Instructor's Guide with detailed list of topics for a first-year graduate course on Linear Systems using the textbook:

LINEAR SYSTEMS
P.J.Antsaklis and A.N.Michel

The descriptions below provide topics and materials from the book that could be the core materials for a semester course (14 weeks) on Linear Systems.

Descriptions emphasize key results. Depending on the class background different background material is included as needed. Depth of coverage varies depending on the pace of the course and the goals set for the course. Sometimes it is particularly illuminating and beneficial to go through a proof of a theorem and this is stated.

Topics of course could be added and subtracted, could be further emphasized or de-emphasized.

Additional information is provided in the introduction of each chapter –guidelines to the reader.

PART I
FUNDAMENTALS

CHAPTERS 1 and 2.

Modeling and Mathematical Descriptions
Primarily section 1.1A (models) pp. 2-3 and sections 1.3A, 1.4 (initial value problems and examples) pp. 10-11, 13-17. Sections 1.1 though 1.4 also contain additional introductory material and could be studied via a reading assignment.

Existence and Uniqueness of solutions.
Section 1.10D. Discuss Theorems 10.8 and 10.9 (Lipschitz conditions and successive approximations) p.47 Systems of differential equations in section 1.12 Theorem 12.1 pp. 54-55.

Linearization
Section 11.1 (linearization theory and examples) pp.47-54. Derivation could be omitted. One could emphasize the use of these results and just outline rationale and derivations.

Solutions of Linear State Equations and the Peano-Baker Series
Section 1.13 (homogeneous and nonhomogeneous equations) pp. 55-58. The state transition matrix. Exp(At) for the time invariant case. Variation of constants formula. Also Section 1.14.

Linear Discrete State Equations (this material can be covered later, together with the discrete time systems material of Chapter 2)
Section 1.15 (difference equations and solutions) pp.60-65. . The state transition matrix. A**k for the time invariant case.

Finding the state transition matrix (this material may be delayed until Chapter 2)
Review of matrix algebra concepts and methods. Eigenvalues and eigenvectors pp. 121-122. Diagonalization of A pp.128-130. The Cayley-Hamilton Theorem pp. 124-126.

Characterization of all solutions of xdot=A(t)x via bases of the solution space
Section 2.3A (the fundamental matrix) Theorem 3.1 (set of solutions forms an n-dimensional vector space) pp. 139-140. Go over proof. Need to introduce/review material on fields and vector spaces (definitions section 1.10A) pp.37-41; linear independence section 2.2B (also independence of functions of time) and bases 2.2C pp.97-100.

At this time also review rank in section 2.2Fpp. 107-108 and solving algebraic equations in section 2.2H pp. 115-116. Also you may want to cover material on linear transformations in 2.2D and E pp. 100-107, and on equivalence and similarity section 2.2I pp. 116-121.

Fundamental and State Transition matrices.
In section 2.3 A, 2.3B definitions and properties pp. 140-145. How to determine the state transition matrix 2.3D pp. 146-148.

Focus on Linear Systems with Constant coefficients.

Properties of the state transition matrix exp(At) in section 2.4A pp.148-150 and how to determine exp(At) in section 2.4B pp. 150-156 (via series, similarity transformations, Cayley-Hamilton theorem, Laplace transform-review Laplace transforms).

Modes and asymptotic behavior in section 2.4C pp. 156-161. Stability of an equilibrium, definitions and tests.

Input-Output Descriptions
Impulse response in section 2.6A pp. 165-168. Review section 1.16 pp. 65-79 for the delta function 1.16C and impulse response basics in 1.16D. Transfer functions in 2.6B pp.168-170. Equivalence of state variable representations and the effect on external descriptions (none) in 2.6C pp. 170-173.

Discrete-time time invariant systems and sampled data systems
Response in section 2.7A pp. 174-177. Transfer function and review of the z transform in section 2.7B pp.177-180. Equivalence of state variable representations in section 2.7C pp.180-181. Sampled data systems in section 2.7D pp. 182-186. Modes, asymptotic behavior and tests for stability of equilibria in section 2.7E pp. 186-190.

It is useful to assign the following exercises from Chapter 2:
2.61-formula for realization of a scalar transfer function
2.62-systems in series, parallel and feedback configurations
2.63-Markov parameters
2.65-frequency response
2.66-Sylvester's Rank Inequality

At this point, typically just before the mid-semester point, the student should have a good understanding of the dynamic behavior of linear continuous and discrete-time systems, the effect of initial conditions and input on the behavior, the role of eigenvalues and modes and how they influence the behavior, the notion of stability and a good understanding of the connections between stability and eigenvalues in the time-invariant case. Note that stability is studied in greater detail in Chapter 6. In addition, the student should have good understanding of concepts and methods in matrix algebra-properties of matrices, eigenvalues/eigenvectors and of transformations- and in abstract algebra-vector spaces, linear independence. Also the student should have good understanding of solving linear ordinary differential and difference equations, of approaches and computer methods.

PART II
CONTROLLABILITY, OBSERVABILITY, REALIZATIONS
STATE FEEDBACK AND STATE OBSERVERS

The topics to be covered now are reachabiliy/controllability, observabity/constructibility, special forms in Chapter 3, and realization theory and algorithms in Chapter 5. In Chapter 4 state feedback and state estimation (observers) are discussed. The emphasis here and the length of time spent on each topic depends greatly on the needs this course serves and on the interests of the instructor. For example one could add material in terms of notes on optimal control, as the coverage of this topic in Chapter 5 is rather brief. Or one could spend more time on stability and Chapter 6 discussing for example Lyapunov functions and stability theory. Or one could select topics from Chapter 7 and discuss fractional descriptions and the Diophantine equation and characterize all stabilizing controllers.

Here we describe a selection of materials that is can be easily covered in the second half of the semester. Note that the descriptions below are more detailed than previous descriptions above. This is done so to facilitate the addition or subtraction of topics.

CHAPTERS 3
Concepts of reachability/controllability and observability/constructibility are introduced in Chapter 3 and tests are derived. State space (internal) realizations of transfer functions and impulse response (external) descriptions are studied in Chapter 5.

Section 3.1A pp. 215-219, introduce reachability/controllability focusing on time invariant, discrete time systems. In this way the role of the controllability matrix C is easily seen and the control inputs that accomplish the transfer of the state are easily calculated. At this point one could continue and finish the discrete time case or study the continuous time case first. Pursuing the first option, in section 3.2C pp. 242-247, prove Theorem 2.18 p.243 and discuss the corollaries with examples. Theorem 2.2 p.245 explains the relation between reachability and controllability. At this point one could continue with discrete time and discuss observabilty-see below. Continuing with reachabilty for continuous time systems in section 3.2B pp.235-241 (using some definitions and results from the time varying case in section 3.2A pp. 227-235) introduce the reachability Gramian (Definition 2.11). Lemma 2.10 shows that reachability of a state does not depend on the time T it takes to reach the state and introduces the controllability matrix. The Gramian is used to calculate the needed control input and these results are given in Theorems 2.11 and 2.13 p.237 and corollaries. Similar results for the time varying case may be found in Theorem 2.4 p. 231. Theorem 2.17 p.240 describes important tests for reachabilty-see also Lemma 2.7 where the role of the Gram matrix is explained.

Section 3.1A pp. 219-223, introduce observability/constructibility focusing on time invariant, discrete time systems. In this way the role of the observability matrix O is easily seen and the initial state is easily calculated. Duality is discussed in pp.222-223. Similarly to the reachability case above, at this point one could continue and finish the discrete time case or study the continuous time case first. Pursuing the first option, in section 3.3C pp. 257-262, prove Theorem 3.11 p.258 (unobservable states) and discuss Corollary 3.12. Theorem 3.16 p.261 explains the relation between observability and constructibility. Continuing with observability for continuous time systems in section 3.3B pp.252-257 (using some definitions and results from the time varying case in section 3.3A pp. 248-252) introduce the observability Gramian (Definition 3.10). Lemma 3.6 shows that unobservability of a state does not depend on the time T one observes the output and input and introduces the observability matrix. The Gramian is used to show how to calculate the initial state (and a point is made why in practice one needs other methodologies-observers are discussed in Chapter 4) and these results are given in Theorems 3.7 p.253 and Corollary 3.8. Similar results for the time varying case may be found in Corollary 3.2. Theorem 3.10 p.256 describes important tests for observability-see Lemma 2.7 where the role of the Gram matrix is explained.

Standard forms for uncontrollable and unobservable systems and the Kalman's decomposition are covered next in section 3.4A pp. 263-272. This is important in relating the state space and input and output descriptions in section 3.4C pp.275-278, and proving a very useful test for controllability and observability in section 3.4 B pp. 272-275.

At this point one could continue in Chapter 3, discuss the controller and observer forms in Section 3.4D pp.278-298 and the Structure Theorems 4.10 and 4.11 p.292 and 297 or bring in and cover these topics as needed. We will follow the latter approach and cover realization in Chapter 5 and state feedback and state estimation in Chapter 4. The order of these topics may be reversed.

CHAPTER 5
Realization Theory and Algorithms.
Focus on the time invariant case, discuss Theorem 2.1 p. 387 in section 5.2, and present the existence Theorem 3.3 p.391. Then discuss minimality of realization in section 5.3B pp.394-397 via Theorems 3.9 (prove) and 3.10. The order of minimal realizations in Section 3.4C pp. 397-401 is very useful, particularly the results involving the pole and minimal polynomial of H(s)-see Theorem 3.11. Several realization algorithms may be found in Section 5.4 pp. 402-424. One could cover Section 3.4C (realizations with A diagonal) pp.417-418 and Section 3.4B (realizations in Controller/Observer Form) pp. 404-416; one could restrict the coverage to the scalar case if there is no time to discuss controller/observer forms by going back to Chapter 3-see above. The Controller/Observer forms are also useful in eigenvalue assignment in state feedback and state estimation as discussed next.

CHAPTER 4
State Feedback and State Observers
State feedback is discussed in Section 4.2 pp. 326-349. Open and closed loop control is discussed p.327. Prove Theorem 2.1 p.329 in Section 4.2B that relates controllability to eigenvalue assignment via state feedback. Several methods for eigenvalue assignment are discussed in pp. 330-342 including methods using he controller form and the desired closed lop eigenvectors.. Exercise 4.2 on p.373 introduces eigenvalue assignment by reducing the systems to a single input controllable system. The Linear Quadratic Regulator is very briefly discussed in section 4.2C pp.342-345 (see pp.348-349 for the discrete time case). Additional material on optimal control enhanced by computer exercises may be added here.
State observers are discussed in Section 4.3 pp.350-363. Full order observers, full and partial state are covered in section 4.3A pp.350-355 and reduced order observers are briefly discussed in section 4.3B. Optimal estimation is introduced in Section 4.3C pp.357-358and the discrete case is covered in Section 4.3F (and 4.3D). Additional material on optimal estimation enhanced by computer exercises may be added here. In Section 4.4 pp.363-370 the state feedback laws are combined with state estimation to derive observer based dynamic controllers. State-space and transfer function analyses in Sections 4.4A and B are used gain insight into the behavior of the compensated system.

PART III
ADDITIONAL TOPICS

In Chapter 6, BIBO Stability and more on Lyapunov Stability
The material in this chapter may be also useful in advanced courses on control and nonlinear systems. The relation between the eigenvalues of A and the poles of H(s) can be used to show that asymptotic stability implies BIBO stability [(Re{all eigenvalues of A}<0) implies (Re{all poles of H(s)}<0)] but not necessarily vice versa.

In Chapter 7, Polynomial Matrices and the Diophantine equation, polynomial and fractional descriptions of systems, feedback control and characterization of all stabilizing controllers. This material in this chapter may be useful in advanced control courses that discuss the H infinity approach to optimal control.

FURTHERMORE
The Appendix pp. 645-660 contains useful material regarding numerical considerations. The detailed Index pp.661-670 greatly helps in locating desirable topics.

Panos Antsaklis