Sensor classification for control and diagnosis problems: a structural approach

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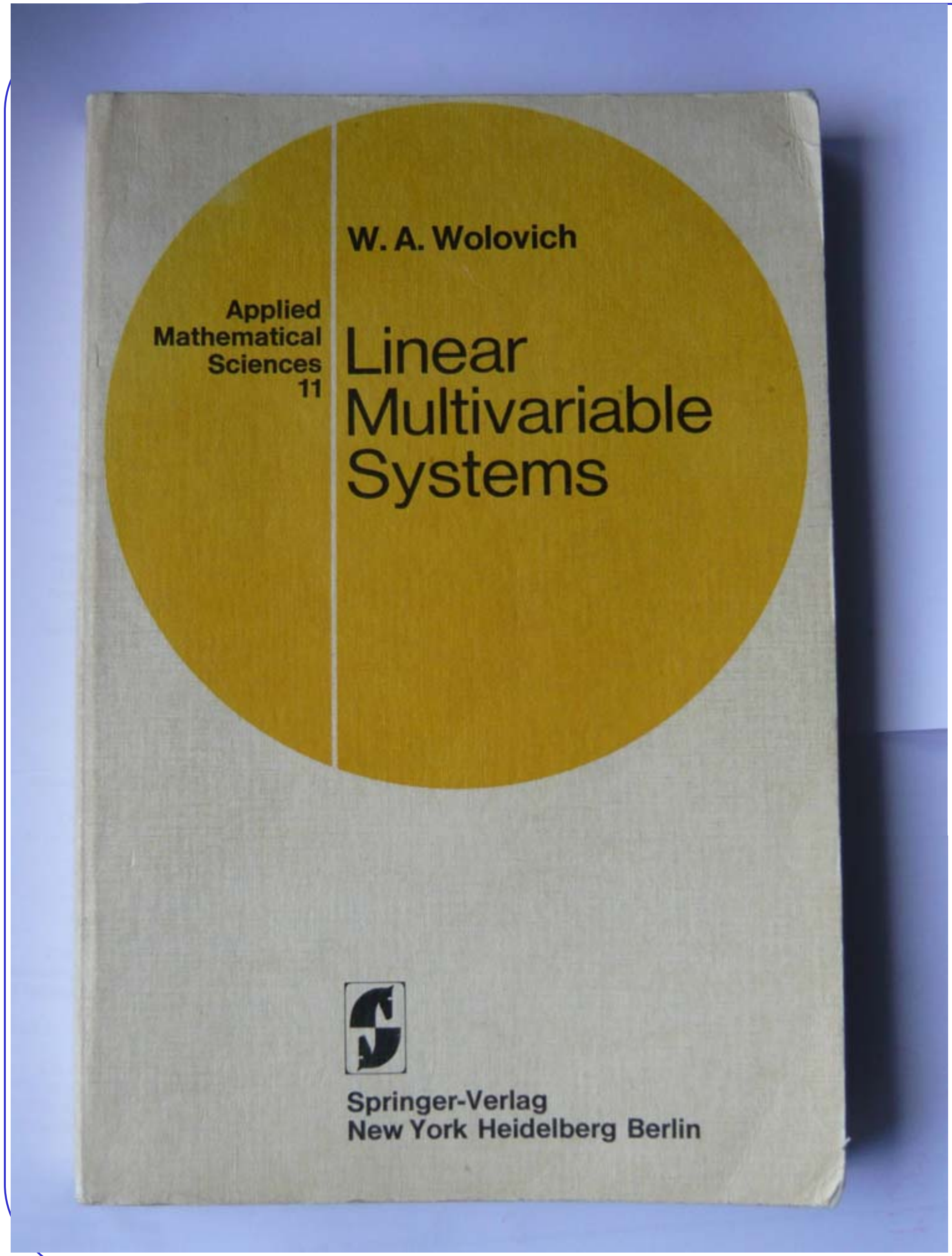


In Honor of Bill Wolovich

My old book

When I was PhD student in 1974

In Honor of Bill Wolovich

- His results on polynomial factorizations was a starting point for me to work on system theory (together with results of V. Kucera)

- His results on the prior kowledge for adaptive control (interactor) was also a starting point for me to work on stability analysis of adaptive systems (together with results of S. Morse)

Introduction

Aim of this talk

- Sensor network design problem for observability and diagnosis
- Structural modeling of dynamical systems
- Sensor classification (useless, essential)
- Quantification of the criticity of useful sensors
- Graph approach with low complexity
- Illustration on an example

References

- FDI: Frank 96, Chen and Patton 99,
- Structural models: Lin 74, Murota 87, van der Woude 00, Dion Commault van der Woude 03
- Observability or FDI in this context: Lin 74, Boukhobza 06, Staroswieki 06, Commault Dion and Trinh 08

Problem formulation

Dynamical system Σ
$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = h(x(t)) \end{cases}$$

$$u \in R^m, x \in R^n, y \in R^p$$

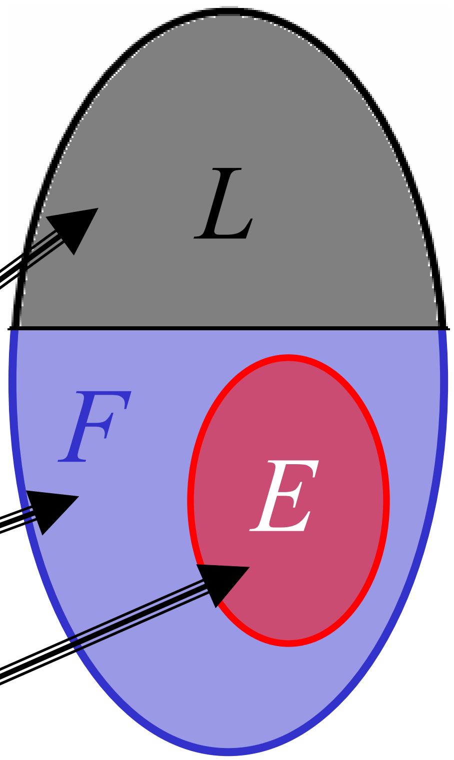
Consider a property P which is TRUE with Σ
(for example observability)

Problem:

Is P preserved under sensor failure?

Sensor classification

- Property *P*
  - Admissible Sensor Set: Subset of sensors V such that P is true for V
- Sensor classification:
  - y* is useless if for any admissible sensor set containing y*, V \ {y*} is an admissible sensor set
  - Useful sensors: not useless
  - y* is essential if y* belong to any admissible sensor set



Sensor classification

- In this talk, *P* will be:
  - Solvability of the Fault Detection and Isolation (FDI) Problem

Structured systems

Linear systems in which the entries of the matrices in a state space representation are:

- *zeros*
- *independent parameters*

Generic properties:

valid for almost any value of the parameters

Associated graph

$$\Sigma : \begin{cases} \dot{x}(t) = A_\lambda x(t) + B_\lambda u(t) \\ y(t) = C_\lambda x(t) + D_\lambda u(t) \end{cases}$$

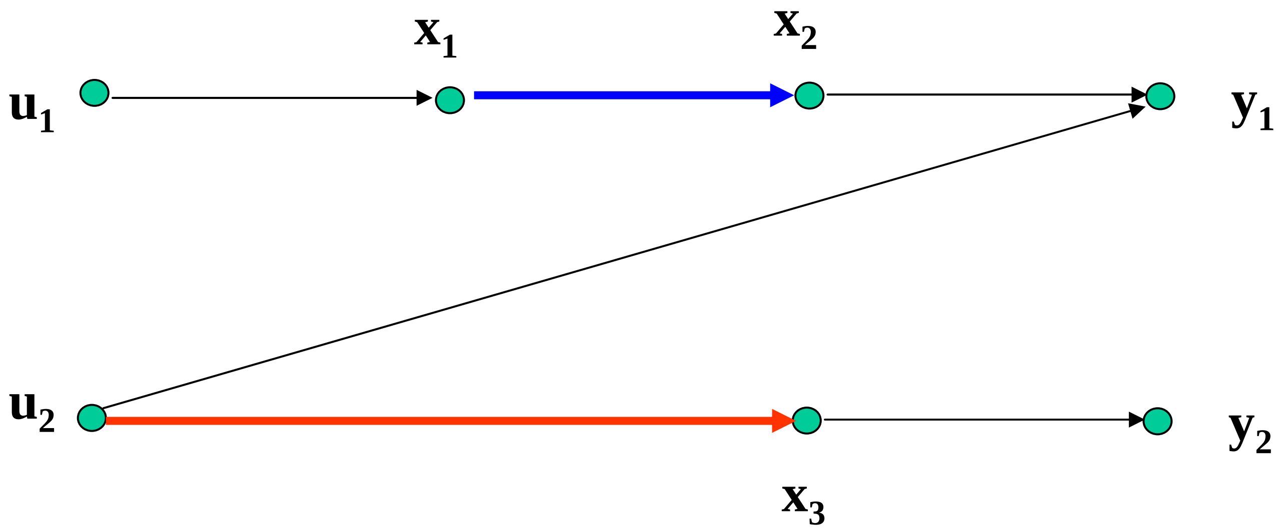
Associated graph:

- vertex set : input, state, output vertices
- edge set : corresponds to non zero entries in matrices (as many edges as parameters λ_i)

Associated graph

Example

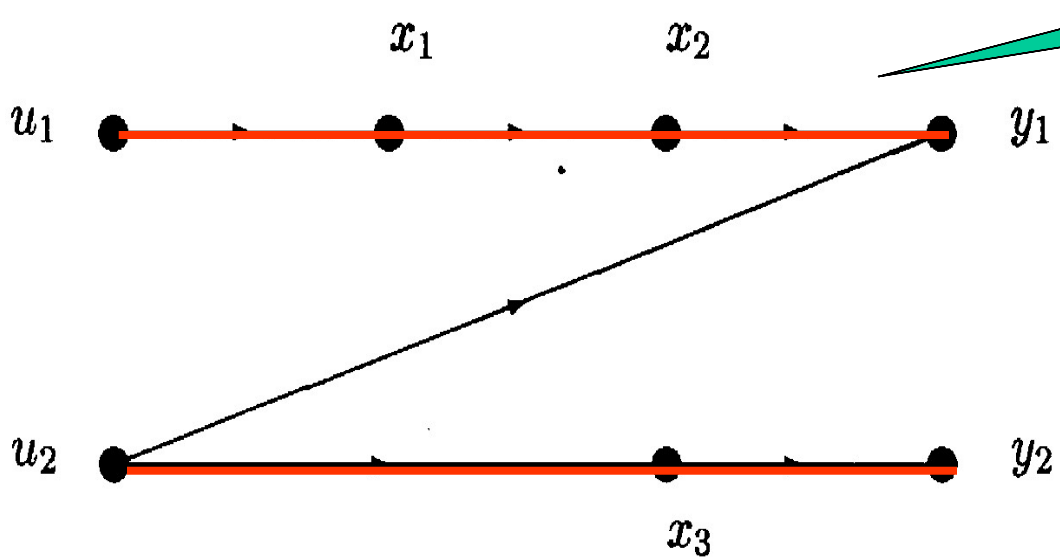
$$A_\lambda = \begin{pmatrix} 0 & 0 & 0 \\ \lambda_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B_\lambda = \begin{pmatrix} \lambda_2 & 0 \\ 0 & 0 \\ 0 & \lambda_3 \end{pmatrix}, \quad C_\lambda = \begin{pmatrix} 0 & \lambda_4 & 0 \\ 0 & 0 & \lambda_5 \end{pmatrix}, \quad D_\lambda = \begin{pmatrix} 0 & \lambda_6 \\ 0 & 0 \end{pmatrix}.$$



Generic rank of a transfer matrix

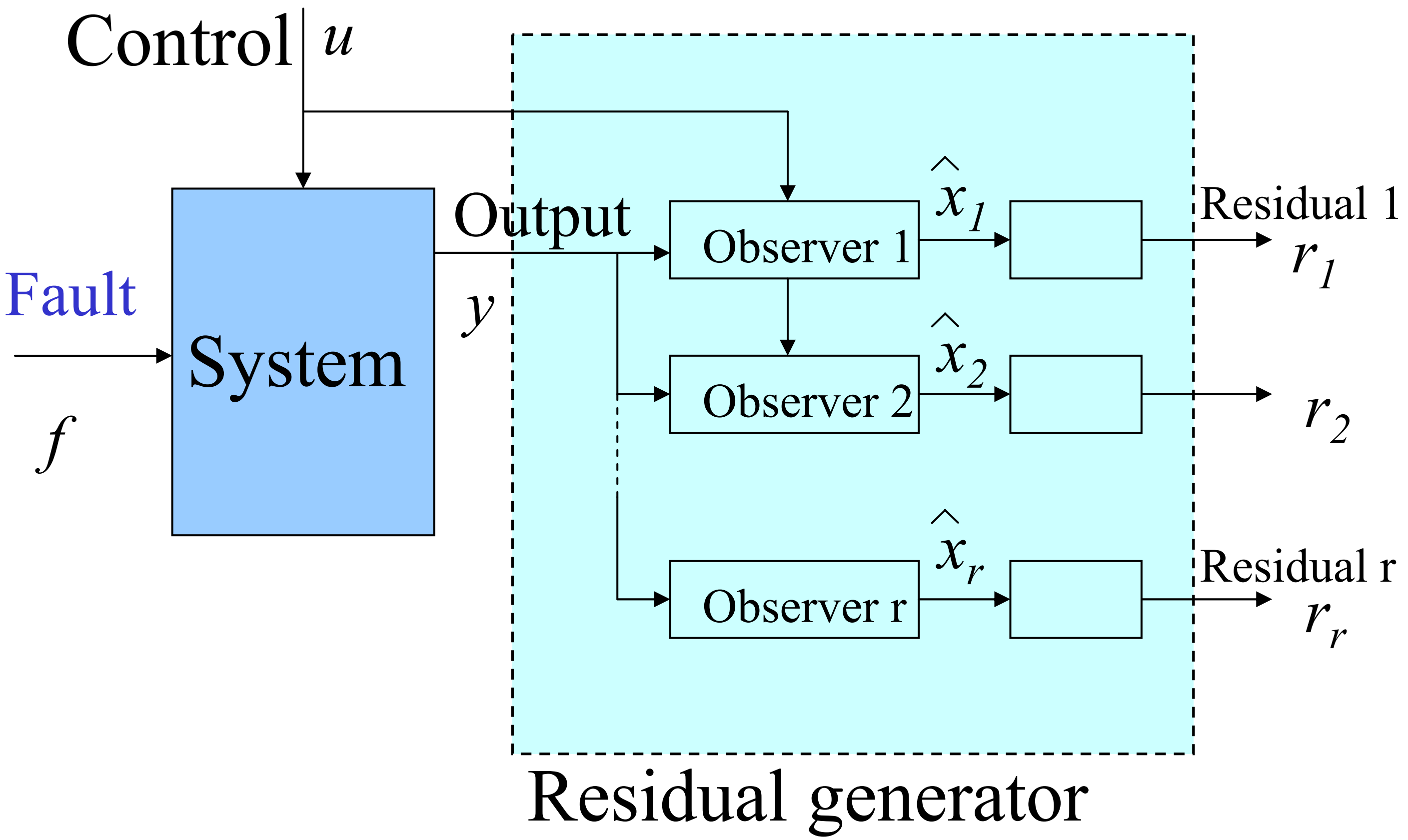
- Generic rank of *T(s)* = Maximal number of vertex disjoint input-output paths (*van der Woude 90)*
- (Maximal linking)

Generic rank = 2



$$T_\lambda(s) = \begin{pmatrix} \dfrac{\lambda_1\lambda_2\lambda_4}{s^2} & \lambda_6 \\ 0 & \dfrac{\lambda_3\lambda_5}{s} \end{pmatrix}$$

Observer-based FDI problem



Observer-based FDI problem

Linear System

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Lf(t) + Bu(t) \\ y(t) = Cx(t) + Mf(t) \end{cases}$$

Observer i

$$\dot{\hat{x}}^i(t) = A\hat{x}^i(t) + K^i(y(t) - C\hat{x}^i(t)) + Bu(t)$$

Residual i

$$r_i(t) = Q^i(y(t) - C\hat{x}^i(t))$$

FDI Problem

With the available sensors, make the system **observable** and

$$r(s)=\begin{bmatrix} t_{11}(s) & 0 & \cdots & 0 \\ 0 & t_{22}(s) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & t_{rr}(s) \end{bmatrix}\left[f(s)\right]$$

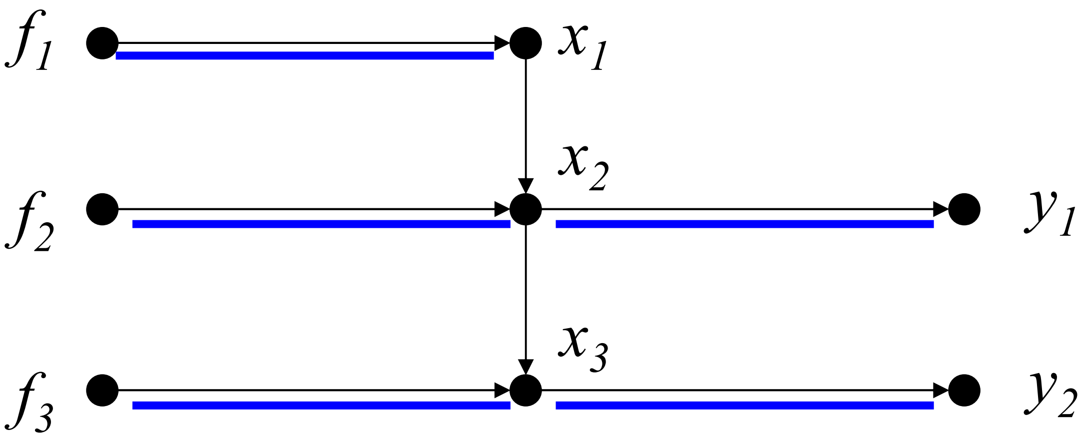
FDI problem structurally solvable iff (*Commault et al 02*):

1. The system is **structurally observable**
2. ***k = r (rank condition)***

r : number of faults

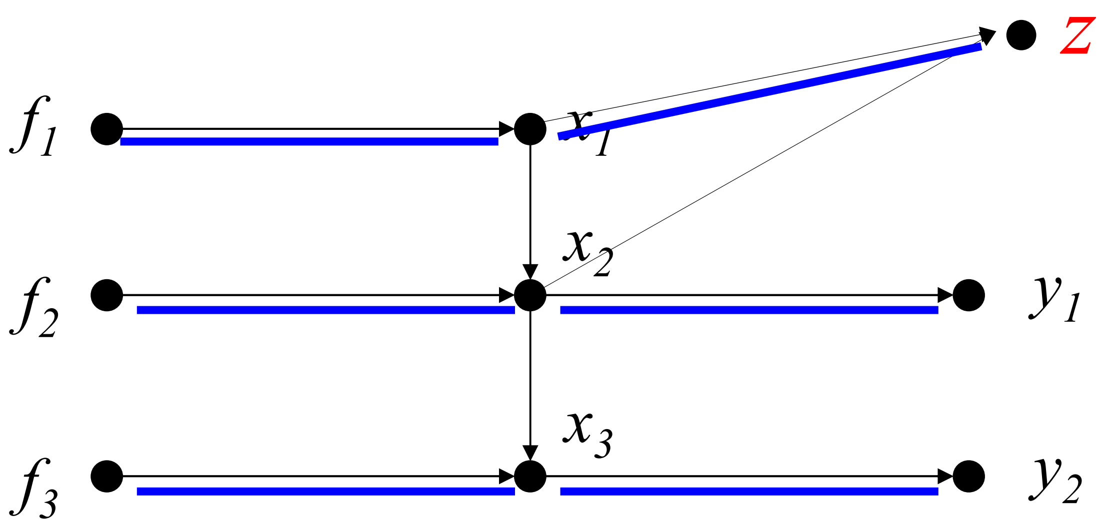
k : maximal number of vertex disjoint fault-ouput paths

Example



Observable but k=2, r=3
Problem not solvable

Example



Additional sensor

Observable and k=3, r=3
problem solvable

The observability conditions

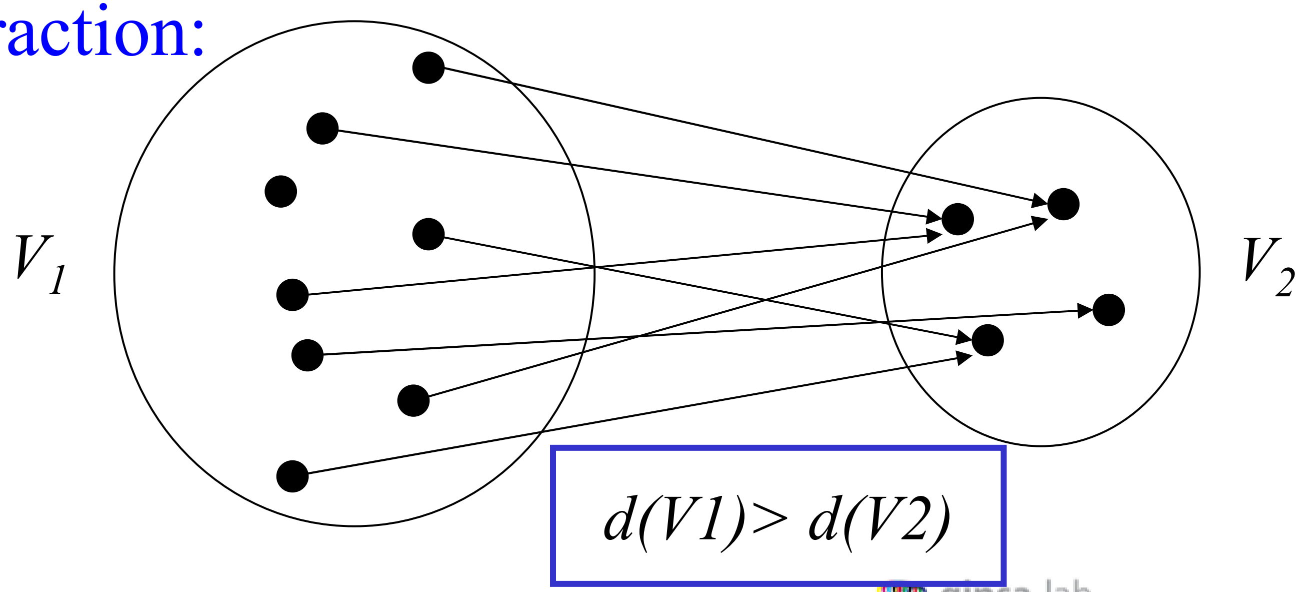
Theorem (*Lin, 74*)

The system is generically observable if and only if:

-There is a path from any state vertex to the outputs **(output connection)**

-There is **no contraction** in the graph

Contraction:



Output connection analysis

Irreducible separators:

Minimal sets of output vertices which when removed, disconnect some states from outputs

Sensor classification for the output connection condition

Theorem:

Useless sensors:

y_i is useless ⇔ does not belong to an irreducible separator

Essential sensors:

y_i is essential ⇔ belongs to an irreducible separator of dimension one

Example

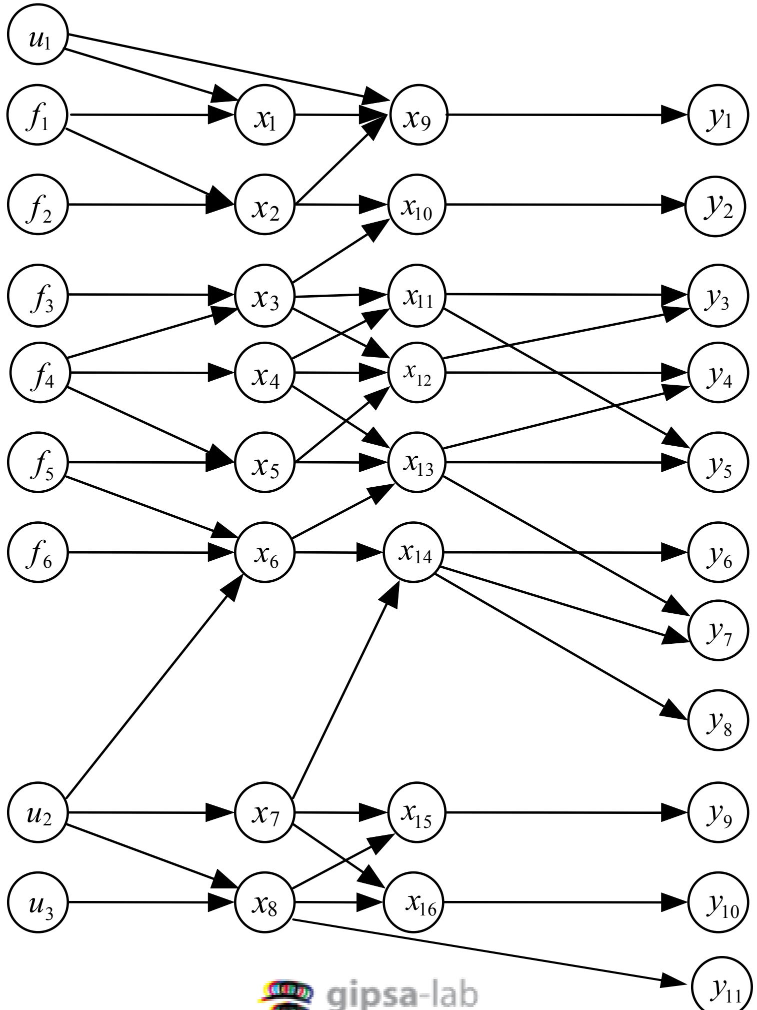
System with:

3 control inputs

6 faults

10 sensors

14 states



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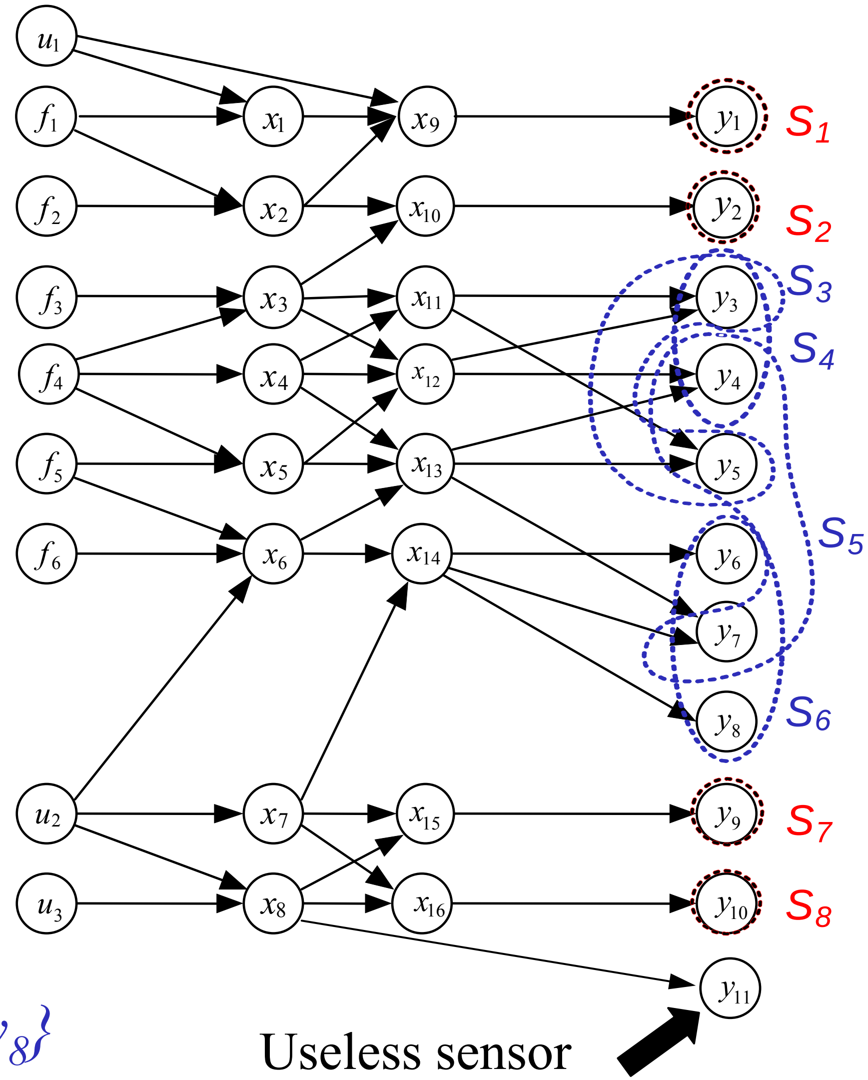
For output connection

Associated graph:



Irreducible separators:

$S_1=\{y_1\}$
$S_2=\{y_2\}$
$S_7=\{y_9\}$
$S_8=\{y_{10}\}$

Essential sensors $\{y_1,y_2,y_9,y_{10}\}$

$S_3=\{y_3,y_5\}$
$S_4=\{y_3,y_4\}$
$S_5=\{y_4,y_5,y_7\}$
$S_6=\{y_6,y_7,y_8\}$

Useful sensors $\{y_1,y_2,y_9,y_{10},$ $y_3,y_4,y_5,y_6,y_7,y_8\}$

Contraction analysis

On the DM decomposition of the bipartite graph associated with the system

We look for matchings of maximum cardinality

No contraction when the maximal matching covers all the state vertices

Sensor classification for the contraction avoidance for FDI

Theorem:

Useless sensor:

y_i is useless ⇔ y_i of no use to build a maximal matching in the bipartite graph

Essential sensor:

y_i is essential ⇔ belongs to the B_i components of the DM decomposition of the bipartite graph

Example

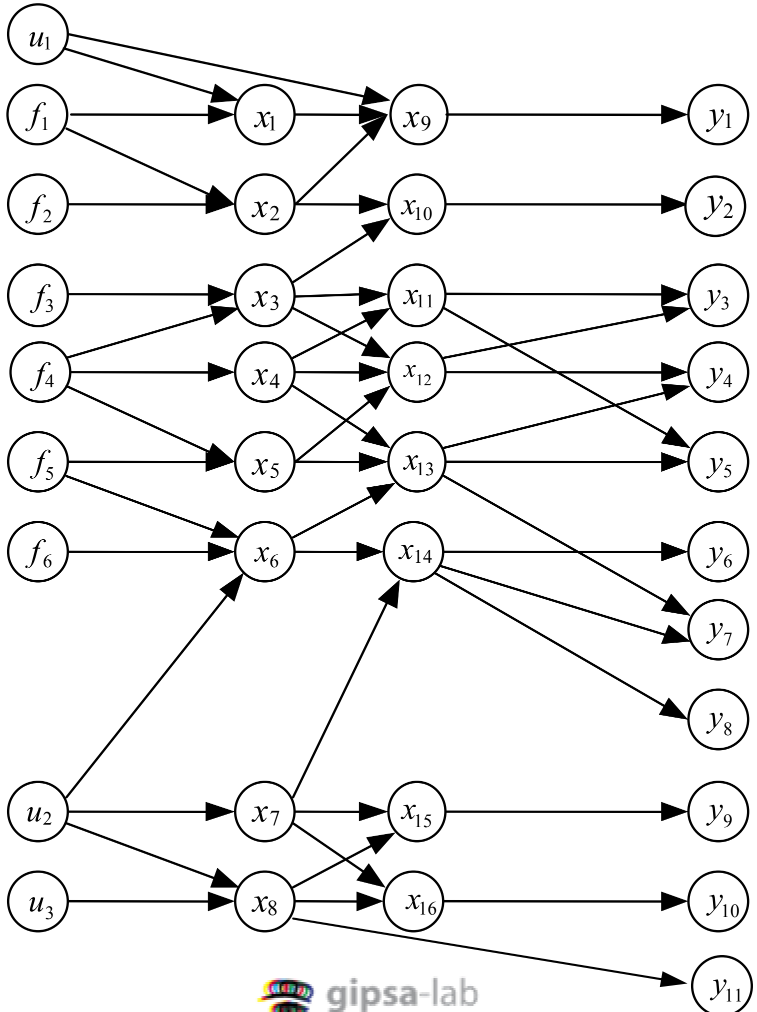
System with:

3 control inputs

6 faults

10 sensors

14 states



For contraction avoidance

Essential sensors

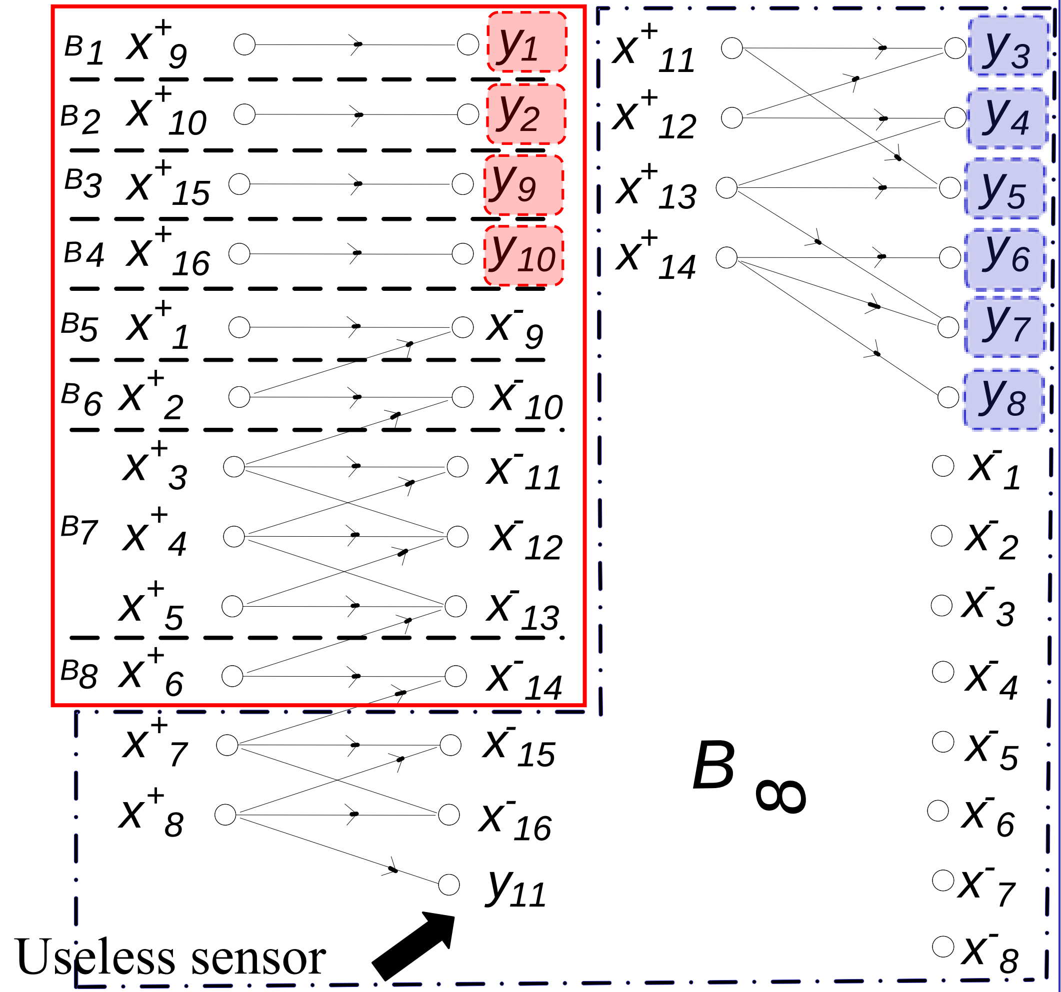
$\{y_1, y_2, y_9, y_{10}\}$

Useful sensors

$\{y_1, y_2, y_9, y_{10}, y_3, y_4, y_5, y_6, y_7, y_8\}$

Useless sensor

$\{y_{11}\}$



The FDI rank condition

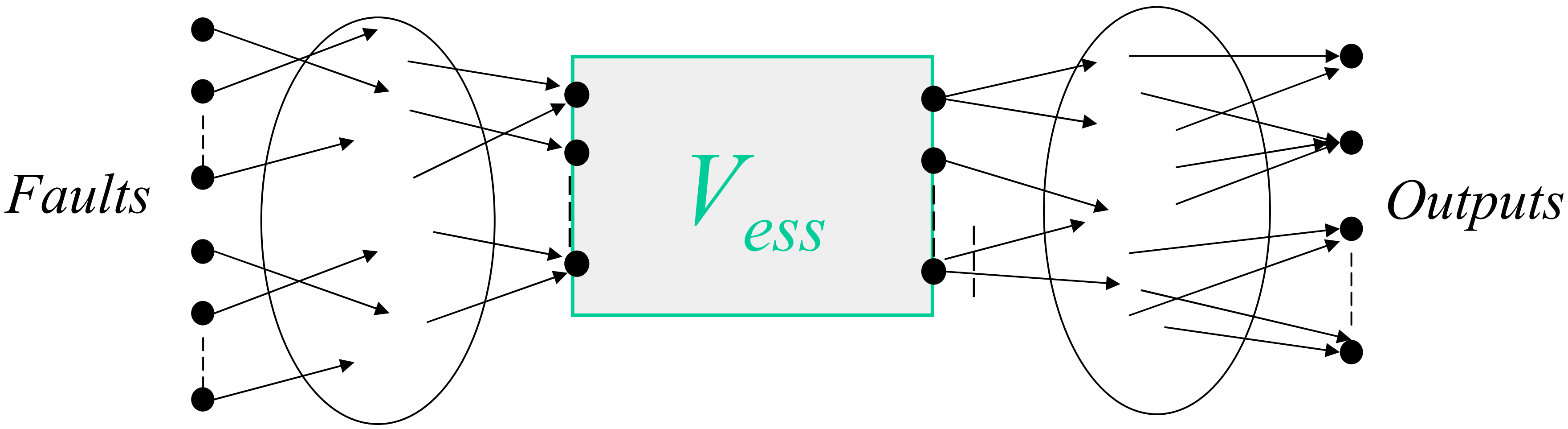
Linear observable system Σ with r faults and sensor set Y :

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Lf(t) + Bu(t) \\ y(t) = Cx(t) + Mf(t) \end{cases}$$

Subset $V \subseteq Y$ admissible sensor set for the rank condition of the FDI problem
⇔ There exists an F-V linking of size r in $G(\Sigma_\Lambda)$ ***(rank condition for FDI)***

Essential vertices

Belong to any maximal size fault-output linking (F-Y linking)



Sensor classification for the rank condition for FDI

Theorem:

Useless sensor:

y_i is useless $\Leftrightarrow$ There is no F-y_i path

Essential sensor:

y_i is essential $\Leftrightarrow$ y_i belongs to V_{ess}

Example

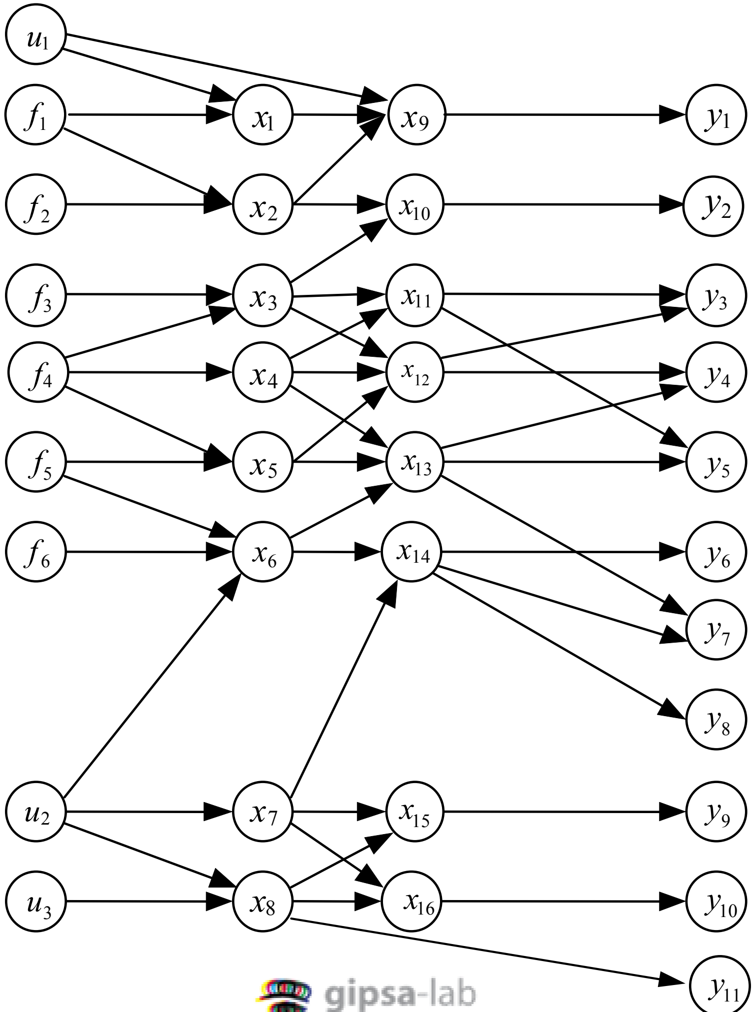
System with:

3 control inputs

6 faults

10 sensors

14 states



FDI rank condition

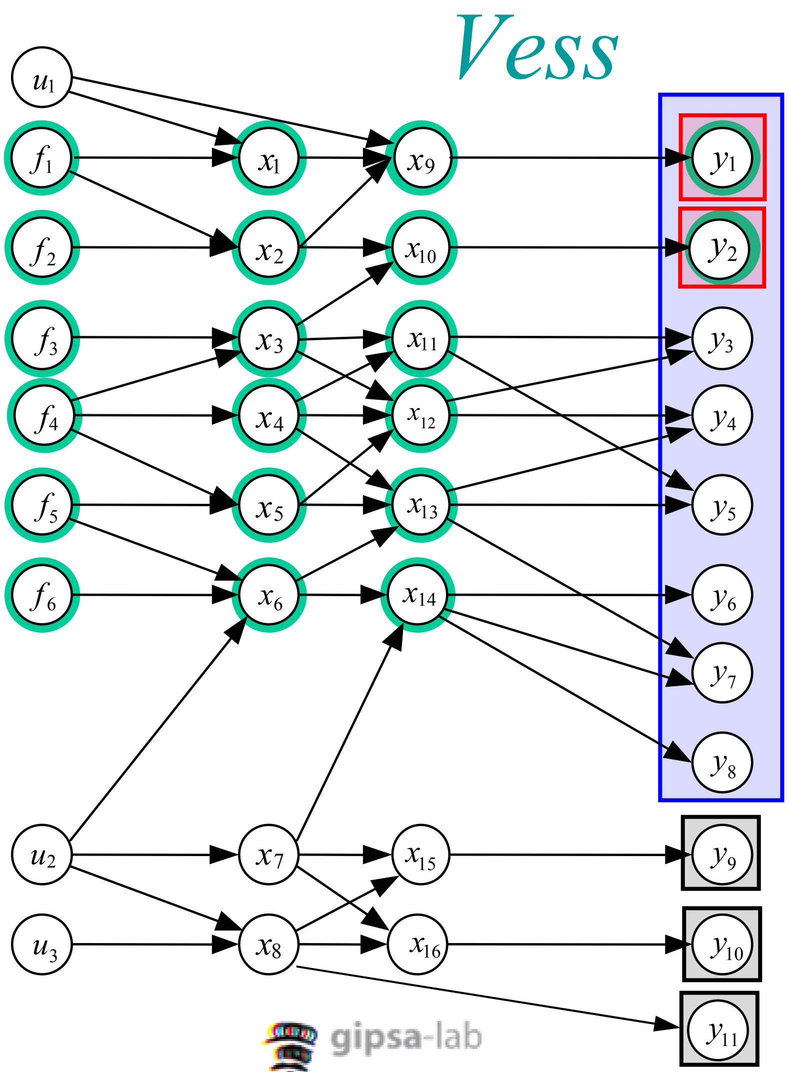
System with:

3 control inputs
6 faults
10 sensors
14 states

Essential sensors: y_1, y_2

Useless sensors: y_9, y_{10}, y_{11}

Useful sensors: $y_1, \dots\ y_8$



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Complexity

For the three problems one gets polynomial time bounded algorithms either for the determination of the set of essential sensors or for the set of useless sensors.

By standard max flow algorithms and by labeling procedures

➔ The classification of sensors is polynomial

Index of criticity for the sensors

K= cardinality of the set of admissible sensor sets containing no useless sensors

K_i*= cardinality of the set of admissible sensor sets of K containing* y_i

Criticity degree of y_i

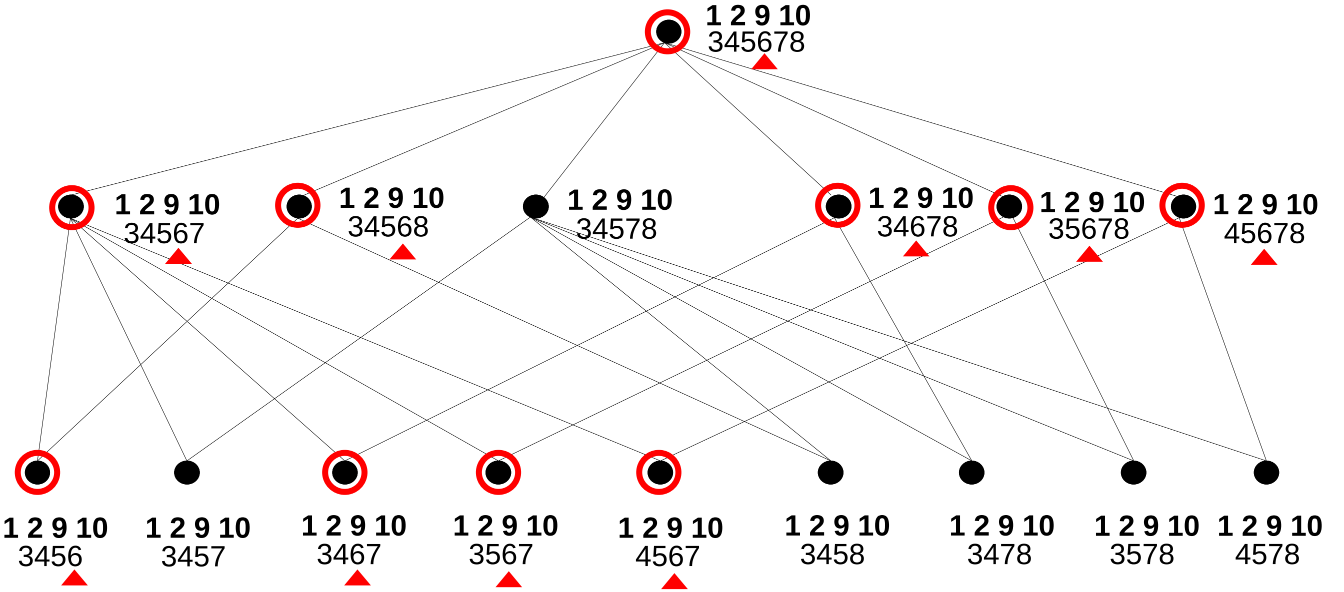
$W(y_i)=K_i/K$

$W(y_i)=1$ *for essential sensors*

$W(y_i)=0$ *for useless sensors*

Index of criticity for the sensors

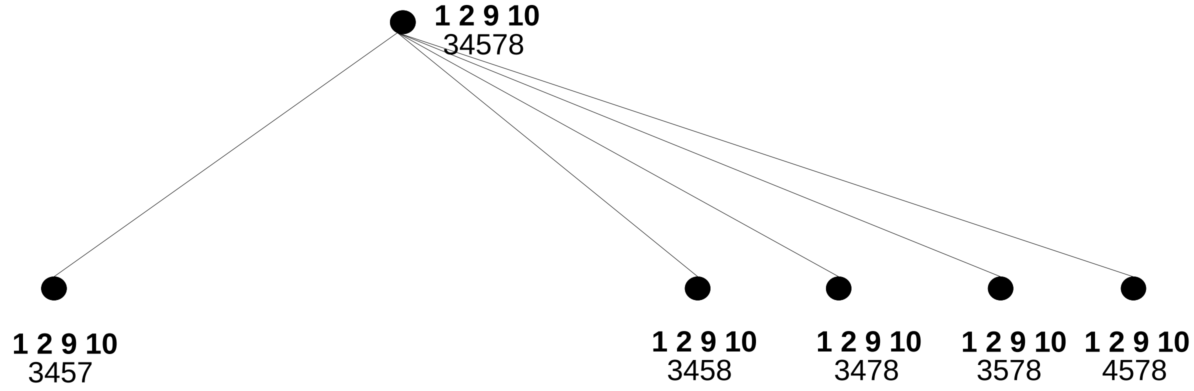
$D(y_6) = 10/16$



Index of criticity for the sensors

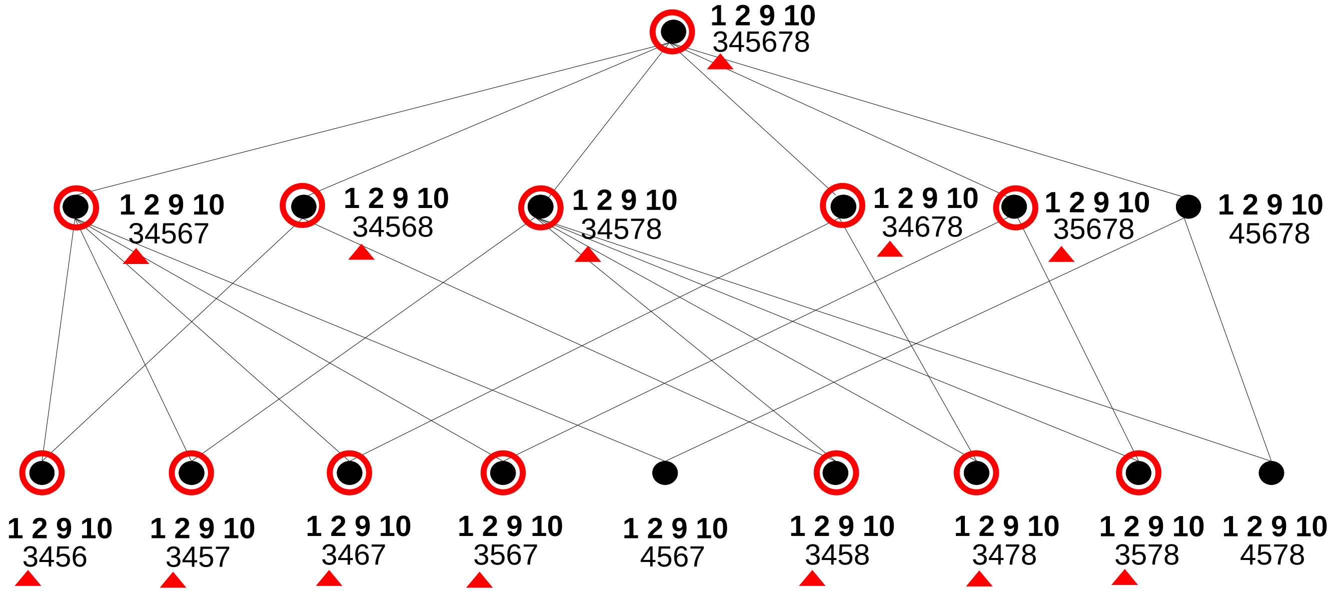
When y_6 is lost:

$1 - D(y_6) = 6/16$



Index of criticity for the sensors

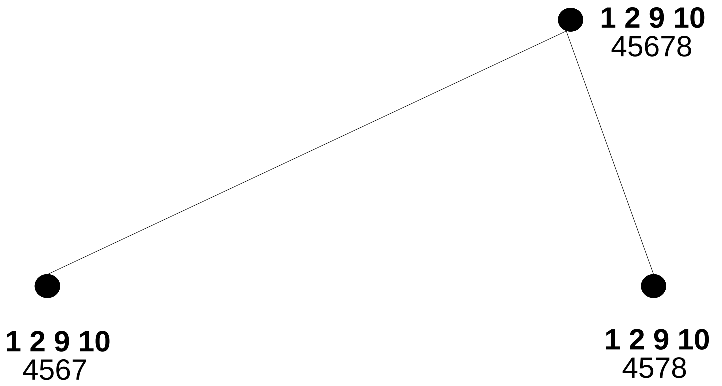
$D(y_3) = 13/16$



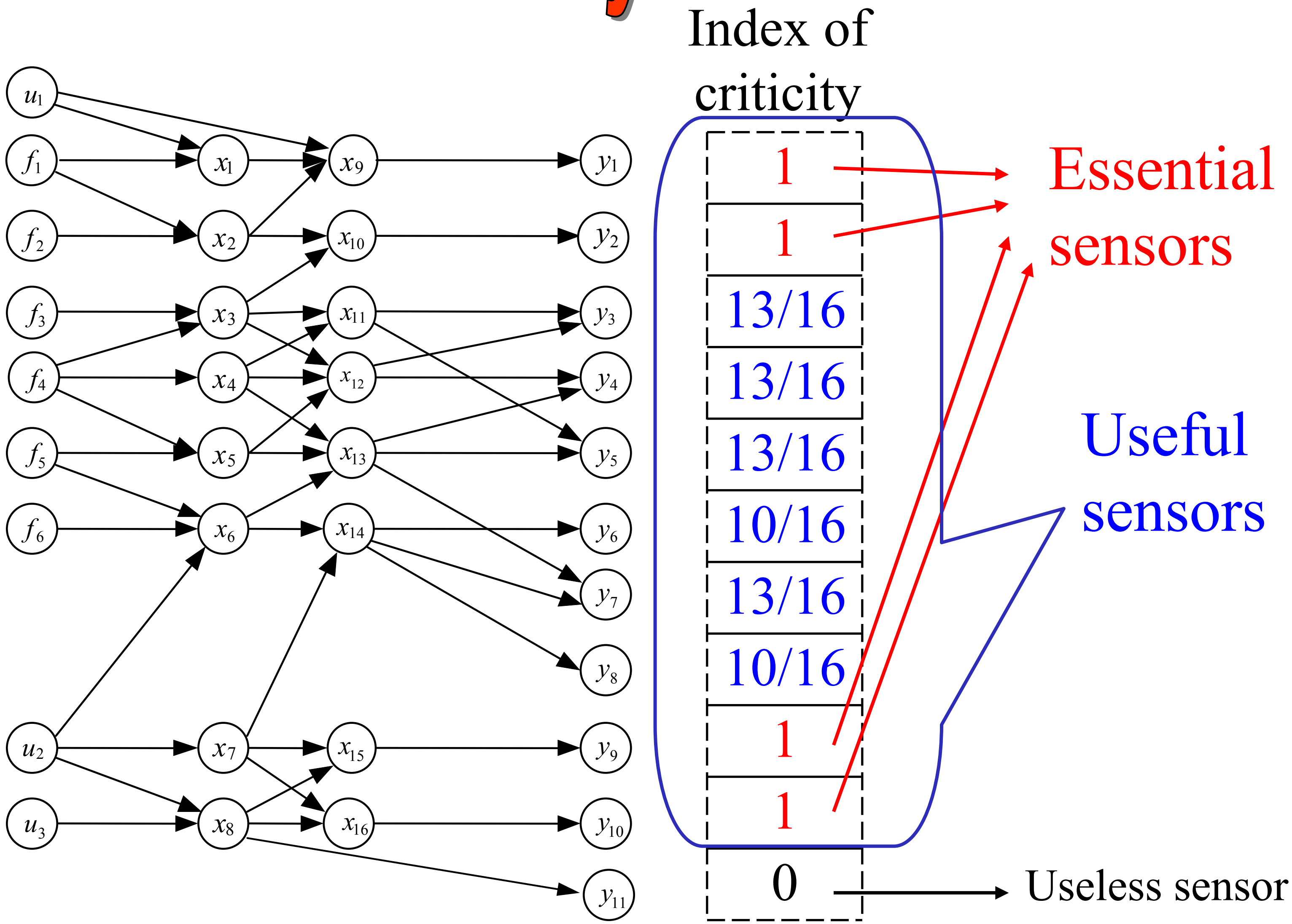
Index of criticity for the sensors

When y_3 is lost:

$1 - D(y_3) = 3/16$



Index of criticity for the sensors



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Conclusion

- Structural modeling of dynamical systems
- Property preservation under sensor failure for structured systems.
- Sensor classification with respect to their critical nature concerning FDI.
- Sensor classification using polynomial time algorithms.
- Quantitative measure of criticity of sensors
- This sensor classification can be extended to other problems (disturbance decoupling, …).

Index of criticity for the sensors

$W(y_1) = 1$ $W(y_2) = 1$
$W(y_9) = 1$ $W(y_{10}) = 1$ Essential

$W(y_3) = 13/16$ $W(y_4) = 13/16$
$W(y_5) = 13/16$ $W(y_7) = 13/16$

$W(y_6) = 10/16$ $W(y_8) = 10/16$

$W(y_{11}) = 0$ Useless

For output connection

$Y_{cn}=\{y_1,y_2,y_9,y_{10}\}$

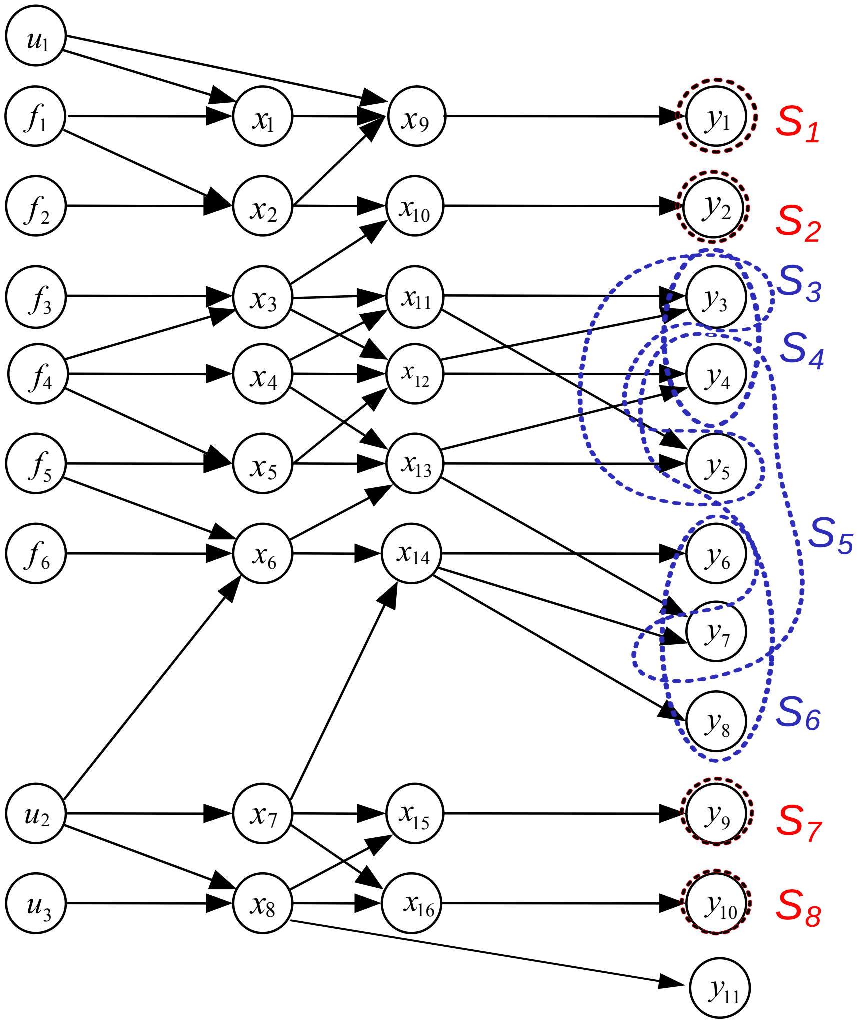
8 minimal sensor sets:

$\{Y_{cn},y_3,y_7\}$ $\{Y_{cn},y_3,y_4,y_6\}$

$\{Y_{cn},y_3,y_5,y_6\}$ $\{Y_{cn},y_3,y_4,y_8\}$

$\{Y_{cn},y_3,y_5,y_8\}$ $\{Y_{cn},y_4,y_5,y_6\}$

$\{Y_{cn},y_4,y_5,y_7\}$ $\{Y_{cn},y_4,y_5,y_8\}$



For contraction avoidance

$Y_{ct}=\{y_1,y_2,y_9,y_{10}\}$

9 minimal sensor sets:

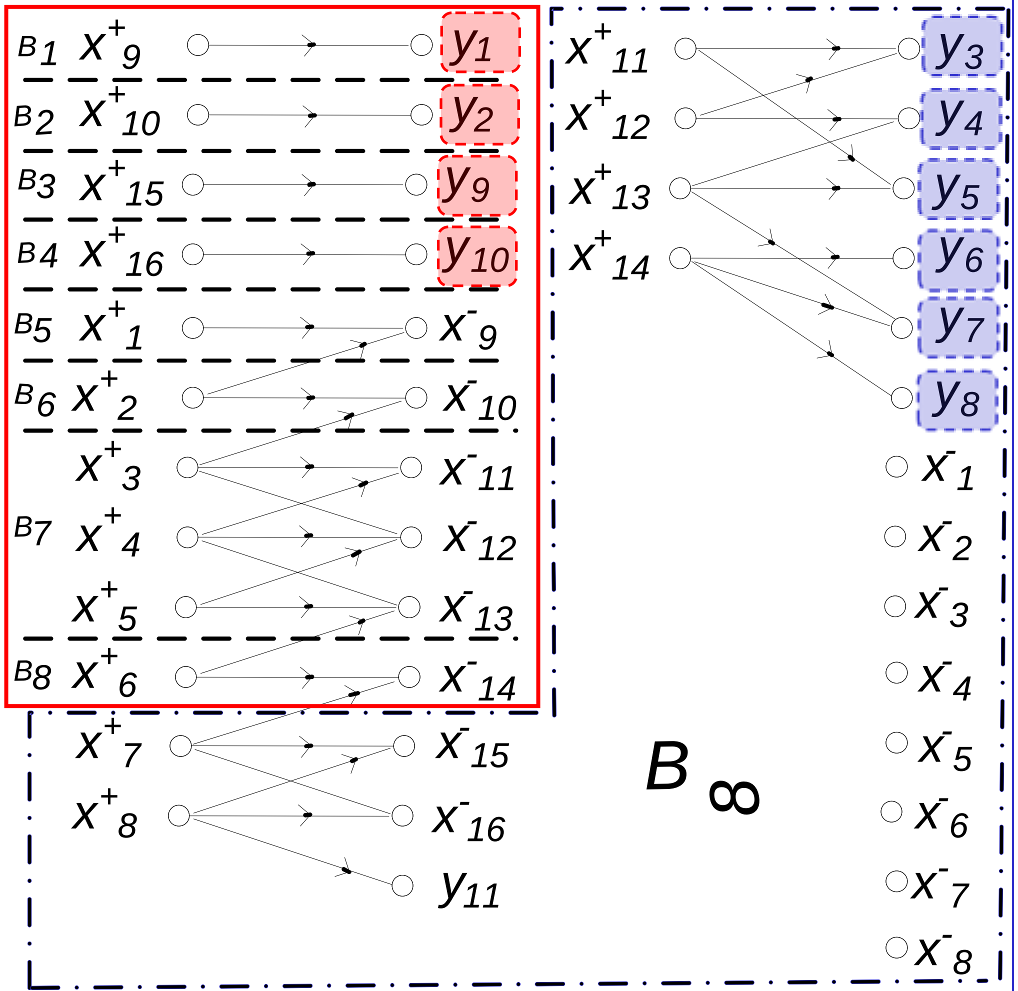
$\{Y_{ct},y_3,y_4,y_5,y_7\}$ $\{Y_{ct},y_3,y_4,y_5,y_6\}$

$\{Y_{ct},y_3,y_4,y_7,y_6\}$ $\{Y_{ct},y_3,y_5,y_7,y_6\}$

$\{Y_{ct},y_5,y_5,y_7,y_6\}$ $\{Y_{ct},y_3,y_4,y_5,y_8\}$

$\{Y_{ct},y_3,y_4,y_7,y_8\}$ $\{Y_{ct},y_3,y_5,y_7,y_8\}$

$\{Y_{ct},y_4,y_5,y_7,y_8\}$



FDI rank condition

$Y_{cf}=\{y_1,y_2\}$

9 minimal sensor sets:

$\{Y_f,y_3,y_4,y_5,y_7\}$ $\{Y_f,y_3,y_4,y_5,y_6\}$

$\{Y_f,y_3,y_4,y_7,y_6\}$ $\{Y_f,y_3,y_5,y_7,y_6\}$

$\{Y_f,y_4,y_5,y_7,y_6\}$ $\{Y_f,y_3,y_4,y_5,y_8\}$

$\{Y_f,y_3,y_4,y_7,y_8\}$ $\{Y_f,y_3,y_5,y_7,y_8\}$

$\{Y_f,y_4,y_5,y_7,y_8\}$