Proceedings of the 34th
Conference on Decision & Control
New Orleans, LA - December 1995

Feedback Petri Net Control Design In the Presence of Uncontrollable Transitions

John O. Moody, Panos J. Antsaklis, and Michael D. Lemmon
Department of Electrical Engineering
University of Notre Dame, Notre Dame, IN 46556
Email: jmoody@maddog.ee.nd.edu

Abstract

This paper describes a computationally efficient method for synthesizing feedback controllers for plants modeled by Petri nets which may contain uncontrollable transitions. The controller, a Petri net itself, enforces a set of linear constraints on the plant. The original set of plant behavioral constraints is transformed to yield a controller which enforces the original constraints without influencing any uncontrollable transitions.

1 Introduction

A method is proposed in this paper for transforming a set of linear constraints on the behavior of a plant modeled by a Petri net into an equivalent set which accounts for uncontrollable transitions; see [3] for an alternative methodology based on integer programming. The control goal is to realize a set of n_c constraints of the form $L\mu_p \leq b$ where μ_p is the marking vector of the Petri net modeling the process, $L \in \mathbb{Z}^{n_c \times m}$, $b \in \mathbb{Z}^{n_c}$, m is the number of places in the plant and $\mathbb{Z}$ is the set of integers. The inequality is with respect to the individual elements of the two vectors $L\mu_p$ and b. In [4] it is shown how these constraints can be enforced by a Petri net controller with incidence matrix D_c and marking μ_c placed in a feedback loop with the plant such that the incidence matrix D and marking μ of the closed loop system are

$$D = \begin{bmatrix} D_p \\ D_c \end{bmatrix} \qquad \mu = \begin{bmatrix} \mu_p \\ \mu_c \end{bmatrix} \tag{1}$$

The controller Petri net is calculated using

$$D_c = -LD_p \qquad \mu_{c_0} = b - L\mu_{p_0} \tag{2}$$

where μ_{c_0} and μ_{p_0} are the initial markings of the controller and plant respectively. The controllers produced are identical to the monitors [2] of Giua et al. derived independantly via an alternative method.

2 Handling Uncontrollable Transitions

Uncontrollable transitions in a plant may not receive any arcs from the places which make up the the external Petri net controller. Let $D_u \in \mathbb{Z}^{m \times n_u}$ be the incidence matrix of the uncontrollable portion of the process net, where n_u is the number of uncontrollable transitions. Assuming no self loops, the Petri net controller given by $D_c = -LD_p$ violates the uncontrollability constraint if LD_u contains any elements greater than zero. If we are unable to meet the uncontrollability constraint, then it is necessary to transform the constraint vector L such that the original constraint of $L\mu_p \leq b$ is still maintained, while obeying the uncontrollability constraint.

Lemma 1.

$$\text{Let } R_1 \in \mathbb{Z}^{n_c \times m} \text{ satisfy } R_1\mu_p \geq 0 \ \forall \ \mu_p. \tag{3}$$

$$\text{Let } R_2 \in \mathbb{Z}^{n_c \times n_c} \text{ p.d. diagonal matrix} \tag{4}$$

If $L'\mu_p \leq b'$ where

$$L' = R_1 + R_2 L \tag{5}$$

$$b' = R_2(b + \mathbf{1}) - \mathbf{1} \tag{6}$$

and $\mathbf{1}$ is an n_c dimensional vector of 1's, then $L\mu_p \leq b$.

Proof. The transformed constraint is $(R_1 + R_2L)\mu_p \leq R_2(b+\mathbf{1}) - \mathbf{1}$. Because all of the elements are integers, the inequality can be transformed into a strict inequality: $(R_1 + R_2L)\mu_p < R_2(b + \mathbf{1})$. Because R_2 is diagonal and positive definite, $R_2^{-1}R_1\mu_p + L\mu_p < b + \mathbf{1}$. Assumptions (3) and (4) imply that all elements of the vector $R_2^{-1}R_1\mu_p \geq 0$, therefore $L\mu_p \leq b$. □

Proposition 2. Let a plant Petri net with incidence matrix D_p be given with a set of uncontrollable transitions, a set of linear constraints $L\mu_p \leq b$ on the net marking. Assume R_1 and R_2 meet (3) and (4) and let

$$\begin{bmatrix} R_1 & R_2 \end{bmatrix} \begin{bmatrix} D_u \\ LD_u \end{bmatrix} \leq 0 \tag{7}$$

Then the controller

$$D_c = -(R_1 + R_2L)D_p \tag{8}$$

$$\mu_{c_0} = R_2(b + \mathbf{1}) - \mathbf{1} - (R_1 + R_2L)\mu_{p_0} \tag{9}$$

causes all subsequent markings of the closed loop system (1) to satisfy the constraint $L\mu_p \leq b$.

Proof. According to (2), equations (8) and (9) define a controller that enforces the constraint $L'\mu_p \leq b'$. Lemma 1 shows that if assumptions (3) and (4) are met then a controller which enforces a particular constraint $L'\mu_p \leq b'$ will also enforce the constraint $L\mu_p \leq b$. Because R_1 and R_2 satisfy inequality (7), no controller arcs are drawn to the uncontrollable transitions. □

0-7803-2685-7/95 $4.00

The usefulness of proposition 2 lies in whether or not it is possible to find R_1 and R_2 which meet the necessary assumptions. If R_1 and R_2 which satisfy (3) and (4) do exist, then they can be found by performing row operations on $\begin{bmatrix} D_u \\ LD_u \end{bmatrix}$.

3 Unreliable Machine Example

The example presented here is partially based on the model of an "unreliable machine" from [1]. The machine is used to process parts from an input queue; completed parts are moved to an output queue. The machine is called unreliable because it is possible that it may break down and damage a part during operation. Damaged parts are moved to a separate queue from the queue for completed parts. The model of the plant is shown in Figure 1; the places are described in Table 1.

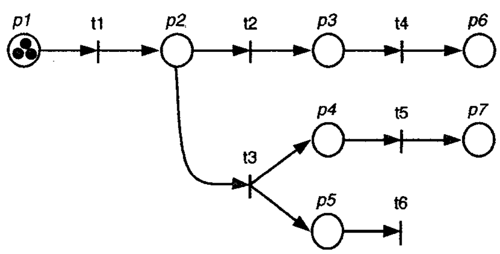

Figure 1: Petri net model of the unreliable machine.

Places

p_1	Input queue
p_2	Machine is "up and busy" processing part
p_3	Part is waiting for transfer to p_6
p_4	Part is waiting for transfer to p_7
p_5	Machine is waiting to be repaired
p_6	Completed parts queue
p_7	Damaged parts queue

Table 1: Place descriptions for the Petri net of Figure 1.

The plant model has two uncontrollable transitions, t_2 and t_3. Transition t_3 represents machine break down and so obviously can not be controlled. Transition t_2 is considered uncontrollable because the controller can not force the machine to instantly finish a part that is not yet completed, nor does it direct the machine to stop working on an unfinished part.

If the machine is broken, we do not want to load a new part until repairs have been completed. This means that places p_2 and p_5 should contain at most one token: $\mu_2 + \mu_5 \leq 1$. Parts waiting to be transferred to a storage queue, whether completed or damaged, wait in the same position on the machine. In order to prevent conflict, the second constraint is $\mu_3 + \mu_4 \leq 1$

A check of the uncontrollability condition shows that LD_u contains positive elements. Row operations are performed to find appropriate values for R_1 and R_2 which yield the transformed constraints that do not influence the uncontrollable transitions. The controlled net is shown in Figure 2.

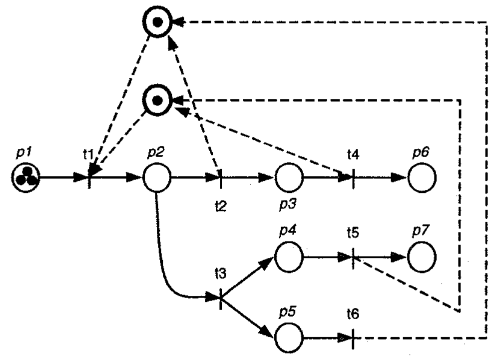

Figure 2: The controlled unreliable machine.

4 Conclusions

This paper has presented a particularly simple method for constructing feedback controllers for untimed Petri nets in the face of uncontrollable plant transitions. The method is based on the idea that row operations on a matrix containing the uncontrollable columns of the plant incidence matrix can be used to eliminate controller use of illegal transitions. The significance of this particular approach to Petri net controller design is that the control net can be computed very efficiently, thus the method shows promise for controlling large systems, or for recomputing the control law online due to a plant failure.

References

[1] A. A. Desrochers and R. Y. Al-Jaar, *Applications of Petri Nets in Manufacturing Systems*, IEEE Press, Piscataway, NJ, 1995.

[2] A. Giua, F. DiCesare, and M. Silva, "Generalized Mutual Exclusion Constraints on Nets with Uncontrollable Transitions", *Proceedings of the 1992 IEEE International Conference on Systems, Man, and Cybernetics*, Chicago, IL, pp. 974–979, October 1992.

[3] Y. Li and W. M. Wonham, "Control of Vector Discrete-Event Systems II – Controller Synthesis," *IEEE Transactions on Automatic Control*, 39(3), March 1994.

[4] J. O. Moody, K. Yamalidou, M. Lemmon, and P. J. Antsaklis, "Feedback Control of Petri Nets Based on Place Invariants", *Proceedings of the 33rd Conference on Decision and Control*, Vol. 3, pp 3104–3109, Lake Buena Vista, FL, December 1994.