A NEW STRATEGY FOR RECONFIGURABLE CONTROL SYSTEMS

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Abstract

An optimization strategy for the problem of control reconfiguration in response to operating condition changes or abrupt system component failures is presented here. The proposed approach provides an output feedback controller that not only stabilizes the new/impaired system, when possible, but also preserves much of the dynamics of the original system. The design is such that the closed-loop system is robust with respect to inevitable uncertainties/modelling errors in the state-space matrices of the impaired system. The approach is applied to an aircraft longitudinal control system, for which two severe cases of failure are considered, first the loss of an actuator and then the loss of a sensor in addition to the actuator loss.

1 Motivation-Previous work

Reconfigurable control systems are control systems that are characterized by the ability to perform in the presence of drastic changes in the system dynamics due, for example, to abrupt system component (actuator/sensor) failures or rapid changes in the operating conditions. Their task is twofold; first they need to guarantee safe performance (stability), when possible, and then recover maximum control performance under impairments. Established techniques exist for the cases of anticipated failures/operating condition changes, for which control laws are precomputed, stored and used according to need. However, the interest here is mainly for the cases of unanticipated scenarios, where an automated on-line failure accommodation technique is needed. Here, we are primarily interested in the reconfiguration part. That is, for the cases of component failures, a failure detection and identification scheme is assumed to provide the dynamics (state-space description) of the impaired system; for the cases of operating condition changes, an online modelling technique is assumed to identify the state-space model that corresponds to the new operating conditions. Once the model of the new/impaired system is available, we present a reconfiguration technique to maintain its stability and performance.

Several approaches for aircraft flight control problems have appeared in the literature, [9], [13], [14], [15]. Note also that an interesting overview of the reconfiguration problem for aircraft flight control systems can be found in [6]. Alternative approaches to reconfigurable control are presented in [2], [4], [19], [20]. In all the papers above, reconfiguration
is either just a part of a more general adaptive or FDI/stability robustness scheme or treated as an uncertainty that enters the system or restricted to some specific classes of failures for which a control law may be stored and used upon need. However, the requirement of maintaining the dynamics of the original closed-loop system is not explicitly included in the reconfiguration design procedure. This is the way reconfiguration is treated in the papers that follow.

In [3], [16], [17], the Pseudo-inverse method (PIM) is used to compute the control law for the impaired system. This method relies on the fact that the new feedback gain based on the pseudo-inverse theory is optimal in the sense that it is the solution of smallest norm for the linear least-squares problem of minimizing the Frobenius norm of the difference matrix between the original and the impaired closed-loop system transition matrices. The main problem, however, is that the stability of the impaired closed-loop system cannot be guaranteed. This problem was overcome in [7], where the reconfigurable control problem was formulated as a constrained minimization problem and a modified pseudo-inverse method (MPIM) was proposed that guarantees the stability of the closed-loop system. This approach, however, has two drawbacks. First it assumes full state-feedback, which can be quite unrealistic at times; and it relies on some stability bounds that may give very conservative results; this results in certain limitations of the proposed scheme. More recently, an approach was presented in [10], where the full-state measurability (state-feedback) is relaxed and the output feedback case is considered. There are, however, two major drawbacks in the proposed technique. First, the stability of only $Max(m, r)$ eigenvalues of the closed-loop system is maintained, where $(m, r)$ are the numbers of inputs, outputs respectively. Note that similar limitations are encountered in [8]. The second drawback concerns the fact that the proposed methodology relies on the assumption that the input matrix $B_I$ of the impaired model is of full column rank. This is a restrictive assumption, considering the common case of actuator loss which corresponds to zeroing a whole column of the input matrix.

Here, we consider the output feedback case and propose an optimization strategy which guarantees the stability of all the closed-loop eigenvalues, even in the case of severe failure scenarios, such as the simultaneous loss of an actuator and a sensor. Note that this happens under the assumption that a stabilizing solution exists for the impaired state-space model. The new stabilizing feedback controller for the impaired system captures as much of the dynamics of the original system as possible, since it is designed to minimize the Frobenius norm of the difference matrix between the original and the impaired closed-loop transition matrices. Another useful feature of the proposed design is that it is robust with respect to modelling errors in the state-space matrices of the new/impaired system. In other words, the realistic possibility of imperfect modelling of the impaired system is incorporated in our design and the controller derived by the proposed algorithm is capable of maintaining the closed-loop stability even in the presence of uncertainty in the state-space matrices of the impaired system.

2 Problem Formulation

We consider the linear multivariable continuous system with the state-space description

\[ \dot{x}(t) = A \ x(t) + B \ u(t), \quad y(t) = C \ x(t) \]  

(1)

and assume static output feedback $u(t) = K \ y(t) = KC \ x(t)$. The above gain matrix has been selected so that it guarantees a certain control performance. Suppose
now that due to system component failures (e.g., actuator or sensor failure/loss) or operating condition changes, the previous state-space representation can no longer model the dynamics of the new/impaired plant, which is now described by

\[
\dot{x}(t) = A_f x(t) + B_f u(t), \quad y(t) = C_f x(t)
\]  

(2)

Our objective is to design fast a new stabilizing output feedback control law \( u(t) = K_f y(t) = K_f C_f x(t) \), so that the new closed-loop system \( A_f + B_f K_f C_f \) captures as much of the dynamics of the nominal closed-loop system \( A + B K C \) as possible. Hence, we need to find a new gain matrix that minimizes the Frobenius norm of the difference between the nominal and the new closed-loop system transition matrices. Therefore, the minimizing quantity is

\[
J_{11} = \| A + B K C - A_f - B_f K_f C_f \|^2_F
= \text{Tr} \left[ (A + B K C - A_f - B_f K_f C_f)^T (A + B K C - A_f - B_f K_f C_f) \right]
\]  

(3)

where \( \| A \|_F \) and \( \text{Tr}(A) \) denote the Frobenius norm and the trace of a matrix \( A \) respectively. We know that given an asymptotically stable matrix \( A \) and a symmetric positive definite matrix \( Q \), there exists a unique symmetric positive definite matrix \( P \), such that \( A^T P + P A + Q = 0 \). The new gain matrix \( K_f \) needs to be stabilizing, that is it has to make \( A_f + B_f K_f C_f \) stable. Therefore, according to above, it suffices to satisfy the following Lyapunov equation

\[
\hat{A}_f^T P + P \hat{A}_f + Q = 0
\]  

(4)

where \( \hat{A}_f = A_f + B_f K_f C_f \). By including (4) in (3), we have a constrained minimization problem. Therefore, the minimizing quantity is given by \( J_1 = J_{11} + \text{Tr} \left[ L_1 (\hat{A}_f^T P + P \hat{A}_f + Q) \right] \), where \( L_1 \in \mathbb{R}^{n \times n} \) is the Lagrange multiplier matrix. In the analysis above, there is the underlying assumption that we know exactly the state-space matrices of the impaired model. This is not usually the case in applications, where we can only approximate the state-space matrices of the impaired system in cases of abrupt operating condition changes or severe failures. Hence, it is imperative that we design a controller that stabilizes the closed-loop system, even in the presence of uncertainty in some or all the state-space matrices \( \{ A_f, B_f, C_f \} \) of the closed-loop system \( A_f + B_f K_f C_f \). We need the following theorem from [5]

**Theorem 2.1** Consider \( \dot{x}(t) = Ax(t) \) with \( A \) a stability matrix; let \( A^T P + PA + Q = 0 \). Suppose that \( A \to A + \Delta A \), then \( \dot{y}(t) = (A + \Delta A) y(t) \) remains asymptotically stable if

\[
(\Delta A) Q^{-1} (\Delta A)^T < \frac{1}{4} P^{-1} Q P^{-1} \quad \text{or} \quad (\Delta A)^T P Q^{-1} P (\Delta A) < \frac{1}{4} Q
\]  

(5)

We can easily see that a sufficient condition for (5) to hold is

\[
\sigma_{\max}(\Delta A) < \frac{1}{2} \frac{\sigma_{\min}(Q)}{\sigma_{\max}(P)}
\]  

(6)

We can use (6) for \( A = \hat{A}_f \) to maintain stability in cases of inevitable uncertainties in the closed-loop system \( A_f + B_f K_f C_f \). Since \( Q \) is selected beforehand, in order to
maximize the stability bound of (6), we simply need to minimize \( \sigma_{\text{max}}(P) \). Since 
\( \sigma_{\text{max}}^2(A) \leq \| A \|_2^2 = \text{Tr}(A^T A) \) for any matrix \( A \), we choose to minimize 
\( J_2 = \text{Tr}(P^T P) = \text{Tr}(P^2) \). Considering \( J_1 \) and \( J_2 \), the overall minimizing quantity is finally given by

\[
J = \text{Tr}
\left[
(\hat{A}_f - B_f K_f C_f)^T (\hat{A}_f - B_f K_f C_f) + L_1 (\hat{A}_f^T P + P \hat{A}_f + Q) + P^2
\right]
\tag{7}
\]

where \( \hat{A}_f = A + BKC - A_f \). We compute the following gradients of the final cost \( J \)

\[
\frac{\partial J}{\partial L_1} = \Delta_{L_1} = \hat{A}_f^T P + P \hat{A}_f + Q
\tag{8}
\]

\[
\frac{\partial J}{\partial P} = \Delta_P = \hat{A}_f L_1^T + L_1^T \hat{A}_f^T + 2P
\tag{9}
\]

\[
\frac{\partial J}{\partial K_f} = \Delta_{K_f} = 2B_f^T B_f K_f C_f C_f^T - 2B_f^T \hat{A}_f C_f^T + B_f^T L_1^T C_f^T + 2B_f^T P L_1 + L_1 C_f^T
\tag{10}
\]

These gradients can be used by the algorithm of [11] to minimize (7). Note that this algorithm uses a version of the Broyden family method of conjugate directions, which is based on the Broyden-Fletcher-Goldfarb-Shanno (BFGS) update rule; a version of this algorithm has been used in [12] for the design of controllers for robust stability and optimal performance of uncertain discrete-time systems.

**Remark 2.1** The minimizing quantity of (7) consists of two components, the reconfiguration term \( (J_1) \) and the robustness term \( (J_2) \). By assigning weights to these terms, we could emphasize the one that is of more interest to us. Without loss of generality, we can always consider a weight \( \omega_1 = 1 \) for \( J_1 \), so that the minimizing quantity is given by \( J_w = J_1 + \omega_2 J_2 \). Note that the introduction of \( \omega_2 \) affects only the gradient of (9), where the term \( 2P \) needs to be substituted by \( 2\omega_2 P \).

**Remark 2.2** We have considered the robustness of the closed-loop system \( \hat{A}_f \) assuming possible uncertainties in all the state-space matrices of the impaired system. If we are certain for some of these matrices, then the bound of (6) can be easily modified. For instance, let's consider a common case in reconfigurable systems, where the state and output matrices remain the same \( A = A_f, C = C_f \) and we only have changes in \( B \). Then, the allowable perturbations in \( B_f \) are given from (6) by

\[
\sigma_{\text{max}}(\Delta B_f) < \frac{1}{2} \frac{\sigma_{\text{max}}(Q)}{\sigma_{\text{max}}(P) \sigma_{\text{max}}(K_f C_f)}. 
\]

By minimizing \( \sigma_{\text{max}}(K_f C_f) \), we can further enlarge the above stability region. This can easily be done by including its upper bound \( \text{Tr}[ (K_f C_f)^T (K_f C_f) ] \) in the minimizing quantity \( J \).

**Remark 2.3** We may also wish to include in \( J \) another term that establishes a specific control performance, such as the familiar LQR cost \( J_3 = \int_0^\infty (x^T Q_1 x + u^T R_1 u) \, dt \), where \( Q_1, R_1 \) are positive definite matrices of appropriate dimensions. Finally note that the results presented here can easily be extended to the dynamic output feedback case, [12].

### 3 An illustrative example

Consider an aircraft longitudinal control system from [10], with the linearized dynamic model
\[
\begin{pmatrix}
\alpha(t) \\
\beta(t) \\
\psi(t) \\
\theta(t)
\end{pmatrix}
= \begin{pmatrix}
-0.0582 & 0.0651 & 0 & -0.171 \\
-0.303 & -0.685 & 1.109 & 0 \\
-0.0715 & -0.658 & -0.947 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix} \begin{pmatrix}
\alpha(t) \\
\beta(t) \\
\psi(t) \\
\theta(t)
\end{pmatrix}
+ \begin{pmatrix}
0 & 1 \\
-0.0541 & 0 \\
-1.11 & 0 \\
0 & 0
\end{pmatrix} \begin{pmatrix}
\eta(t) \\
\tau(t)
\end{pmatrix}
\]

\[y(t) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{pmatrix} \begin{pmatrix}
\alpha(t) \\
\beta(t) \\
\psi(t) \\
\theta(t)
\end{pmatrix}
\]  \hspace{1cm} (11)

where \(\alpha(t)\) and \(\beta(t)\) are the forward and vertical speeds, \(\psi(t)\) is the pitch rate and \(\theta(t)\) is the pitch angle. The control inputs \(\eta(t)\) and \(\tau(t)\) are the elevator angle and throttle position respectively. When we consider static output feedback, the controller that assigns the closed-loop eigenvalues at \(\{-2, -0.5973, -1.5 \pm 2j\}\) is given by

\[
K = \begin{pmatrix}
-0.00031 & 4.77004 & 1.70457 \\
-2.01505 & -1.13002 & 0.02904
\end{pmatrix}
\]  \hspace{1cm} (12)

Next, we suppose that the system dynamics change due to operating condition variations and at the same time severe failures happen at the actuators or sensors. First, we study the case of actuator loss and then the case of both actuator and sensor losses.

### 3.1 Actuator loss

The state-space matrices of the impaired model are given below. Note the loss of the second actuator.

\[
A_f = \begin{pmatrix}
-0.0582 & 0.10 & 0.0 & -0.171 \\
-0.103 & -0.685 & 1.109 & 0 \\
-0.0715 & -0.658 & 1.98 & 0 \\
0 & 0 & 1.5 & 0
\end{pmatrix}, \quad B_f = \begin{pmatrix}
0 & 0.0 \\
-0.09 & 0.0 \\
-1.11 & 0.0 \\
0 & 0.0
\end{pmatrix}
\]

\[
C_f = \begin{pmatrix}
0.9 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.7 \\
0 & 0 & 1 & 0
\end{pmatrix}
\]  \hspace{1cm} (13)

Note that the state-space matrices given above correspond to new operating conditions, as given in [10]; in addition, we imposed the loss of one of the actuators. We use the algorithm of [12] for different weights to the robustness term, so that the minimizing quantity is given by \(J_w\). In Table 1, we give the closed-loop eigenvalues and the first row of the stabilizing output feedback gain for the original and several impaired models. Note that for the impaired models the second row of the controller becomes irrelevant due to the actuator loss. In Table 2, we give the results for the reconfiguration and the robustness terms, and the robustness bound, which is the maximum singular value of the variations of the closed-loop system that can be allowed so that the asymptotic stability is maintained. We also restrict these variations in perturbations in the input matrix \(B\)


Table 1: Actuator loss: eigenvalues and feedback gain for the nominal and the impaired models.

<table>
<thead>
<tr>
<th>model</th>
<th>eigenvalues</th>
<th>gain K (first row)</th>
</tr>
</thead>
<tbody>
<tr>
<td>nominal</td>
<td>{-2.0, -0.5973, -1.50 \pm 2.00j}</td>
<td>{-0.0031 4.77004 \ 1.70458}</td>
</tr>
<tr>
<td>imp (\omega_2 = 0.01)</td>
<td>{-0.0770, -0.5591, -1.4610 \pm 2.5498j}</td>
<td>{0.0680 6.79671 \ 4.31982}</td>
</tr>
<tr>
<td>imp (\omega_2 = 0.1)</td>
<td>{-0.0797, -0.5576, -1.4603 \pm 2.5491j}</td>
<td>{-0.0651 6.79624 \ 4.31961}</td>
</tr>
<tr>
<td>imp (\omega_2 = 1)</td>
<td>{-0.0947, -0.5491, -1.4575 \pm 2.5352j}</td>
<td>{-0.45799 6.75011 \ 4.32028}</td>
</tr>
<tr>
<td>imp (\omega_2 = 10)</td>
<td>{-0.1371, -0.5194, -1.4840 \pm 2.4060j}</td>
<td>{-1.37154 6.31477 \ 4.37946}</td>
</tr>
<tr>
<td>imp (\omega_2 = 50)</td>
<td>{-0.1779, -0.4819, -1.6283 \pm 2.1596j}</td>
<td>{-1.92021 5.75013 \ 4.64244}</td>
</tr>
</tbody>
</table>

Table 2: Actuator loss: results for different weight factors considered for the impaired model.

<table>
<thead>
<tr>
<th>impaired model</th>
<th>(J_1) (reconfiguration term)</th>
<th>(J_2) (robustness term)</th>
<th>bound (general)</th>
<th>bound for (\Delta B_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega_2 = 0.01)</td>
<td>5.7469</td>
<td>2.3324</td>
<td>0.0668</td>
<td>0.0104</td>
</tr>
<tr>
<td>(\omega_2 = 0.1)</td>
<td>5.7534</td>
<td>2.2092</td>
<td>0.0687</td>
<td>0.0107</td>
</tr>
<tr>
<td>(\omega_2 = 1)</td>
<td>5.9737</td>
<td>1.7149</td>
<td>0.0785</td>
<td>0.0122</td>
</tr>
<tr>
<td>(\omega_2 = 10)</td>
<td>7.8258</td>
<td>1.1772</td>
<td>0.0956</td>
<td>0.0151</td>
</tr>
<tr>
<td>(\omega_2 = 50)</td>
<td>10.3066</td>
<td>1.0538</td>
<td>0.1012</td>
<td>0.0159</td>
</tr>
</tbody>
</table>

and compute the same bound for \(\sigma_{max}(\Delta B_f)\). Note that the same can easily be done for the case of structured perturbations in \(B_f\). It is quite obvious that for larger \(\omega_2\), we enhance the robustness of the closed-loop system, which translates into smaller \(J_2\) and larger perturbation bounds (for \(\Delta B_f\) or general); at the same time, however, the reconfiguration term \(J_1\) increases, which results in the deterioration of the closeness of the impaired closed-loop system \(A_f + B_fK_fC_f\) to the nominal closed-loop system \(A + BK_C\).

In Fig. 1-2, we compare the state-responses of the original and the impaired closed-loop systems. For the latter, we have chosen the controller obtained by our algorithm for \(\omega_2 = 1\). The initial conditions vector was chosen as \((0.1\ 0.5\ 0.3\ 1)^T\). The two plots are very similar, which implies that despite the severity of the actuator loss, we were able to recover quite successfully the dynamics of the nominal model.

### 3.2 Actuator/sensor losses

We consider the even more severe case of losing both the second actuator and the first sensor. Hence, \(A_f\) and \(B_f\) remain the same as before, whereas the output matrix changes to
Figure 1: Nominal system: state trajectories for the closed-loop system.

Figure 2: Actuator loss: state trajectories for the impaired closed-loop system.

<table>
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<td>{-0.00031 4.77004 1.70458}</td>
</tr>
<tr>
<td>imp ((\omega_2 = 0.01))</td>
<td>{-0.0773, -0.5589, -1.4610 \pm 2.5497j}</td>
<td>{-0.00031 6.79669 4.31982}</td>
</tr>
<tr>
<td>imp ((\omega_2 = 1))</td>
<td>{-0.0773, -0.5590, -1.4594 \pm 2.5527j}</td>
<td>{-0.00031 6.80620 4.31707}</td>
</tr>
<tr>
<td>imp ((\omega_2 = 10))</td>
<td>{-0.0772, -0.5598, -1.4488 \pm 2.5762j}</td>
<td>{-0.00031 6.88535 4.29861}</td>
</tr>
</tbody>
</table>

Table 3: Actuator/sensor losses: eigenvalues/feedback gain of original/impaired models.

<table>
<thead>
<tr>
<th>impaired model</th>
<th>(J_1) (reconfiguration term)</th>
<th>(J_2) (robustness term)</th>
<th>bound (general)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega_2 = 0.01)</td>
<td>5.7471</td>
<td>2.3202</td>
<td>0.0670</td>
</tr>
<tr>
<td>(\omega_2 = 1)</td>
<td>5.7472</td>
<td>2.3200</td>
<td>0.0670</td>
</tr>
<tr>
<td>(\omega_2 = 10)</td>
<td>5.7525</td>
<td>2.3190</td>
<td>0.0670</td>
</tr>
</tbody>
</table>

Table 4: Actuator/sensor losses: different weight factors for the impaired model.

\[
\hat{C}_f = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 \\ 0 & 0 & 1 & 0 \end{pmatrix}
\] (14)

Note that these losses make the second row and the (1,1) element of the first row of the controller gain irrelevant. In Tables 3-4, we give the same results as before for the present case. Comparing with the actuator loss case, we see that the sensor loss, in addition to the actuator loss, affects the robustness of the closed-loop system. Unlike the actuator case, we can not remove the pole at -0.0773, see Table 3, no matter how much we increase \(\omega_2\) in the minimizing quantity; in the actuator loss case we were able to enhance the robustness of the closed-loop system by assigning a large weight to the robustness term \(J_2\), which resulted in removing the problematic pole from -0.0770 to -0.1779. Comparing the reconfiguration terms \(J_1\) of Tables 2 and 4, we see that the loss of the first sensor did not affect at all the reconfiguration aspect of our design. Finally note that the state-responses of the impaired closed-loop system after the loss of both the actuator and the sensor for the same initial vector as before are almost identical to the ones of Fig. 2, which shows that despite the loss of the sensor, in addition to the actuator loss, our scheme was capable of recovering the dynamics of the original system.

In Table 5, we compare the Frobenius norm of the difference between the original and the impaired closed-loop transition matrices \(||A + BKC - A_f - B_f K_f C_f||_F\) for the controllers derived in the examples of [7] and [10] and the ones derived by the proposed algorithm here for the cases of \(\omega_2 = 0.1\) and \(\omega_2 = 1\). The present approach, in addition to maintaining closed-loop stability, is more successful in preserving the characteristics of the original system compared to the techniques presented in the papers above.
Table 5: Comparing $\|A + BKC - A_f - B_fK_fC_f\|_F$ for literature examples.

4 Conclusions

The problem of control reconfiguration in response to operating condition changes or abrupt system component failures has been studied here. An optimization algorithm has been presented that provides an output feedback controller that not only stabilizes the new/impaired system but also preserves much of the dynamics of the original/unfailed system. The design is such that the closed-loop system is robust with respect to uncertainties/modelling errors in the state-space model of the impaired system.

Although the interest here is in continuous-time systems, a similar approach can be applied to the discrete-time case. Results concerning eigenstructure assignment can be included in our design so that additional restrictions/specifications can be introduced in the minimizing quantity with respect to the eigenspace where the closed-loop eigenvectors are desired/needed to vary. In that respect, the requirement of closeness to the original closed-loop system can be specialized to the case where the closeness to the eigenstructure of the original system becomes the main objective of the reconfiguration design.

References


