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ON MULTIVARIABLE REGULATORS. AN EXPLICIT DESCRIPTION

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The problem of tracking and regulation in multivariable systems is considered and the class of output dymanic compensators which regulate and at the same time stabilize the closed loop system is explicitly determined. Different versions of the problem have been studied by a number of authors including W.M. Wonham and J.B. Pearson [1]; G. Bengtsson [2]; L. Cheng and J.B. Pearson [3]; W.A. Wolovich and P. Ferreira [4]; P.J. Antsaklis and J.B. Pearson [5]. This work deals with a generalized version of the problem (more general model); necessary and sufficient conditions for the existence of solutions are derived and the appropriate compensators are parametrically characterized thus allowing the study of their structural properties.

More specifically, the system

$$y = -H_1 u + D_1 w$$

$$z = H_2 u + D_2 w$$

is considered where y is the (pxl) measured output, z is the (rxl) output to be regulated, w is the (qxl) disturbance and u the (mxl) control input. H_1, H_2, D_1 and D_2 are rational matrices. Let $H_1 = B_1 A_1^{-1}$ with B_1, A_1 rel. right prime polynomial matrices; a control law $u = Cy$ $(=Q_c^{-1}P_c y)$ must be determined so

(a) $Q_c A_1 + P_c B_1 = A_d$ a stable polynomial matrix

and (b) the transfer matrix from w to z is stable.

Condition (a) deals with the stability of the closed loop system while (b) with the regulation. Under the assumption that $H_2 A_1$ is stable, it is shown that this problem has a solution iff

(i) $Q_2^{-1} Q_1 = N$ a polynomial matrix

and (ii) there exist polynomial matrices a_1 and a_2 such that

$$B_3 a_1 + a_2 \tilde{Q}_1 = D$$

where the polynomial matrices Q_1, Q_2, B_3, $\tilde{Q}_1$ and D are determined from H_1, H_2, D_1 and D_2 using prime factorizations. Note that (i) and (ii) are greatly simplified if less general versions of the problem are considered (e.g. In the work by G. Bengtsson and by W.A. Wolovich, (i) is always satisfied and the equation in (ii) has the identity matrix on the right hand side). In order to characterize all the solutions, it is shown that (ii) is satisfied iff there exist polynomial matrices d_1 and d_2 which satisfy

$$\hat{C}_v d_1 + d_2 \tilde{Q}_1 = -\hat{A}_v D$$

where $(H_2 A_1) A_d^{-1} = \hat{A}_v^{-1} \hat{C}_v$ with $\hat{A}_v$, $\hat{C}_v$ rel. left prime polynomial matrices and A_d *any* stable polynomial matrix. Using this result, it is then shown that if a solution exists then Q_c, P_c of the compensator are given by:

$$[Q_c, P_c] = [A_d, d_1] U \; ; \quad U = \begin{bmatrix} X_1 & Y_1 \\ -B & A \end{bmatrix}$$

where d_1 satisfies the above equation and the (unimodular) matrix U satisfies

$$U \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}.$$ In view of the fact that

$$U^{-1} = \begin{bmatrix} A_1 & -Y \\ B_1 & X \end{bmatrix}$$ [6] it is clear that Q_c, P_c satisfy

$$Q_c A_1 + P_c B_1 = A_d$$

$$Q_c(-Y) + P_c X = d_1$$

which clearly shows the dual role of the compensator i.e. stabilization (first equation) and regulation (second equation). Using these expressions it is then shown that for any choice of A_d, the compensator must be of the form

$$C = G_d^{-1} \hat{Q}_c^{-1} \hat{P}_c G_n$$

where G_n and G_d introduce in the loop characteristics of the exogenous signal necessary for regulation. This result generalizes known results of the single-input single-output case. Finally it is shown that appropriate compensators are given by

$$Q_c = A_d \bar{X}_1 - \hat{W}(\tilde{Q}_1 B)$$

$$P_c = A_d \bar{Y}_1 + \hat{W}(\tilde{Q}_1 A)$$

where $\hat{W}$ is an arbitrary polynomial matrix; methods to derive proper compensators are also discussed.

A simple example is now given as an illustration of the above.

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Example Given

$$y = \frac{1}{s-1}u + \left[\frac{1}{(s-1)(s-2)}, \frac{1}{s^2}\right]w$$

$$z = \frac{1}{s-1}u + \left[\frac{1}{(s-1)(s-2)}, 0\right]w$$

conditions (i) and (ii) are satisfied since:

$$\text{(i)} \quad Q_2^{-1}Q_1 = \begin{pmatrix} s-2, & 0 \\ 0, & 1 \end{pmatrix} = N$$

and (ii) $a_1 + a_2 s^2(s-2) = -\frac{1}{4}(3s+2)s-2)$

Appropriate compensators will be of the form

$$C = \frac{s^2}{s-2}\hat{Q}_c^{-1}\hat{P}_c \quad (G_n = s^2;\ G_d = s-2)$$

for any choice of A_d. Actually Q_c, P_c are given by:

$$Q_c = A_d[\tfrac{1}{4}(3s+2)(s-2)] + \hat{W}[s^2(s-2)]$$

$$P_c = A_d[\tfrac{1}{4}(3s-7)s^2] + \hat{W}[(s-1)(s-2)s^2]$$

For closed loop poles at $-1\pm j$, -2, -3(i.e. $A_d = s^4 + 7s^3 + 18s^2 + 22s + 12$)

$$C = \frac{10s^2(s-7)}{(s-2)(s^2+20s+6)}$$

which is of minimal order and proper.

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