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On Strongly Stabilizing Controller Synthesis

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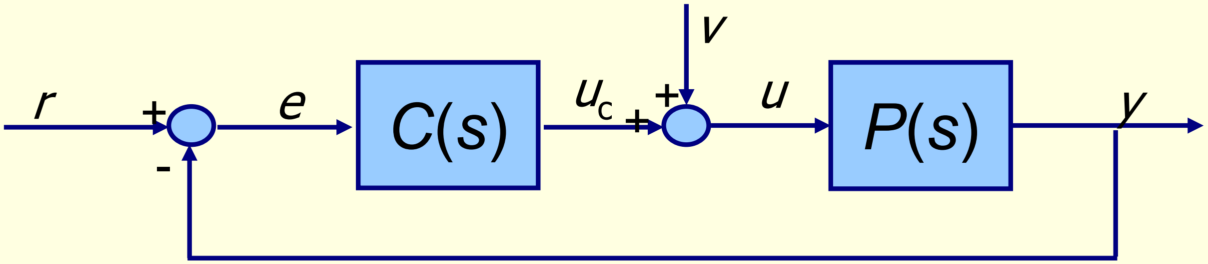
OUTLINE

- Definition of the Strong Stabilization Problem
- MIMO Finite Dimensional Plants Case
  - A parameterization of strongly stabilizing controllers
  - An LMI based design for reduced conservatism
- Strongly Stabilizing Controllers for Systems with Delays
- PD-like Stable Controllers
- Conclusions

Strong Stabilization Problem



We say that *C strongly stabilizes P* if

(i) *C is stable and*

(ii) *the closed-loop system (C,P) is stable.*

Why strong stabilization?

- Simultaneous stabilization of two plants is equivalent to strong stabilization of another plant.
- Robustness to sensor failures in the feedback path.
- The capability to test the stand-alone controller off-line.

Strong stabilization ←→ Parity Interlacing Property (PIP) :

For a given plant *P*, there exists a strongly stabilizing controller if and only if the number of poles of *P* (counted according to their McMillan degrees) between every pair of real blocking zeros of *P* in the extended right half plane is even.

Controller Design (SISO Case):
Let $P = NM^{-1}$, where $N, M \in H_\infty$ are coprime factors, and z_i, $i = 1, \ldots, \ell$ denote the extended right half plane zeros of P.
There exists a strongly stabilizing controller C
$\Longleftrightarrow$
there exists a *unit* U in H_∞ satisfying $U(z_i) = M(z_i)$ for $i = 1, \ldots, \ell$.
Moreover, a strongly stabilizing controller C is given by,

$$C = \frac{U - M}{N}.$$

An iterative construction method for computing such a unit is given by Youla *et al.* (1974). Parameterization of all strongly stabilizing controllers is given by Vidyasagar (1985) using an interpolation approach.

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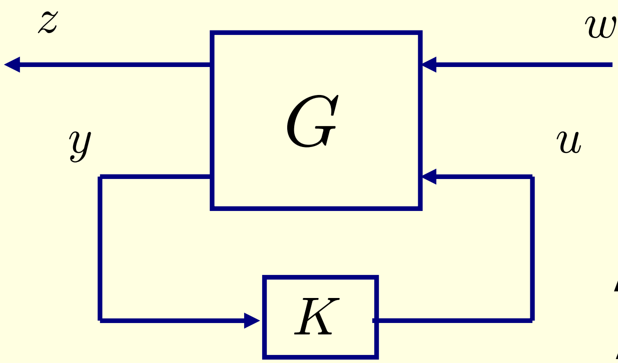
State Space Approach for MIMO Plants



$$G(s) = \left[\begin{array}{c|cc} A & B_1 & B \\ \hline C_1 & D_{11} & D_{12} \\ C & D_{21} & 0 \end{array}\right]$$

Assume (A, B) stabilizable, (C, A) detectable.
Assume G_{22} does not have poles on the Im-axis.

A coprime factorization (RCFND):

$$P(s) = G_{22}(s) = C(sI - A)^{-1}B = NM^{-1}, \text{ where}$$

$$\left[\begin{array}{c} M \\ N \end{array}\right] = \left[\begin{array}{c|c} A_X & B \\ \hline F & I \\ C & 0 \end{array}\right] \in \mathcal{RH}^\infty,$$

with $F = -B^T X$, and $X = X^T \geq 0$ being the solution of

$$A^T X + XA - XBB^T X = 0$$

so that $A_X := (A + BF)$ is stable. Note that M is inner.

A stable controller *K* is strongly stabilizing if and only if

$$U = M + KN = I + F(sI - A_X)^{-1}B + K(s)\ C(sI - A_X)^{-1}B$$

is a unit in $\mathcal{RH}^\infty$.

So, if

$$\inf_{Q \in \mathcal{RH}^\infty} \| [F(sI - A_X)^{-1}B \quad 0] - Q(s)[C(sI - A_X)^{-1}B \quad \frac{1}{\rho} I] \|_\infty < 1$$

for some $\rho > 0$, then there exists a strongly stabilizing controller whose norm is bounded by ρ.

This is a two-block H∞ control problem and if it is solvable for some $\rho = \rho_o > 0$, then it is solvable for all $\rho \geq \rho_o$.

Zeren and Özbay (Automatica, 2000):

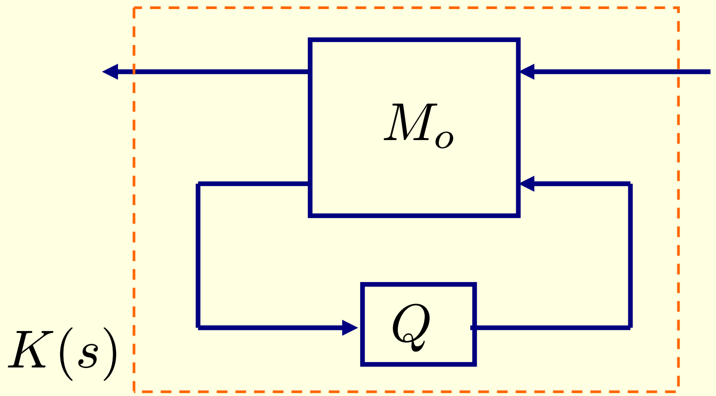
There exists a strongly stabilizing stabilizing controller if there exists $\rho > 0$ such that

$$A_X Y + Y A_X^T - Y(\rho^2 C^T C - X B B^T X) Y + B B^T = 0$$

has a stabilizing positive definite solution, i.e. $Y = Y^T \geq 0$ with stable $A_X + LC$. Under this sufficient condition, a stable controller K, with $\|K\|_\infty \leq \rho$, is given by

$$K = -F(sI - A_K)^{-1} L \;; \quad A_K = (A + BF + LC),\; F = -B^T X,\; L = -\rho^2 Y C^T.$$

Moreover, when this condition holds, all controllers in the form given below with $Q \in \mathcal{RH}^\infty, \|Q\|_\infty < \rho$ are strongly stabilizing:



$$M_0(s) = \left[\begin{array}{c|cc} A + BF + LC & -L & B \\ \hline F & 0 & I \\ -C & I & 0 \end{array}\right].$$

A dual result can be obtained starting from a LCFND instead of a RCFND.

An LMI Based Design for Reduced Conservatism

Gümüşsoy-Özbay (IEEE T-AC, 2005):

Let X and A_X be as defined before.

There exists a strongly stabilizing controller, K with $\|K\|_\infty \leq \gamma_K$ if there exist $X_K = X_K^T > 0$ and Z satisfying the LMIs

$$A^T X_K + X_K A + C^T Z + ZC < 0$$

$$\begin{bmatrix} A_X^T X_K + X_K A_X + C^T Z + ZC & -Z & -XB \\ -Z^T & -\gamma_K I & 0 \\ -B^T X & 0 & -\gamma_K I \end{bmatrix} < 0$$

Under the above sufficient condition a strongly stabilizing controller is given by

$$K_G(s) = \left[\begin{array}{c|c} A_X + X_K^{-1} ZC & -X_K^{-1} Z \\ \hline -B^T X & 0 \end{array} \right]$$

Reduces to Zeren-Özbay (2000) with $X_K = (\gamma_K Y)^{-1} \quad Z = -\gamma_K C^T$

Moreover, all controllers in the set

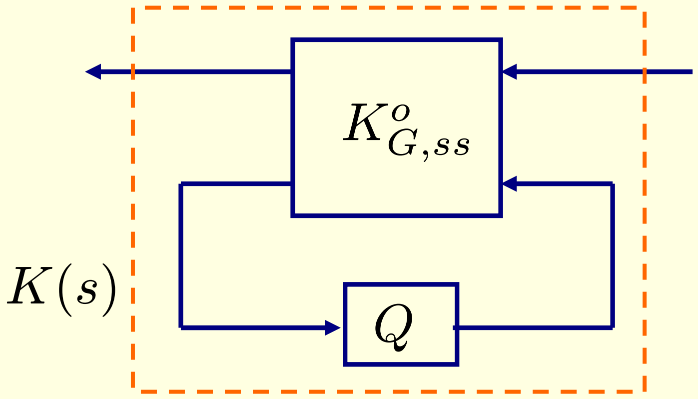
$$\mathsf{K}_{G,ss} := \{K = \mathcal{F}_l(K^0_{G,ss}, Q) : Q \in \mathcal{RH}^\infty, \|Q\|_\infty < \gamma_Q\}$$

are strongly stabilizing, where

$$K^0_{G,ss}(s) = \left[\begin{array}{c|cc} A_X + X_K^{-1}ZC & -X_K^{-1}Z & B \\ \hline -B^T X & 0 & I \\ -C & I & 0 \end{array}\right]$$

and

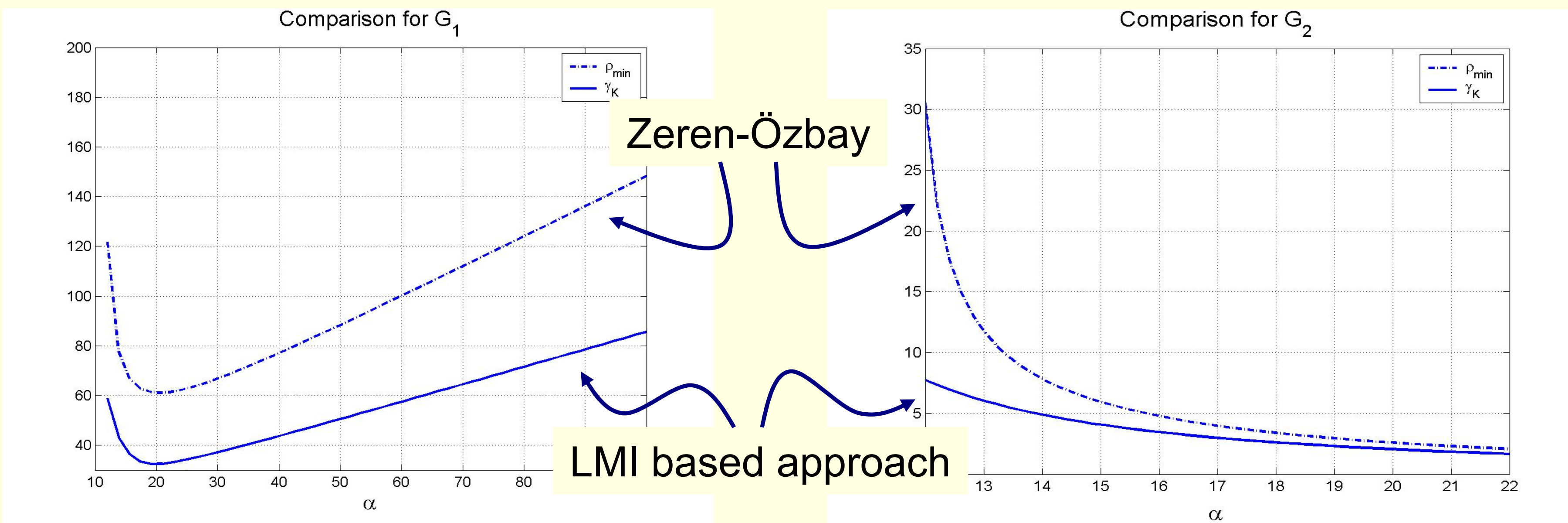
$$\gamma_Q = \left(\|C(sI - (A_X + X_K^{-1}ZC))^{-1}B\|_\infty\right)^{-1}.$$



Examples

$$G_1(s) = \begin{bmatrix} \frac{(s+5)(s-1)(s-5)}{(s+2+j)(s+2-j)(s-\alpha)(s-20)} \\ \\ \frac{(s+1)(s-1)(s-5)}{(s+2+j)(s+2-j)(s-\alpha)(s-20)} \end{bmatrix} \quad G_2(s) = \begin{bmatrix} \frac{(s+1)(s-2-j\alpha)(s-2+j\alpha)}{(s+2+j)(s+2-j)(s-1)(s-5)} \\ \\ \frac{(s+5)(s-2-j\alpha)(s-2+j\alpha)}{(s+2+j)(s+2-j)(s-1)(s-5)} \end{bmatrix}$$

- Stabilize G_1 and G_2 by stable controllers, K where $|K|_\infty < \gamma_K$ using LMI-based result.
- Solve the same problem using Zeren-Özbay (2000), with $|K|_\infty < \rho_{min}$.
- Compare γ_K and ρ_{min} (comparison of conservatism)
- Note that for G_1 as $\alpha \searrow 5$ and for G_2 as $\alpha \searrow 0$, p.i.p. is close to being violated



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Strongly Stabilizing Controllers for Systems with Time Delays

Özbay (IFAC 2008): Zeren-Özbay approach can be extended to plants of the form $P = D_p^{-1} N_p$, where $D_p, N_p \in \mathcal{H}_\infty$ strongly coprime, and D_p is finite dimensional $N_p(s) = N_{pi}(s) N_{po}(s) N_{p1}(s)$ with N_{pi} inner, N_{po} outer finite dimensional, and N_{p1} is right invertible in H_∞.

Example:

$$P(s) = \frac{(s-2)e^{-\tau s}}{(s+a-ke^{-hs})} \begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+2} & \frac{1}{s+3} \\ 0 & 0 & \frac{e^{-2\tau s}}{s+4+e^{-s}} \end{bmatrix}$$

$a, h, k, \tau > 0$ with $kh < 1$ and $k > a$.

The plant contains a single pole in $\mathbb{C}_+$

This pole, denoted by α, is the unique solution of $\alpha = ke^{-h\alpha} - a$.

Blocking zeros in the extended right half plane: $\{2, \infty\}$

The plant satisfies the PIP if and only if $\alpha < 2$.

$$N_{pi}(s) = \frac{(s-2)}{(s+2)} e^{-\tau s} \begin{bmatrix} 1 & 0 \\ 0 & e^{-2\tau s} \end{bmatrix}$$

$$N_{po}(s) = \frac{1}{s+1} I$$

$$N_{p1}(s) = \frac{(s-\alpha)}{(s+a-ke^{-hs})} \begin{bmatrix} \frac{s+2}{s+1} & 1 & \frac{s+2}{s+3} \\ 0 & 0 & \frac{s+2}{s+4+e^{-s}} \end{bmatrix}$$

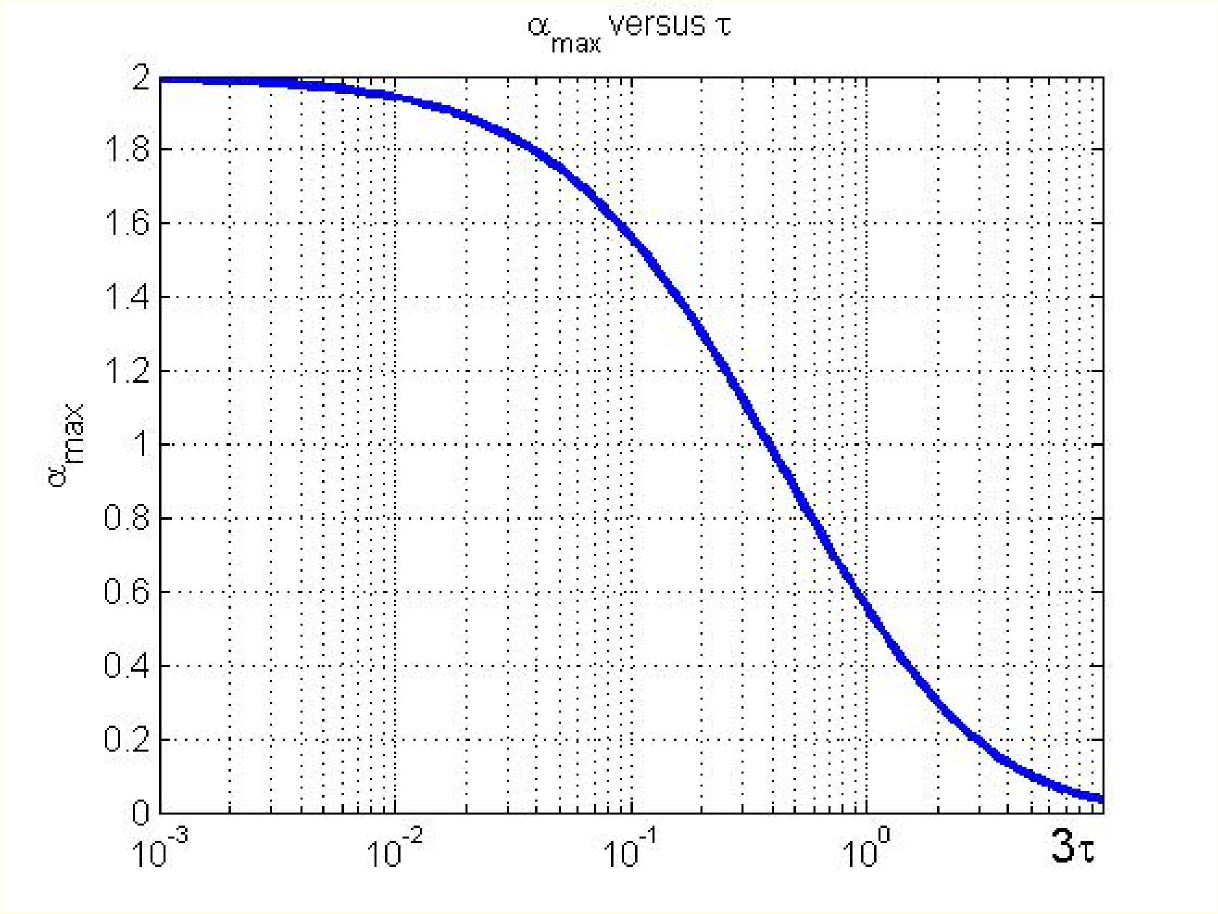
$$N_{p1}^{\dagger}(s) = \frac{(s+a-ke^{-hs})}{(s-\alpha)} \begin{bmatrix} 2\frac{s+1}{s+2} & 0 \\ -1 & -\frac{s+4+e^{-s}}{s+3} \\ 0 & \frac{s+4+e^{-s}}{s+2} \end{bmatrix}$$

For the plant given here, we can find a strongly stabilizing controller using this approach if and only if

$$\inf_{q_2 \in \mathcal{H}_\infty} \| - \frac{(\alpha+1)}{s+1} + \frac{(s-2)}{(s+2)(s+1)} e^{-3\tau s} q_2(s) \|_\infty < 1$$

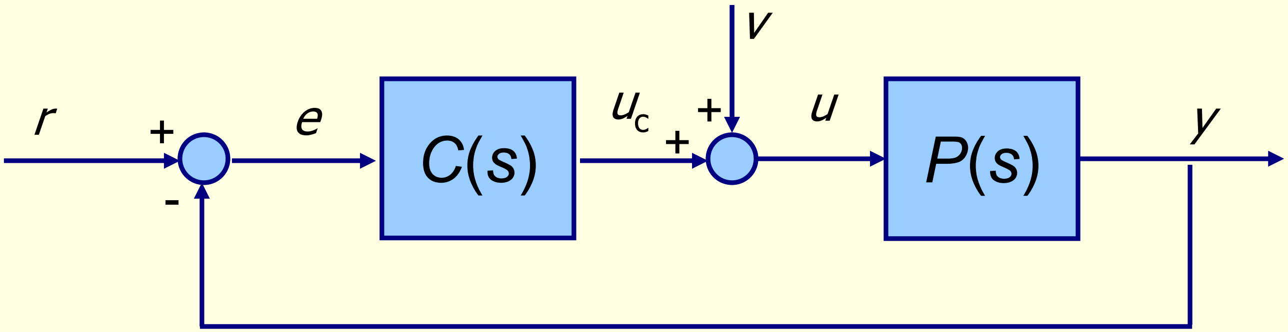
For $\tau = 0$ the problem is solvable if and only if $\frac{1}{3} < \frac{1}{\alpha+1} \iff \alpha < 2$ which is equivalent to PIP.

For $\tau > 0$ largest α for which we can find a strongly stabilizing controller using this approach is shown in the figure.



Sensitivity Minimization by Stable Controllers

Gümüşsoy-Özbay (CDC2007, IEEE T-AC to appear)



Feedback system is stable if and only if $S = (1+PC)^{-1}$, PS and CS are in $\mathcal{H}_\infty$

If a controller is stable, $C \in \mathcal{H}_\infty$ and stabilizes the feedback system with plant *P* then we say that it is a strongly stabilizing controller. The set of all strongly stabilizing controller for the plant *P* is denoted by $\mathcal{S}_\infty(P)$

WSMSC: given *W* and *P*, find

$$\begin{aligned} \gamma_{ss} &= \inf_{C \in \mathcal{S}_\infty(P)} \|W(1+PC)^{-1}\|_\infty, \\ &= \|W(1+PC_{\gamma_{ss}})^{-1}\|_\infty \end{aligned}$$

The Class of Plants Considered in This Section:

$$P(s) = \frac{m_n(s)}{m_d(s)} N_o(s)$$

inner, finite dimensional

inner, possibly infinite dimensional

outer, possibly infinite dimensional

Examples from time delay systems will be given.

Factorization of Unstable Time Delay Systems

Assumption:

$$P(s) = \frac{R(s)}{T(s)} = \frac{\sum_{i=1}^{n} R_i(s) e^{-h_i s}}{\sum_{j=1}^{m} T_j(s) e^{-\tau_j s}} \qquad P(s) = \frac{m_n(s)}{m_d(s)} N_o(s)$$

$$0 = h_1 < \ldots < h_n \qquad 0 = \tau_1 < \ldots < \tau_m$$

R_i and T_j are finite dimensional stable proper functions with no Im-axis zeros.
$R(s)$ has finitely many zeros in the right half plane.
$T(s)$ may have infinitely many zeros in the right half plane.

Example:

$$\begin{aligned} \dot{x}(t) + 2\dot{x}(t-2) &= -x(t) + 2x(t-2) + u(t), \\ y(t) &= 4x(t-3) - 2\dot{x}(t-2) + 2x(t-2) + u(t) \end{aligned}$$

$$P(s) = \frac{(s+1) + 4e^{-3s}}{(s+1) + 2(s-1)e^{-2s}} = \frac{1e^{-0s} + \left(\frac{4}{s+1}\right) e^{-3s}}{1e^{-0s} + \left(\frac{2(s-1)}{s+1}\right) e^{-2s}}$$

Let $s_1, \ldots, s_{n_r}$ be the zeros of $R(s)$ in the right half plane.

Define an inner function M_R(finite Blaschke product) from $s_1, \ldots, s_{n_r}$

Then $\dfrac{R(s)}{M_R(s)}$ is stable and minimum phase.

We can do the same for T when it has finitely many zeros in the right half plane. But in this paper we consider a more general case where *T* *is allowed to have inifinitely many zeros in the right half plane*.

Define the conjugate of T as $\bar{T}(s) := e^{-\tau_m s} T(-s) M_C(s)$
where $M_C(s)$ is a finite dimensional inner function whose zeros are the poles of T.

$\bar{T}(s)$ is like *R(s),* it has finitely many zeros in the right laf plane.

Define $M_{\bar{T}}(s)$ a finite Blaschke product from these zeros, so that

$\dfrac{M_{\bar{T}}}{\bar{T}}$ is outer. Then we have:

$$P(s) = \frac{m_n(s)}{m_d(s)} N_o(s) \qquad m_d = M_{\bar{T}} \frac{T}{\bar{T}}, \quad m_n = M_R, \quad N_o = \frac{R}{M_R} \frac{M_{\bar{T}}}{\bar{T}}.$$

Example:

$$R(s) = 1 + \frac{4}{s+1} e^{-3s} \qquad T(s) = 1 + 2\frac{(s-1)}{(s+1)} e^{-2s}$$

Zeros of R(s) in the right half plane: $0.3125 \pm j0.8548$

$$\bar{T}(s) = 2 + \left(\frac{s-1}{s+1}\right) e^{-2s}$$ it has no zeros in the right half plane.

$$\begin{aligned} m_d(s) &= \frac{T(s)}{\bar{T}(s)}, \\ m_n(s) &= M_R(s) = \frac{s^2 - 0.6250s + 0.8283}{s^2 + 0.6250s + 0.8283}, \\ N_o(s) &= \frac{R(s)}{M_R(s)} \frac{1}{\bar{T}(s)}. \end{aligned}$$

Interpolation Approach for the Solution of WSMSC

$$P(s) = \frac{m_n(s)}{m_d(s)} N_o(s)$$

$s_1, \ldots, s_N$ zeros of m_n in $\mathbb{C}_+$

Let W be minimum phase

WSMSC: given *W* and *P*, find

$$\gamma_{ss} = \inf_{C \in \mathcal{S}_\infty(P)} \|W(1+PC)^{-1}\|_\infty,$$
$$= \|W(1+PC_{\gamma_{ss}})^{-1}\|_\infty$$

Feedback system is stable and $\|W(1+PC)^{-1}\|_\infty \leq \gamma$

if and only if there exists $F \in \mathcal{H}_\infty$ such that $F^{-1} \in \mathcal{H}_\infty$ and

$$F(s_i) = \frac{W(s_i)}{\gamma m_d(s_i)} =: \frac{\omega_i}{\gamma}, \quad i = 1, \ldots, N$$

Once such an F is found the corresponding the stable controller is

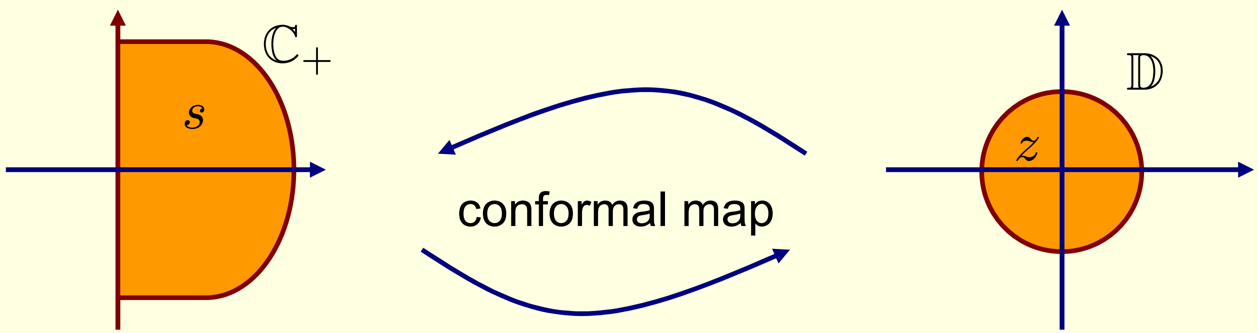
$$C_\gamma = \frac{(W - \gamma\ m_d\ F)}{\gamma\ m_n}\ N_o^{-1}\ F^{-1} \in \mathcal{H}_\infty$$

Ganesh-Pearson (ACC, 1986) solution via the Nevanlinna-Pick interpolation:

$$F(s) = e^{-G(s)} \qquad G(s) = -\ln F(s)$$

Find and analytic function $G \; : \; \mathbb{C}_+ \to \mathbb{C}_+$ such that

$$G(s_i) = -\ln \omega_i + \ln \gamma - j2\pi m_i =: \nu_i \qquad m_i \in \mathbb{Z}$$



$$g(z) = G(\varphi^{-1}(z))$$

$$z = \varphi(s) = \frac{s-a}{s+a},\; a > 0 \qquad z_i = \varphi(s_i), i = 1, \ldots, N$$

Optimal solution γ_{ss} is the smallest γ (over all possible integers $m_i, m_k \in \mathbb{Z}$) such that the Pick matrix whose entries are given below is positive semidefinite:

$$P_{i,k} = \frac{2\ln\gamma - \ln\omega_i - \ln\bar{\omega}_k + j2\pi m_{k,i}}{1 - z_i \bar{z}_k} \qquad m_{k,i} = m_k - m_i$$

Example

$$R(s) = 1 + \frac{4}{s+1} e^{-3s} \qquad s_{1,2} = 0.31 \pm j0.85$$

$$T(s) = 1 + 2\frac{(s-1)}{(s+1)} e^{-2s}$$

$$m_d(s) = \frac{T(s)}{\bar{T}(s)},$$

$$N_o(s) = \frac{R(s)}{M_R(s)} \frac{1}{\bar{T}(s)}.$$

$$m_n(s) = M_R(s) = \frac{s^2 - 0.625s + 0.828}{s^2 + 0.625s + 0.828}.$$

Let $W(s) = \frac{1+0.1s}{1+s}$ then $\omega_{1,2} = 0.79 \mp j0.85$

$\gamma_{ss} = 1.07$ (computed from the 2x2 Pick matrix)

Resulting “optimal” interpolating function is

$$F(s) = e^{-0.57s}$$

Clearly $F^{-1} \neq \mathcal{H}_\infty$ and the corresponding controller includes a time advance.

Modified Interpolation Problem

Now we add an extra condition: $\|F^{-1}\|_\infty \leq \rho$

This also puts a bound on the H_∞ norm of the controller:

$$\|C_\gamma\|_\infty \leq \|N_o\|_\infty^{-1} \left(1 + \frac{\rho}{\gamma}\|W\|_\infty\right).$$

Recall that in the unrestricted problem we are looking for an $F(s) = e^{-G(s)}$

for some analytic $G : \mathbb{C}_+ \to \mathbb{C}_+$ satisfying $G(s_i) = \nu_i$, $i = 1, \ldots, n$.

In this case we have $|F(s)^{-1}| = |e^{\mathrm{Re}(G(s))}|$ for all $s \in \mathbb{C}_+$

So we need $0 < \mathrm{Re}(G(s)) < \ln(\rho) =: \sigma_o$ and now we seek an analytic function

$G : \mathbb{C}_+ \to \mathbb{C}_+^{\sigma_o}$ satisfying $G(s_i) = \nu_i$, $i = 1, \ldots, n$, where

$$\mathbb{C}_+^{\sigma_o} := \{s \in : 0 < \mathrm{Re}(s) < \sigma_o\}$$

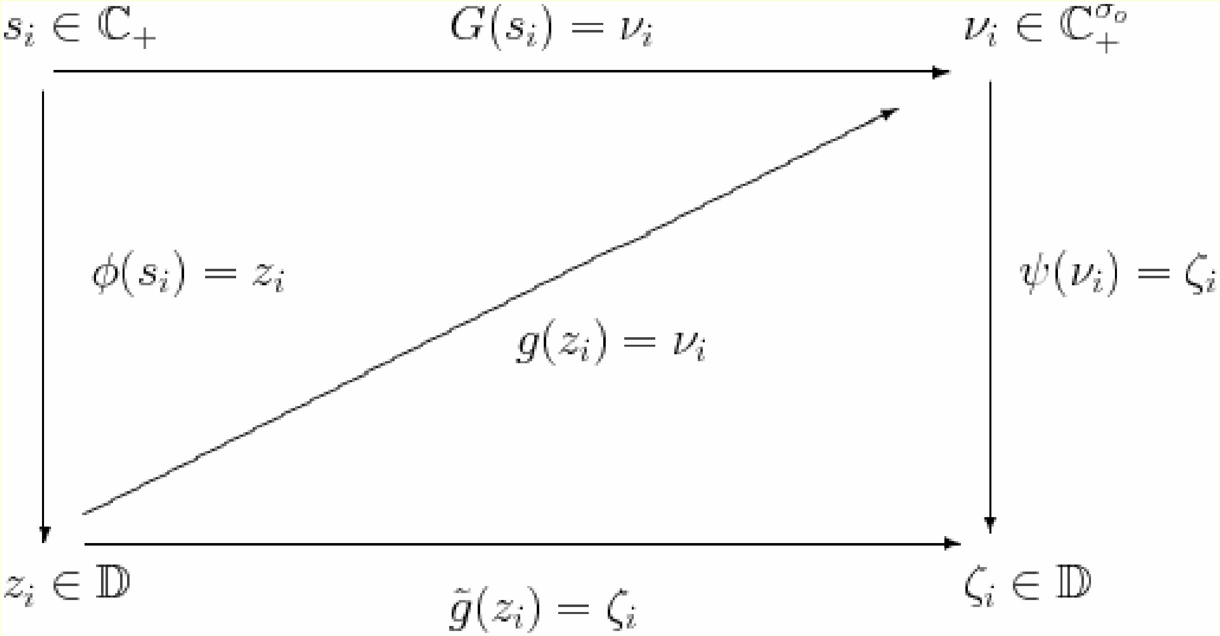
Define $\gamma_{\mathrm{ss},\rho}$ as the smallest γ for which a feasible solution exists.

This problem can be solved by using Nevanlinna-Pick interpolation using the following conformal maps

$$z = \phi(s) = \frac{s-1}{s+1} \qquad s = \phi^{-1}(z) = \frac{1+z}{1-z}$$

$$z = \psi(s) = \frac{je^{-j\pi s/\sigma_o} - 1}{je^{-j\pi s/\sigma_o} + 1}$$

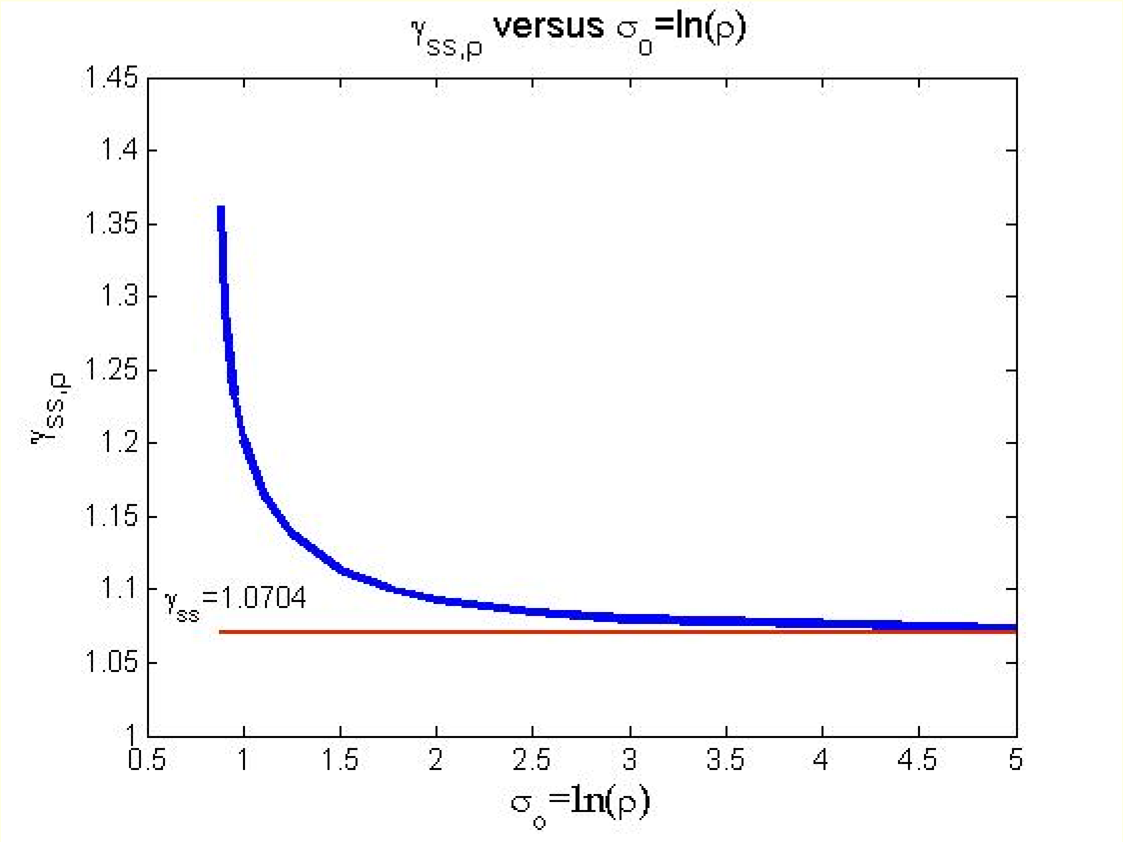
$$s = \psi^{-1}(z) = \frac{\sigma_o}{\pi}\left(\frac{\pi}{2} + j\ln(\frac{1+z}{1-z})\right),$$



Back to the example:

Recall that $\gamma_{ss} = 1.07$ now as $\rho \to \infty$ we have $\gamma_{ss,\rho} \to \gamma_{ss}$

But there is a lower bound on how small ρ can get: $\rho_{\text{min}} = e^{0.88} = 2.41$



For $\rho = e^3 \approx 20$, we have $\gamma_{\text{ss},\rho} = 1.08$, corresponding optimal interpolant is

$$\tilde{G}(s) := \tilde{g}(\phi(s)) = j\frac{-0.99794(s-3.415)(s+1)}{(s+3.406)(s+1.001)}.$$

which results in

$$F(s) = \exp(-\frac{\sigma_o}{2} - \frac{j\sigma_o}{\pi}\ln(\frac{1+\tilde{G}(s)}{1-\tilde{G}(s)})).$$

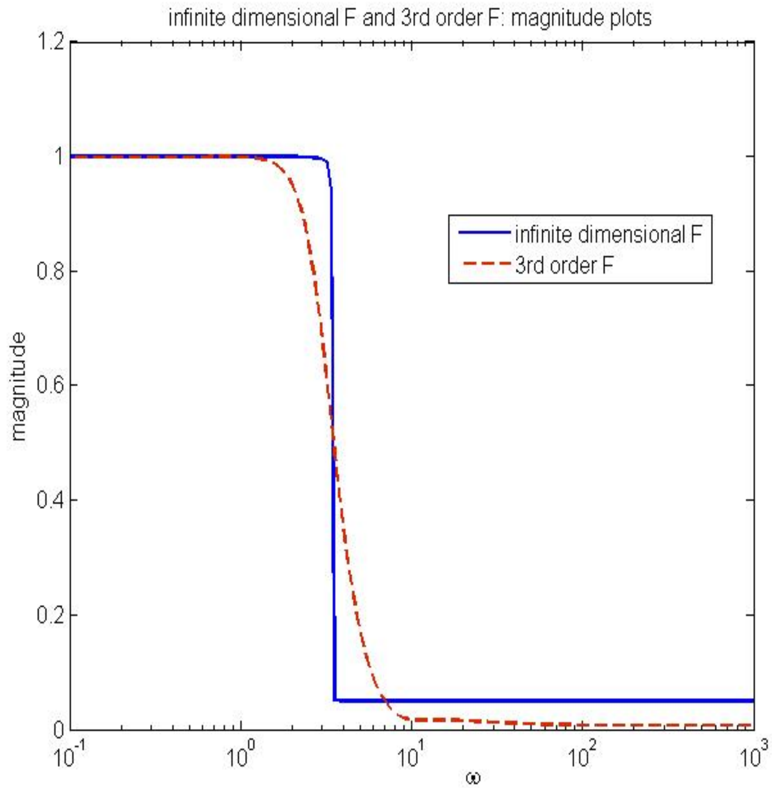
This is an infinite dimensional transfer function whose rational approximations can be obtained.

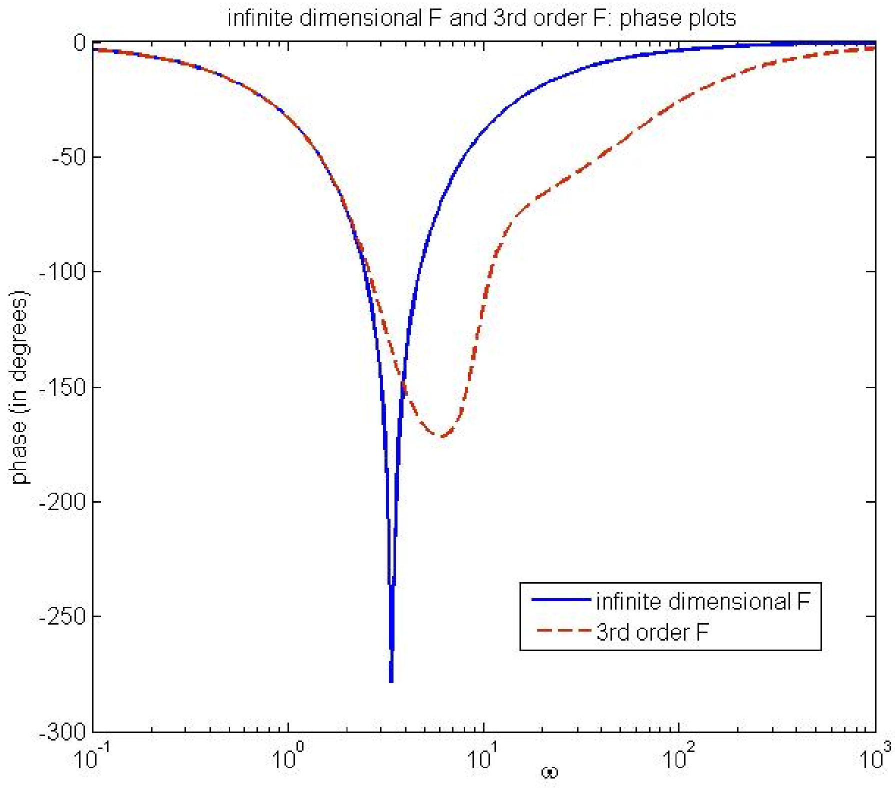
For $\gamma = 1.08$ we find a feasible suboptimal solution

$$F(s) = \frac{0.068s^3 + 3.77s^2 + 21.45s + 295.84}{9.93s^3 + 62.77s^2 + 187.25s + 296.27}. \qquad \|F\|_\infty = \frac{295.84}{296.27} < 1$$

$$F(s_i) = \frac{\omega_i}{1.08}, \quad \text{for } i = 1, 2$$

$$\texttt{zero}(\texttt{F}) = -50.9245, -2.2583 \pm j\ 8.9628$$
$$\texttt{pole}(\texttt{F}) = -3.3510, -1.4851 \pm j\ 2.5881$$





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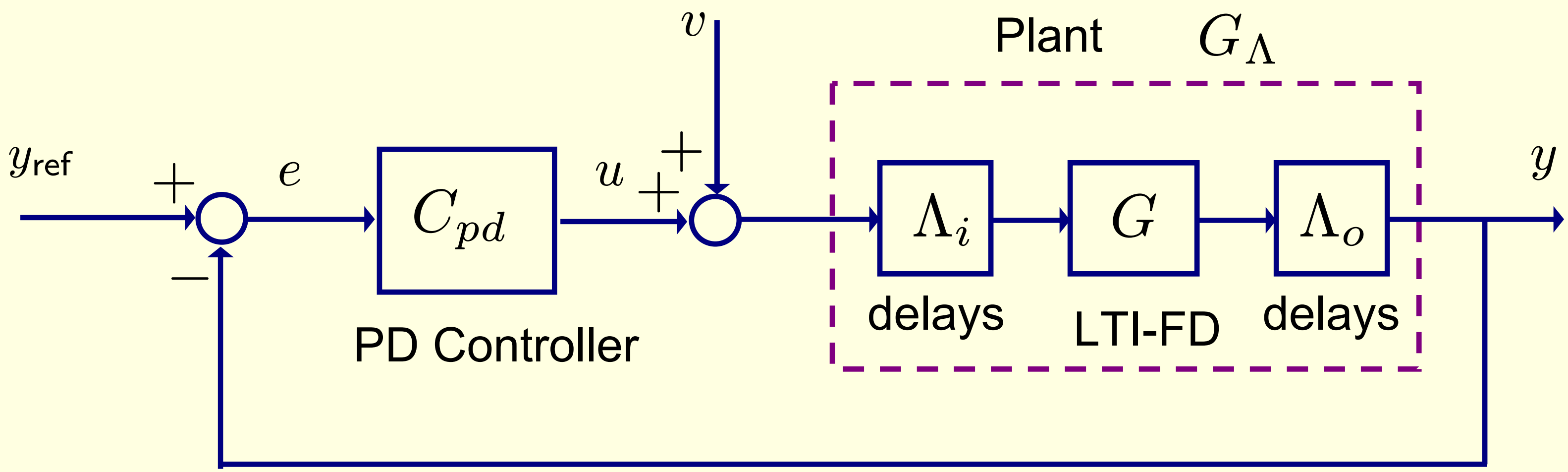
PD-like Controllers for MIMO Systems with Delays

Gündeş-Özbay-Özgüler (Automatica 2007)



$$G(s) = C(sI - A)^{-1}B + D \qquad C_{pd} = K_p + \frac{K_d\ s}{\tau_d s + 1}$$

{ very special stable controller

$$\Lambda_\star(s) = \text{diag}\left[e^{-T_1^\star s}, \cdots, e^{-T_r^\star s}\right] \qquad T_j^\star \in \Theta_j^\star = [0\ ,\ T_{\max}^{j\star}) \subset \mathbb{R}_+$$

Define $\mathcal{T}^\star := (T_1^\star, \ldots, T_r^\star)$ and $\Theta^\star := (\Theta_1^\star, \ldots, \Theta_r^\star)$

As a shorthand notation we will write $(\mathcal{T}^i, \mathcal{T}^o) = \mathcal{T} \in \Theta = (\Theta^i, \Theta^o)$

to represent all possibilities $T_j^\star \in \Theta_j^\star,\ 1 \le j \le r$

Consider plants $G(s)$ with "single unstable pole" $p \geq 0$, and define coprime factors as follows:

Let $rank G(s) = r$, and let $X(s) = \frac{(s-p)}{as+1} G(s)$ be stable for $a > 0$.

Assume $rank X(p) = rank (s-p) G(s)|_{s=p} = r$, and

let $X(0) = (s-p) G(s)|_{s=0}$ be nonsingular, $G^{-1}(0) = -p\, X(0)^{-1}$.

Further Structural Assumption:

$$G = Y^{-1} X \ , \quad X, Y \in \mathcal{H}_\infty$$ Y is diagonal, and hence

$$G_\Lambda = Y^{-1} X_\Lambda \qquad X_\Lambda = \Lambda_o X \Lambda_i$$

PD-like controller synthesis:

Choose any $\widehat{K}_d \in \mathbb{R}^{r\times r}$, $\tau_d > 0$. Define

$$\widehat{C}_{pd}(s) := X(0)^{-1} + \frac{\widehat{K}_d\, s}{\tau_d s + 1}$$

$$\Phi_\Lambda := \frac{(s-p)G_\Lambda(s)\widehat{C}_{pd}(s) - I}{s} \quad , \quad \widetilde{\Phi}_\Lambda := \frac{\widehat{C}_{pd}(s)(s-p)G_\Lambda(s) - I}{s}.$$

If

$$0 \le p < \max\{\min_{\mathcal{T}\in\Theta} \|\Phi_\Lambda\|^{-1}, \min_{\mathcal{T}\in\Theta} \|\widetilde{\Phi}_\Lambda\|^{-1}\} =: \phi,$$

then for any positive $\alpha \in \mathbb{R}$ satisfying

$$p < \ \alpha + p \ < \phi$$

a PD-controller that stabilizes G_Λ for all $\mathcal{T} \in \Theta$ is given by

$$C_{pd}(s) = (\,\alpha + p\,)\widehat{C}_{pd}(s).$$

If $\widehat{K}_d = 0$, above is a P-controller.

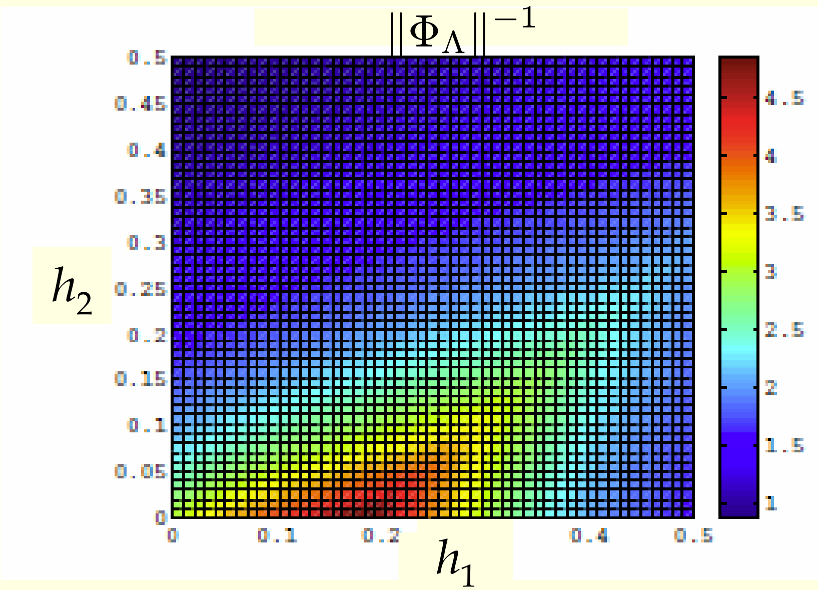
Example: Consider $G(s) = \frac{1}{s}\, G_o\, G_1(s)$ with

$$G_o = \begin{bmatrix} 3.04 & -278.2/180 \\ 0.052 & 206.6/180 \end{bmatrix} \quad G_1(s) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{180}{(s+6)(s+30)} \end{bmatrix}.$$

An LCF of the plant is $G(s) = Y(s)^{-1} X(s)$, with

$$X(s) = \frac{1}{as+1} G_o G_1(s) \qquad Y(s) = \frac{s}{as+1} I, \quad a > 0.$$

Assume that the delays in the input channels are h_1 and h_2, and consider P control.



Largest value = 4.86, for h_1=0.18 and h_2=0

$\|\widetilde{\Phi}_\Lambda\|^{-1} = 1/\max\{h_1, 0.2 + h_2\} = 5$
for $h_2 = 0$ and $0 \le h_1 \le 0.2$.
So, $\alpha_{\max} = 5$.

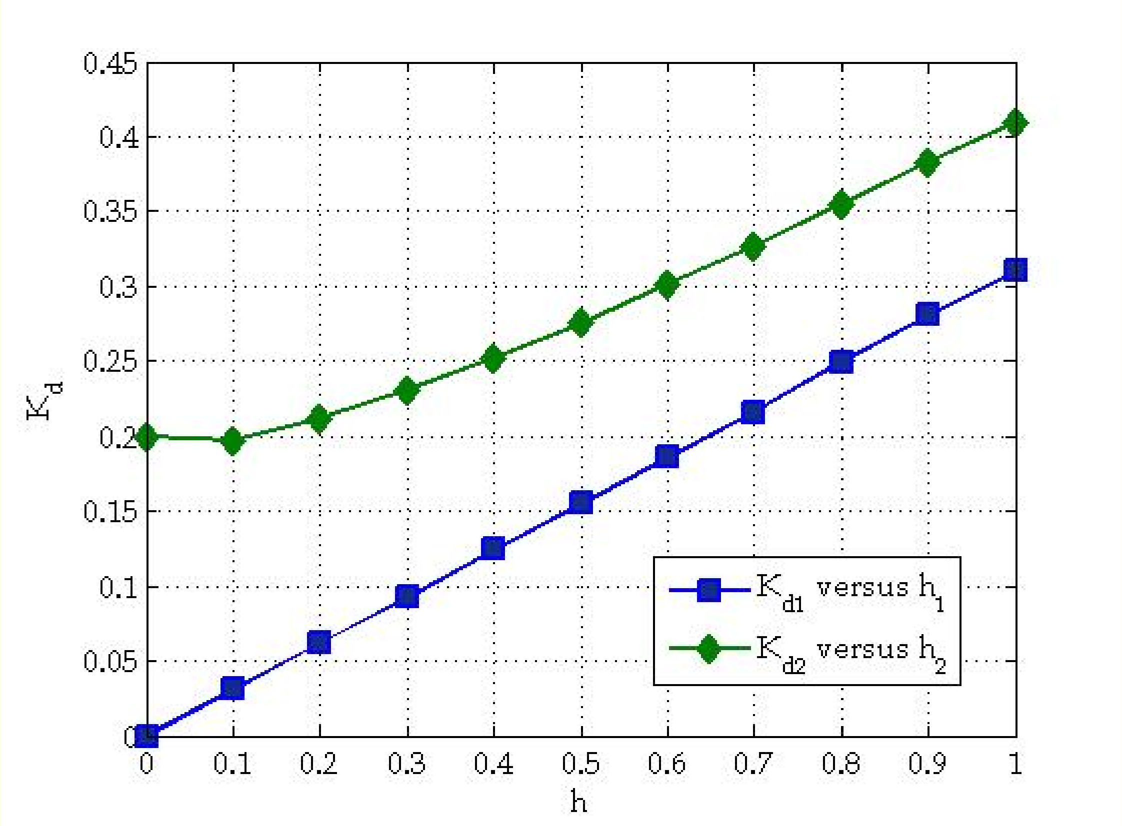
For $h_2 = 0$ and $h_1 > 0.2$ we find $\alpha_{\max} = \frac{1}{h_1}$

Actual largest allowable gain is $\frac{\pi}{2h_1}$.

Now consider the PD-controller $\alpha(I + \frac{\tilde{K}_d s}{\tau_d s+1})G_o^{-1}$ where $\hat{K}_d =: \tilde{K}_d G_o^{-1}$.

The optimal $\hat{K}_d = \tilde{K}_d G_o^{-1}$ is the one which minimizes $\|\tilde{\Phi}_\Lambda\|$.

Since $\tilde{\Phi}_\Lambda$ is diagonal, we restrict $\tilde{K}_d$ to be in the form $diag(K_{d,1}, K_{d,2})$.



Conclusions

- Strongly stabilizing controllers are obtained using a special factorization of the MIMO plants.
- Further optimization (e.g. H_∞ and H_2) in the paramaterized family of strongly stabilizing controllers is possible.
- An LMI-based technique reduces conservatism in this approach.
- The method can be extended to systems with time delays.
- Nevanlinna-Pick interpolation can be used for sensitivity minimization in the set of all strongly stabilizing controllers.
- PD- like controllers can be derived for MIMO plants with restricted number of poles in the right half plane (input/output delays are allowed in the plant).

Ackowledgements:

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- S. Gumussoy and H. Özbay, "Sensitivity Minimization by Strongly Stabilizing Controllers for a Class of Unstable Time-Delay Systems," *IEEE Transactions on Automatic Control*, vol. 53, no. 12, December 2008, to appear. See also *Proc. of the 46th IEEE Conference on Decision and Control*, New Orleans, LA, December 2007, pp. 6071—6076.

- A. N. Gündeş, H. Özbay, A. B. Özgüler, "PID controller synthesis for a class of unstable MIMO plants with I/O delays", *Automatica,* vol. 43, No. 1, January 2007, pp. 135—142.

- S. Gumussoy and H. Özbay, "Remarks on strong stabilization and stable H∞ controller design," *IEEE Transactions on Automatic Control*, vol. 50, No. 12, December 2005, pp. 2083—2087.

- M. Zeren and H. Özbay, "On the strong stabilization and stable H∞ controller design problems for MIMO systems," *Automatica*, vol. 36 (2000), no. 11, pp. 1675–1684.

- H. Özbay, "On Strongly Stabilizing Controller Synthesis for Time Delay Systems," *Proc. of the 17th IFAC World Congress*, Seoul, Korea, July 2008, pp. 6342—6346.

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