I.K. Konstantopoulos and P.J. Antsaklis, "An Eigenstructure Assignment Approach to Control Reconfiguration," P roc o f 4 th I EEE M editerranean S ymposium o n C ontrol a nd A utomation , pp. 328-333, Chania, Crete, Greece, June 10-14, 1996.

AN EIGENSTRUCTURE ASSIGNMENT APPROACH TO CONTROL RECONFIGURATION

Ioannis K. Konstantopoulos
Department of Engineering
University of Leicester
Leicester, LE1 7RH, U.K.
ikk@sun.engg.le.ac.uk

Panos J. Antsaklis
Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556, U.S.A.
antsakli@saturn.ee.nd.edu

Abstract

An optimization approach to control reconfiguration, based on eigenstructure assignment, for control systems with output feedback is presented. The proposed scheme preserves the $\max(r, q)$ most dominant eigenvalues of the nominal closed-loop system and determines their associated closed-loop eigenvectors as close to the corresponding eigenvectors of the nominal closed-loop system as possible. Additionally, the stability of the remaining closed-loop eigenvalues is guaranteed by the satisfaction of an appropriate Lyapunov equation. The overall design is robust with respect to uncertainties in the state-space matrices of the reconfigured system. The approach is applied to an aircraft control example, where it is shown to recover the shape of the transient response.

1. Introduction

Eigenstructure assignment is a powerful technique that has developed considerably over the last fifteen years or so; see for instance [1], [3], [11], [12], [13], [14], [15], [17], [19], [20]. This technique is concerned with the placing of eigenvalues and their associated eigenvectors, via feedback control laws, to meet closed-loop design specifications. Specifically, the method alllows the designer to directly satisfy damping, settling time and mode decoupling specifications by appropriately selecting the closed-loop eigenvalues and eigenvectors. The most popular approach to eigenstructure assignment has appeared in [1], where both cases of state and output feedback are studied and a design technique for eigenstructure assignment with output feedback is presented. Note that a list of papers dealing with eigenstructure assignment can be found in [19] and the review paper of [16].

The interest here is in control reconfiguration and the main objective is the design of a feedback law that preserves the eigenstructure characteristics describing the nominal closed-loop system. In other words, we assume changes in the operating conditions or system component failures occurring in the nominal system whose performance is determined by the nominal closed-loop eigenvalues/eigenvectors. The new control law needs to be designed such that as much of this nominal performance is recovered as possible; this can be done by recovering as much of the nominal eigenstructure information as possible. Similar to the approach presented in [6], [7], the overall design needs to be robust with respect to the matrices of the impaired state-space model. Note that the design scheme here is different from the design scheme of [6], [7], where no information regarding the eigenstructure of the nominal closed-loop system was taken into account.

2. Reconfiguration and eigenstructure assignment

2.1. Problem formulation

We consider the linear multivariable continuous system with the state-space description

$$\dot{x}(t) = A\,x(t) + B\,u(t) \qquad (1)$$
$$y(t) = C\,x(t) \qquad (2)$$

where $x \in \Re^n$ is the state vector, $u \in \Re^r$ the input vector, and $y \in \Re^q$ the output vector; $A \in \Re^{n\times n}$, $B \in \Re^{n\times r}$, $C \in \Re^{q\times n}$ are the system matrices. The above system is assumed to be both controllable and observable, that is

$$\operatorname{rank}\,[\,B \;\; AB \;\; \cdots \;\; A^{n-1}B\,] = n \qquad (3)$$
$$\operatorname{rank}\,[\,C^T \;\; A^TC^T \;\; \cdots \;\; (A^T)^{n-1}C^T\,] = n \qquad (4)$$

We also assume that the input and output matrices are of full rank, that is $\operatorname{rank}(B) = r$ and $\operatorname{rank}(C) = q$. Also, as is usually the case in aircraft problems, it is assumed that $r < q < n$. For this system, an output feedback control gain

I.K. Konstantopoulos and P.J. Antsaklis, "An Eigenstructure Assignment Approach to Control Reconfiguration," P roc o f 4 th I EEE M editerranean S ymposium o n C ontrol a nd A utomation , pp. 328-333, Chania, Crete, Greece, June 10-14, 1996.

$$u(t) = K\, y(t) = KC\, x(t) \tag{5}$$

has been selected such that the closed-loop eigenvalues are located at $\{\lambda_i, i = 1,..,n\}$ and the shape of the response is determined by the set of their associated eigenvectors $\{v_i, i = 1,..,n\}$. Note that the eigenstructure specified above characterizes the behavior of the closed-loop system since the eigenvalues determine the stability of the system and the eigenvectors the contribution of each system mode to the system (state or output) response.

Suppose that a system component (e.g. actuator or sensor) failure occurs in the system or that the operating conditions change. The state-space model of (1), (2) can no longer model the dymanics of the system, which is now described by

$$\begin{aligned} \dot{x}(t) &= A_f\, x(t) + B_f\, u(t) \qquad (6)\\ y(t) &= C_f\, x(t) \qquad (7) \end{aligned}$$

where the state-space matrices of the impaired system are of the same dimensions with the matrices of the nominal state-space model, and the previous assumptions still hold. Our objective is to design fast a new stabilizing output feedback control law

$$u(t) = K_f\, y(t) = K_f C_f\, x(t) \tag{8}$$

such that the new closed-loop system $A_f + B_f K_f C_f$ can capture as much of the eigenstructure information characterizing the nominal closed-loop system $A + BKC$ as possible. In other words, the new output feedback matrix has to be such that the shape of the response of the impaired system closely approximates the shape of the response of the nominal system.

Without loss of generality, we assume that the nominal closed-loop eigenvalues are arranged in decreasing order with respect to their real parts, that is real$(\lambda_1) \geq$ real$(\lambda_2) \geq \cdots \geq$ real(λ_n). As shown in [18], with output feedback we can only choose q closed-loop eigenvalues and partially assign the same number of closed-loop eigenvectors. Therefore, in order to maintain the performance of the nominal closed-loop system, we should determine the new control law (8) such that the set of the impaired closed-loop eigenvalues includes the q most dominant eigenvalues of the nominal closed-loop system, $\{\lambda_i, i = 1,..,q\}$. On the other hand, the eigenvectors of the impaired system that correspond to the above identical eigenvalues have to be as close to the corresponding eigenvalues of the nominal system, $\{v_i, i = 1,..,q\}$ as possible. Therefore, if we denote by $\{(\lambda_i^f, v_i^f), i = 1,..,n\}$ the closed-loop eigenvalues/eigenvectors for the impaired system, the above objectives are translated into

$$\lambda_i^f = \lambda(A_f + B_f K_f C_f) = \lambda_i = \lambda(A + BKC), \qquad i = 1,..,q \tag{9}$$

$$\min \left[\sum_{i=1}^{q} \|v_i^f - v_i\|^2 \right] \tag{10}$$

For reasons explained in [1], [8], [9], we consider the state-transformation matrix $T_f = (\, B_f \quad S_f \,)$, where S_f is selected such that rank$(T_f) = n$. In the new state-coordinates specified by T_f above, the impaired system is described by the matrices $(\tilde{A}_f, \tilde{B}_f, \tilde{C}_f)$, with

$$\tilde{B}_f = T_f^{-1} B = \begin{pmatrix} I_r \\ O_{n-r,r} \end{pmatrix} \tag{11}$$

where $O_{n-r,r}$ is defined as an $[(n-r) \times r]$ zero matrix. Note the special structure of the input matrix $\tilde{B}_f$. The desired closed-loop eigenvectors, $\{v_i, i = 1,..,q\}$ together with the actual closed-loop eigenvectors of the impaired system, $\{v_i^f, i = 1,..,q\}$ need also to be transformed to the new state-coordinates. Define

$$\begin{aligned} \tilde{v}_i &= T_f^{-1} v_i \qquad (12)\\ \tilde{v}_i^f &= T_f^{-1} v_i^f \qquad (13) \end{aligned}$$

as the desired and actual closed-loop eigenvectors for the transformed impaired system respectively. From now on, we continue our discussion considering the impaired system in the new state-coordinates specified above. Therefore, the objective of (10) for the transformed impaired system is given by

$$\min \left[\sum_{i=1}^{q} \|\tilde{v}_i^f - \tilde{v}_i\|^2 \right] \tag{14}$$

As discussed in [1], [8], [9], all achievable eigenvectors $\tilde{v}_i^f$ that correspond to the closed-loop eigenvalue λ_i^f must lie in the subspace spanned by the columns of $(\lambda_i^f I_n - \tilde{A}_f)^{-1} \tilde{B}_f$. Define

$$\tilde{\Pi}_i = (\lambda_i^f I_n - \tilde{A}_f)^{-1} \tilde{B}_f \tag{15}$$

All achievable closed-loop eigenvectors of the impaired system that correspond to the eigenvalue λ_i^f should be of the form

$$\tilde{v}_i^f = \tilde{\Pi}_i \mu_i \tag{16}$$

I.K. Konstantopoulos and P.J. Antsaklis, "An Eigenstructure Assignment Approach to Control Reconfiguration," Proc of 4th IEEE Mediterranean Symposium on Control and Automation, pp. 328-333, Chania, Crete, Greece, June 10-14, 1996.

where μ_i is an $(r \times 1)$ vector. Note that μ_i is a real vector if λ_i^f is a real eigenvalue or a complex vector if λ_i^f is a complex eigenvalue. In view of (16), the objective of (14) is rewritten as

$$\min \left[\sum_{i=1}^{q} ||\tilde{\Pi}_i \mu_i - \tilde{v}_i||^2 \right] \tag{17}$$

and the minimizing quantity is defined as

$$J_1^{'} = \mathrm{Tr}\left[\sum_{i=1}^{q} (\tilde{\Pi}_i \mu_i - \tilde{v}_i)^H \, (\tilde{\Pi}_i \mu_i - \tilde{v}_i) \right] \tag{18}$$

where v^H denotes the complex conjugate transpose of a vector v. Each pair of closed-loop eigenvalues/eigenvectors should satisfy, ([8], [9])

$$(\tilde{A}_f^1 + K_f \tilde{C}_f - \lambda_i^f I_{r,n}) \, \tilde{\Pi}_i \mu_i = 0 \tag{19}$$

where $\tilde{A}_f^1$ contains the first r rows of $\tilde{A}_f$ and

$$I_{r,n} = (I_r \qquad O_{r,n-r}) \tag{20}$$

We see that the vectors $\{\mu_i, i = 1,..,q\}$ that minimize (18) also need to satisfy the eigenstructure condition of (19). Therefore, we need to include this condition for the q eigenvectors of interest in the minimizing quantity, which becomes

$$\begin{aligned} J_1 \;\; = \;\; & \mathrm{Tr}\left\{ \sum_{i=1}^{q} (\tilde{\Pi}_i \mu_i - \tilde{v}_i)^H \, (\tilde{\Pi}_i \mu_i - \tilde{v}_i) \right. \\ & \left. + \sum_{i=1}^{q} M_i \, \left[(\tilde{A}_f^1 + K_f \tilde{C}_f - \lambda_i^f I_{r,n}) \, \tilde{\Pi}_i \mu_i \right] \right\} \end{aligned} \tag{21}$$

where $\{M_i, i = 1,..,q\}$ are $(1 \times r)$ Lagrange multiplier vectors, which are real if they correspond to a real eigenvalue or complex if they correspond to a complex eigenvalue. So far, we have concentrated on the q closed-loop eigenvalues that we wish to preserve with the procedure outlined above. Although we have no control upon the remaining $(n - q)$ eigenvalues of the closed-loop system, we need to ascertain that they remain stable. Therefore, the output feedback gain needs to be such that the closed-loop system $\tilde{A}_f + \tilde{B}_f K_f \tilde{C}_f$ is stable. In other words, it suffices to satisfy the Lyapunov equation

$$(\hat{A}_f)^T P + P \, \hat{A}_f + Q = 0 \tag{22}$$

where

$$\hat{A}_f = \tilde{A}_f + \tilde{B}_f K_f \tilde{C}_f \tag{23}$$

As discussed in [6], [7], we also need to safeguard against possible uncertainties in the state-space matrices of the impaired system. It can be shown, [6], [7], that this can be done by including the term $\mathrm{Tr}(P^2)$ in the minimizing quantity. Therefore, the overall minimizing quantity is finally given by

$$\begin{aligned} J \;\; = \;\; & \mathrm{Tr}\left\{ \sum_{i=1}^{q} (\tilde{\Pi}_i \mu_i - \tilde{v}_i)^H \, (\tilde{\Pi}_i \mu_i - \tilde{v}_i) \right. \\ & + L_1 \, \left[\hat{A}_f^T P + P \hat{A}_f + Q \right] \\ & \left. + \sum_{i=1}^{q} M_i \, \left[(\tilde{A}_f^1 + K_f \tilde{C}_f - \lambda_i^f I_{r,n}) \, \tilde{\Pi}_i \mu_i \right] + P^2 \right\} \end{aligned} \tag{24}$$

where $L_1 \in \Re^{n \times n}$ is another Lagrange multiplier matrix. To summarize the approach outlined above, we should state that with the minimization of the quantity in (24) above we seek an output feedback matrix K_f such that

- The q most dominant eigenvalues of the nominal closed-loop system belong to the set of the eigenvalues of the impaired closed-loop system $A_f + B_f K_f C_f$.
- The eigenvectors of the impaired system that correspond to the above set of closed-loop eigenvalues are as close to the corresponding eigenvalues of the nominal system as possible.
- The remaining $(n - q)$ closed-loop eigenvalues are stable.
- Possible uncertainties in the state-space matrices of the impaired system are taken care of by maximizing the stability margin allowed to the closed-loop system.

2.2. Algorithmic approach

Without loss of generality, we assume that the set of desired eigenvalues, that is the set of q most dominant eigenvalues of the nominal closed-loop system consists of a complex conjugate pair, that is $\lambda_1^f = (\lambda_2^f)^\star \in \mathcal{C}$, and $(q - 2)$ real eigenvalues, that is $\{\lambda_i^f \in \Re, i = 3,..,q\}$. Then, $\tilde{v}_1^f = (\tilde{v}_2^f)^\star$. The generalization to the case of more complex conjugate pairs of eigenvalues is straightforward.

We need to compute the partial derivatives of the minimizing quantity of (24) with respect to all the matrix parameters entailed. These parameters are the Lagrange multiplier vectors $\{M_i, i = 1,..,q\}$, the Lagrange multiplier matrix L_1, the positive definite matrix P, the output

I.K. Konstantopoulos and P.J. Antsaklis, "An Eigenstructure Assignment Approach to Control Reconfiguration," Proc of 4th IEEE Mediterranean Symposium on Control and Automation, pp. 328-333, Chania, Crete, Greece, June 10-14, 1996.

feedback matrix K_f, and the vectors $\{\mu_i, i = 1,..,q\}$ that specify the closed-loop eigenvectors. Using the properties of [2] we have

$$\frac{\partial J}{\partial M_i} = \left[(\tilde{A}_f^1 + K_f\tilde{C}_f - \lambda_i^f I_{r,n})\,\tilde{\Pi}_i\,\mu_i\right]^T, \quad i = 1,..,q \tag{25}$$

$$\frac{\partial J}{\partial L_1} = \hat{A}_f^T P + P\hat{A}_f + Q \tag{26}$$

$$\frac{\partial J}{\partial P} = \hat{A}_f L_1^T + L_1^T \hat{A}_f^T + 2P \tag{27}$$

$$\frac{\partial J}{\partial K_f} = \tilde{B}_f^T P L_1 \tilde{C}_f^T + \tilde{B}_f^T P L_1^T \tilde{C}_f^T + \sum_{i=1}^{q} M_i^T \mu_i^T \tilde{\Pi}_i^T \tilde{C}_f^T \tag{28}$$

$$\begin{aligned} \frac{\partial J}{\partial \mu_1} &= 2\,\tilde{\Pi}_2^H\,\tilde{\Pi}_2\,\mu_2 - 2\,\tilde{\Pi}_2^H\,\tilde{v}_2 \\ &\quad + \tilde{\Pi}_1^T\,(\tilde{A}_f^1 + K_f\tilde{C}_f - \lambda_1^f I_{r,n})^T\,M_1^T && (29) \\ &= \left(\frac{\partial J}{\partial \mu_2}\right)^\star && (30) \end{aligned}$$

$$\begin{aligned} \frac{\partial J}{\partial \mu_i} &= 2\,\tilde{\Pi}_i^T\,\tilde{\Pi}_i\,\mu_i - 2\,\tilde{\Pi}_i^T\,\tilde{v}_i \\ &\quad + \tilde{\Pi}_i^T\,(\tilde{A}_f^1 + K_f C_f - \lambda_i^f I_{r,n})^T\,M_i^T, \\ &\qquad i = 3,..,q && (31) \end{aligned}$$

The derivation of (29) and the equivalence of (30) are shown in [8], [9]. To minimize (24) we use a version of the BFGS (Broyden-Fletcher-Goldfarb-Shanno) optimization method of conjugate directions. Note that there are significant changes compared to similar algorithms used in [5], [6], [7], [10]. This is due to the structure of the present problem, since now we update the vectors $\{\mu_i, i = 1,..,q\}$ instead of the output feedback matrix. On the other hand, the existence of complex eigenvalues/eigenvectors imposes certain modifications to the algorithmic scheme. The proposed algorithm is presented in [8], [9]. Note that the optimal K_f determined by the above approach is the optimal gain for the impaired system in the original state-coordinates as well, whereas the optimal vectors $\{\tilde{v}_i^f, i = 1,..,q\}$ need to be transformed back to the original state-coordinates using (13). In [8], [9], the cases of $q < r$, state-feedback, and dynamic compensation are also studied.

3. An illustrative example

Consider the aircraft longitudinal control system of [4], whose linearized dynamic model is given by

$$A = \begin{pmatrix} -0.0582 & 0.0651 & 0 & -0.171 \\ -0.303 & -0.685 & 1.109 & 0 \\ -0.0715 & -0.658 & -0.947 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{32}$$

$$B = \begin{pmatrix} 0 & 1 \\ -0.0541 & 0 \\ -1.11 & 0 \\ 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{33}$$

$$x(t) = (\alpha(t) \quad \beta(t) \quad \psi(t) \quad \theta(t))^T \tag{34}$$

$$u(t) = (\eta(t) \quad \tau(t))^T \tag{35}$$

where $\alpha(t)$ and $\beta(t)$ are the forward and vertical speeds, $\psi(t)$ is the pitch rate and $\theta(t)$ is the pitch angle. The control inputs $\eta(t)$ and $\tau(t)$ are the elevator angle and throttle position respectively. When we consider the static output feedback law of (5), the controller that assigns the closed-loop eigenvalues at $\{-0.5973, -1.5 \pm j2, -2\}$ and their corresponding eigenvectors at

$$\begin{aligned} V &= (v_1 \quad v_2 = v_3^\star \quad v_4) \\ &= \begin{pmatrix} -0.1887 & 0.1465 + j0.0958 & 0.9680 \\ -0.9634 & 0.2257 - j0.2492 & 0.1441 \\ -0.0977 & 0.3790 + j0.6047 & 0.0905 \\ 0.1636 & 0.1025 - j0.2664 & -0.0453 \end{pmatrix} && (36) \end{aligned}$$

is given in [4] by

$$K = \begin{pmatrix} -0.00031 & 4.77004 & 1.70457 \\ -2.01505 & -1.13002 & 0.02904 \end{pmatrix} \tag{37}$$

Next, we suppose that the system dynamics change due to operating condition variations. The state-space matrices of the impaired model are given below.

$$A_f = \begin{pmatrix} -0.0582 & 0.10 & 0.0 & -0.171 \\ -0.103 & -0.685 & 1.109 & 0 \\ -0.0715 & -0.658 & 1.98 & 0 \\ 0 & 0 & 1.5 & 0 \end{pmatrix} \tag{38}$$

$$B_f = \begin{pmatrix} 0 & 0.9 \\ -0.09 & 0.0 \\ -1.11 & 0.0 \\ 0 & 0.0 \end{pmatrix}, \quad C_f = \begin{pmatrix} 0.9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{39}$$

The algorithmic approach discussed above is used to find the optimal output feedback matrix K_f, that is the controller gain that minimizes J of (24), for the impaired system. Our objective is to preserve the first 3 most dominant eigenvalues of the nominal closed-loop system, that

I.K. Konstantopoulos and P.J. Antsaklis, "An Eigenstructure Assignment Approach to Control Reconfiguration," Proc of 4th IEEE Mediterranean Symposium on Control and Automation, pp. 328-333, Chania, Crete, Greece, June 10-14, 1996.

is $\{-0.5973, -1.5 \pm j2\}$, and achieve closed-loop eigenvectors as close to their corresponding eigenvectors of (36) as possible. First we need to transform the impaired system (A_f, B_f, C_f) to new state-coordinates. Select

$$T = \begin{pmatrix} 0 & 0.9 & 0 & 0 \\ -0.09 & 0 & 1 & 0 \\ -1.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{40}$$

The best results, with regard to closeness of the closed-loop eigenvectors of the impaired system to the desired eigenvectors specified in (36) are obtained when we assign a weight factor of 0.1 to the term $\left\{(\tilde{\Pi}_1\mu_1 - \tilde{v}_1)^T\,(\tilde{\Pi}_1\mu_1 - \tilde{v}_1)\right\}$ of the minimizing quantity of (24). This is the term that corresponds to the real eigenvalue -0.5973. By assigning this weight, we are able to emphasize the task of achieving optimal eigenvectors for the complex conjugate pair of eigenvalues, $(-1.5 \pm j2)$. Note that this task is the most difficult to achieve due to the complex nature of the corresponding eigenvectors. The introduction of this weight factor only affects (31), whose first 2 terms need to be multiplied by this weight factor.

The algorithm yields

$$\begin{aligned} ||\tilde{v}_1^f - \tilde{v}_1||^2 &= 0.0242 && (41)\\ ||\tilde{v}_2^f - \tilde{v}_2||^2 &= ||\tilde{v}_3^f - \tilde{v}_3||^2 = 0.0230 && (42)\\ Tr(P^2) &= 0.0788 && (43) \end{aligned}$$

The output feedback gain that achieves these results is

$$K_f = \begin{pmatrix} -4.42776 & 5.95419 & 5.59306 \\ -4.15014 & -0.71481 & 0.49365 \end{pmatrix} \tag{44}$$

With the above controller, the fourth closed-loop eigenvalue is placed at -4.7358. Note that the above results concern the impaired system in the new state-coordinates specified by (40). However, the controller remains the same in the original state-coordinates, as discussed before. The obtained eigenvectors transformed back to the original coordinates of the impaired system are given by

$$\begin{aligned} V_f &= \begin{pmatrix} v_1^f & v_2^f = (v_3^f)^\star \end{pmatrix} \\ &= \begin{pmatrix} -0.0674 & 0.1424 + j0.0945 \\ -0.9950 & 0.1834 - j0.1722 \\ -0.0453 & 0.3519 + j0.5667 \\ 0.1136 & 0.1453 - j0.3729 \end{pmatrix} \end{aligned} \tag{45}$$

where obviously the first column is the eigenvector that corresponds to the real eigenvalue -0.5973, and the sec-

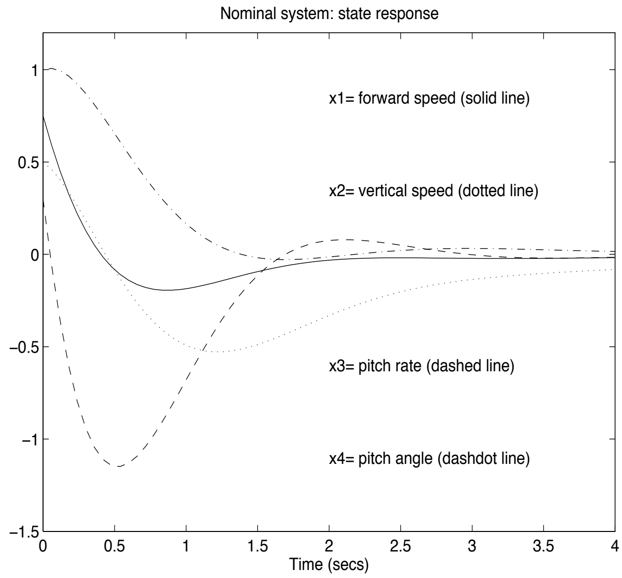

Figure 1: Nominal system. Closed-loop state response.

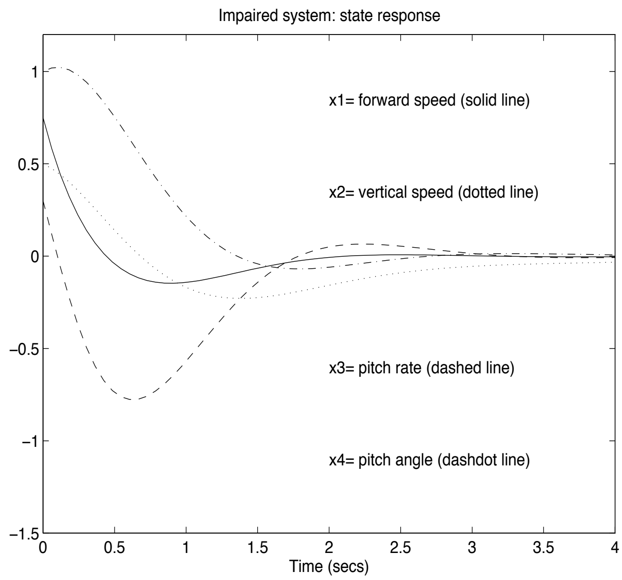

Figure 2: Impaired system. Closed-loop state response.

I.K. Konstantopoulos and P.J. Antsaklis, "An Eigenstructure Assignment Approach to Control Reconfiguration," Proc of 4th IEEE Mediterranean Symposium on Control and Automation, pp. 328-333, Chania, Crete, Greece, June 10-14, 1996.

ond column the eigenvector that corresponds to the complex conjugate pair of eigenvalues $(-1.5 \pm j2)$. As we see, the above eigenvectors are indeed very close to the desired eigenvectors of (36), as suggested by (41)-(42) above. This can also be shown by computing

$$\begin{aligned} \|v_1^f - v_1\|^2 &= 0.0231 && (46) \\ \|v_2^f - v_2\|^2 &= \|v_3^f - v_3\|^2 = 0.0210 && (47) \end{aligned}$$

In Figures 1-2, we compare the state response of the nominal system (32)-(35) with the output feedback matrix K of (37) and the state response of the impaired system of (38)-(39) with the output feedback matrix K_f of (44). The initial conditon vector is chosen as $V_{in}^1 = (0.75 \quad 0.5 \quad 0.3 \quad 1)^T$. As we see, the algorithm is capable of recovering the performance of the nominal system. This should be expected, since the eigenvectors of the impaired closed-loop system are assigned very close to the eigenvectors of the nominal closed-loop system, as shown in (46)-(47) above.

4. Conclusions

An eigenstructure assignment approach to control reconfiguration for systems with output feedback has been presented. The emphasis has been on the recovery of the nominal closed-loop performance, which is determined by the closed-loop eigenvalues and eigenvectors. The overall design is robust with respect to uncertainties in the state-space matrices of the impaired/reconfigured system. The approach has been applied to an aircraft control example.

References

[1] A. N. Andry, E. Y. Shapiro, and J. C. Chung, "Eigenstructure assignment for linear systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 19, no. 5, pp. 711-729, 1983.

[2] M. Athans, "The matrix minimum principle," *Informat. and Contr.*, vol. 11, no. 5/6, pp. 592-606, 1967.

[3] J. R. Calvo-Ramon, "Eigenstructure assignment by output feedback and residue analysis," *IEEE Trans. Automat. Contr.*, vol. 31, no. 3, pp. 247-249, 1986.

[4] J. Jiang, "Design of reconfigurable control systems using eigenstructure assignments," *Int. J. Contr.*, vol. 59, no. 2, pp. 395-410, 1994.

[5] I. K. Konstantopoulos and P. J. Antsaklis, "Optimal design of robust controllers for uncertain discrete-time systems," in *Proc. 3rd IEEE Mediterranean Symp. on New Directions in Contr. and Automation*, Limassol, Cyprus, vol. II, pp. 285-292, July 1995.

[6] I. K. Konstantopoulos and P. J. Antsaklis, "An optimization strategy for reconfigurable control systems," *Technical Report of the ISIS Group at the University of Notre Dame*, ISIS-95-006, September 1995.

[7] I. K. Konstantopoulos and P. J. Antsaklis, "A new strategy for reconfigurable control systems," in *Proc. 33rd Annual Allerton Conf. on Communication, Contr., and Computing*, Monticello, IL, pp. 69-78, October 1995.

[8] I. K. Konstantopoulos, *Optimal controller design and reconfigurable control under stability robustness.* Ph.D. Dissertation, University of Notre Dame, January 1996.

[9] I. K. Konstantopoulos and P. J. Antsaklis, "Eigenstructure assignment in reconfigurable control systems," *Technical Report of the ISIS Group at the University of Notre Dame*, ISIS-96-001, January 1996.

[10] I. K. Konstantopoulos and P. J. Antsaklis, "Optimal design of robust controllers for uncertain discrete-time systems," *Int. J. Contr.* To appear.

[11] R. J. Patton and G. P. Liu, "Robust control design via eigenstructure assignment, genetic algorithms and gradient-based optimisation," *IEE Proc. - Contr. Theory Appl.*, vol. 141, no. 3, pp. 202-208, 1994.

[12] K. M. Sobel and F. J. Lallman, "Eigenstructure assignment for the control of highly augmented aircraft," *J. Guidance, Contr., Dynamics*, vol. 12, no. 3, pp. 318-324, 1989.

[13] K. M. Sobel and E. Y. Shapiro, "Application of eigenstructure assignment to flight control design: some extensions," *J. Guidance, Contr., Dynamics*, vol. 10, no. 1, pp. 73-81, 1987.

[14] K. M. Sobel and E. Y. Shapiro, "Flight control examples of robust eigenstructure assignment," in *Proc. 26th IEEE Conf. Decision Contr.*, Los Angeles, CA, pp. 1290-1291, December 1987.

[15] K. M. Sobel and E. Y. Shapiro, "An extension to the pseudo control strategy with application to an eigenstructure assignment yaw pointing/lateral translation control law," in *Proc. 30th IEEE Conf. Decision Contr.*, Brighton, UK, pp. 515-516, December 1991.

[16] K. M. Sobel, E. Y. Shapiro, and A. N. Andry, "Eigenstructure assignment," *Int. J. Contr.*, vol. 59, no. 1, pp. 13-37, 1994.

[17] K. M. Sobel, W. Yu, and F. J. Lallman, "Eigenstructure assignment with gain suppression using eigenvalue and eigenvector derivatives," *J. Guidance, Contr., Dynamics*, vol. 13, no. 6, pp. 1008-1013, 1990.

[18] S. Srinathkumar, "Eigenvalue/eigenvector assignment using output feedback," *IEEE Trans. Automat. Contr.*, vol. 23, no. 1 , pp. 79-81, 1978.

[19] R. F. Wilson, J. R. Cloutier, and R. K. Yedavalli, "Lyapunov-constrained eigenstructure assignment for the design of robust mode-decoupled roll-yaw missile autopilots," in *Proc. of 1st IEEE Conf. Contr. Appl.*, Dayton, OH, pp. 994-999, September 1992.

[20] W. Yu and K. M. Sobel, "Robust eigenstructure assignment with structured state space uncertainty," *J. Guidance, Contr., Dynamics*, vol. 14, no. 3, pp. 621-628, 1991.