W.A. Wolovich and P. J. Antsaklis, "Output Feedback Compensation in Multivariable Systems," Paper No. WP5.5, Proc. of the 1974 IEEE Conf. on Decision and Control, Phoenix, AZ, Nov. 20-24, 1974.

PAPER NO. WP5.5

OUTPUT FEEDBACK COMPENSATION IN MULTIVARIABLE SYSTEMS

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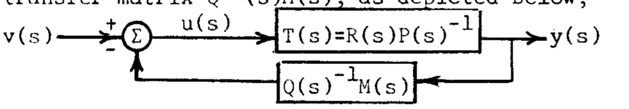
Abstract

The question of dynamic feedback compensation in linear multi-variable systems is considered from a frequency domain point of view. Two related approaches to this problem are presented, both of which employ "dual relatively prime factorizations"; i.e. either $R(s)P(s)^{-1}$ or $\bar{P}(s)^{-1}\bar{R}(s)$, of the transfer matrix, T(s), of the system. The first approach employs the notion of the "eliminant matrix" of two relatively right prime (r.r.p.) polynomial matrices, such as R(s) and P(s), while the second uses the properties of the class of unimodular matrices which "reduce" the composite matrix $\begin{bmatrix} R(s) \\ P(s) \end{bmatrix}$ to an upper right identity, $\begin{bmatrix} I \\ 0 \end{bmatrix}$.

More specifically, employing the first approach, the transfer matrix, T(s), of a p-output, m-input linear, time invariant dynamical system is first expressed in the "factored form" $R(s)P(s)^{-1}$, with R(s) and P(s) relatively right prime and the degree of the determinant of P(s) equal to n; i.e. $n = \partial|P(s)|$. A method is then presented for obtaining a proper (output to input) feedback compensator, with transfer matrix $Q^{-1}(s)M(s)$, as depicted below,



such that all n+q $(=\partial|P(s)|+\partial|Q(s)|)$ closed loop system poles, which are shown to be equal to the zeros of the determinant of M(s)R(s) + Q(s)P(s), can be arbitrarily positioned in the asymptotically stable half-plane $\mathcal{Re}\ (s) < 0$. The method outlined is a constructive one and is based on manipulation of the "eliminant matrix" of R(s) and P(s).

We also constructively show that when the given system is single input controllable, a proper dynamic compensation of total order ν-1, where ν is the observability index of the system, can be used to completely and arbitrarily position all n+ν-1 poles of the closed loop system. The dual of this result, from the viewpoint of single output observability and the controllability index, μ is also presented and an extension based on a result due to F. M. Brasch and J. B. Pearson ("Pole Placement using Dynamic Compensators", IEEE Transactions on Automatic Control, Vol. AC-15, No. 1 (1970) pp. 34-43) then enables us to present a direct, frequency domain proof of the fact that all (n+q) closed loop poles of any minimal system can be arbitrarily assigned via a proper feedback compensator of total order q=min (ν-1, μ-1).

By employing the second approach, the class of unimodular matrices which "reduces" the composite matrix $\begin{bmatrix} P(s) \\ R(s) \end{bmatrix}$ to an upper right identity, $\begin{bmatrix} I \\ 0 \end{bmatrix}$, is shown to play an important role in characterizing dual factorizations, $\bar{P}^{-1}(s)\bar{R}(s)$, with $\bar{P}(s)$ and $\bar{R}(s)$ relatively left prime, of a given transfer matrix, $T(s)=R(s)P^{-1}(s)$. It is then shown that if $\bar{M}(s)\bar{Q}^{-1}(s)$ is any dual factorization of $Q^{-1}(s)M(s)$, then the determinant of the composite matrix $\begin{bmatrix} P(s) & -\bar{M}(s) \\ R(s) & \bar{Q}(s) \end{bmatrix}$ is equal to the determinant of $M(s)R(s)+Q(s)P(s)$, except for a nonzero scalar multiplier. A relation between an appropriate class of unimodular matrices and the output feedback compensation problem is then established. Some important properties of these unimodular matrices are presented, and their significance in the treatment of the output feedback compensation problem is discussed. The significance of this result is further exploited through the development of a synthesis technique for achieving a more general class of feedback compensators, including those which involve output differentiation. This synthesis technique is shown to be capable of producing virtually any number of closed loop poles, all of which can be arbitrarily assigned. The application of this synthesis technique to the important "static output pole placement question:; i.e. the question of whether or not the closed loop poles of a given system can be arbitrarily assigned via constant gain output feedback is then demonstrated by example.