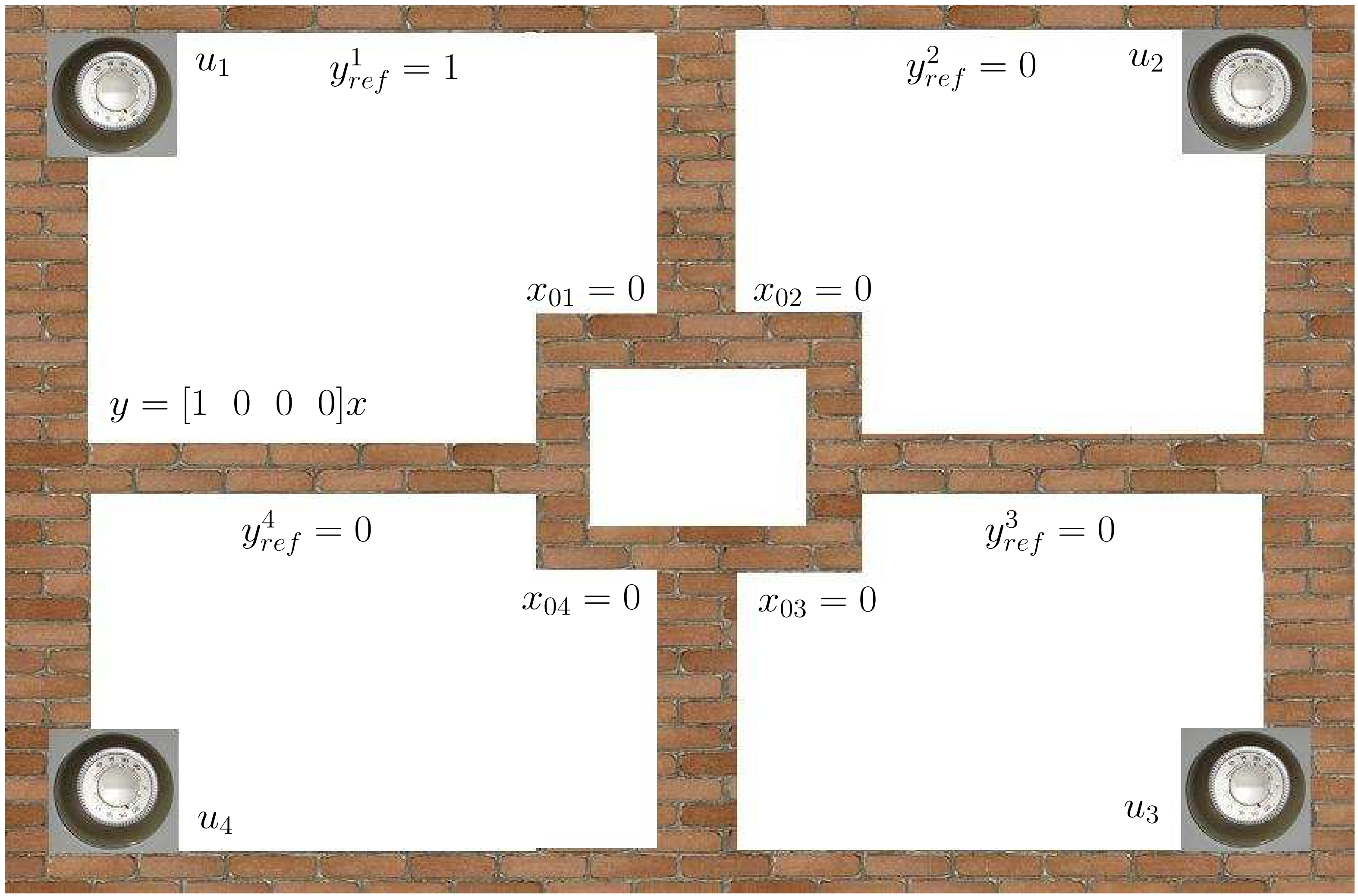

* no constraints are applied, $f\omega = 0.5,\ y_{ref} = 1,\ e_\omega = 0$
* violation occurs!

# Results for Optimal Control: Water Tanks Example (Figure 6)



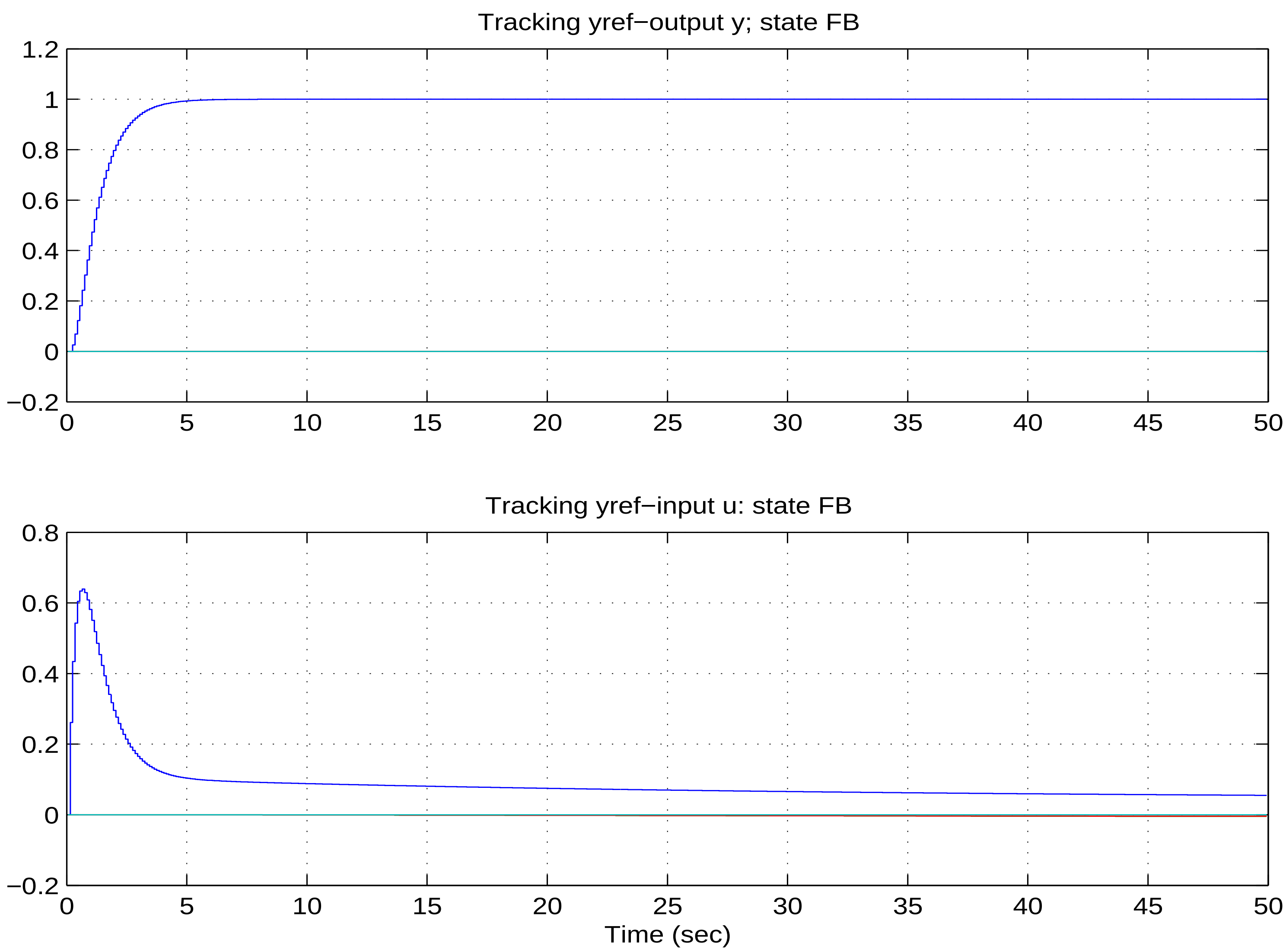

* same as Figure 5, but input constraint is satisfied

# Results for Optimal Control: Building Example




$\omega_{outside}$ (decrease of outside temperature)

# Results for Optimal Control: Building Example



# References: Bartek Roszak Edward Davison


- “The servomechanism problem for unknown MIMO LTI positive systems: feedforward and robust tuning regulators”. *American Control Conference*, June 11-13, 2008, Seattle, USA.
- “The servomechanism problem for unknown SISO positive systems using clamping”, *17th IFAC World Congress*, July 6-11, 2008, Seoul, Korea.
- “Tuning regulators for tracking SISO positive linear systems”, *Proceedings of the European Control Conference* pp.540-547, July 2007

# Conclusion:

- Motivation - why positive systems? ✓
- Introduction and background to positive LTI systems ✓
- SISO results and examples ✓
- MIMO results and examples ✓
- Extensions and future development ✓
- References ✓