Proceedings of the 4th IEEE Mediterranean Symposium
on New Directions in Control and Automation
Maleme, Krete, GREECE June 10-13, 1996

MATRIX INTERPOLATION IN CONTROL DESIGN

Panos J. Antsaklis
Dept. of Electrical Engr., Univ. of Notre Dame
Notre Dame, IN 46556 USA
(219) 631-5792; (219) 631-4393 Fax
antsakli@saturn.ee.nd.edu
antsaklis@nd.edu

ABSTRACT

In this paper, recent developments of the matrix interpolation method and its applications in control design are reported and references are given. Certain fundamental results from the theory of polynomial and rational matrix interpolation are reviewed.

Introduction

Many system and control problems can be formulated in terms of matrix equations where polynomial or rational solutions with specific properties are of interest. It is known that equations involving just polynomials can be solved by either equating coefficients of equal power of the indeterminate s or equivalently by using the values obtained when appropriate values for s are substituted in the given polynomials; in the latter case one uses results from the theory of polynomial interpolation. Similarly one may solve polynomial matrix equations using the theory of polynomial matrix interpolation discussed here; polynomial matrix interpolation of the type $Q(sj)aj = bj$, where $Q(s)$ is a matrix and aj, bj vectors, has been introduced recently as a generalization of the scalar polynomial interpolation of the form $q(sj) = bj$. Rational matrix interpolation can be seen as a special case of polynomial matrix interpolation.

Details of the theory of polynomial and rational matrix interpolation can be found in [1]. The development of the methodology from the results on pole placement using static output and state feedback (1976-77), to the characteristic values and vector results (1980), to the Diophantine equation (1983) and the development of the full theory (1993) can be traced through [2]-[10]. Recent applications of interpolation on model matching, loop gain design, identification of the transfer matrix, self-tuning and control reconfiguration are discussed in [11]-[20]. Note that the interpolation algorithms have been successfully implemented in Matlab. The numerical aspects of the interpolation method are explored in [21]-[24]. The approach is extended to the 2-D case in [26]-[28]. Applications of certain interpolation results have also been developed independently: In [25] matrix interpolation is used in J-spectral factorization; in [29]-[31] eigenstructure assignment is explored with emphasis on aircraft applications; in [32]-[33] (directional) matrix interpolation is used in H-inf optimization problems; interpolation is also used in [34] to solve similar problems; finally in [35]-[36] questions that deal with finding and parameterizing rational matrices of minimal (low) McMillan degree that satisfy interpolation constraints are studied, with emphasis on realizations in state-space or fractional form.

Below, certain fundamental results from the theory of polynomial and rational matrix interpolation are reviewed.

Polynomial Matrix Interpolation

The basic polynomial interpolation problem can be stated as follows:

Given l distinct complex scalars s_j $j = 1, \cdots, l$ and l corresponding complex values b_j, there exists a unique polynomial $q(s)$ of degree $l-1$ for which

$$q(s_j) = b_j \; j = 1, \cdots, l \tag{1}$$

That is, an nth degree polynomial $q(s)$ can be uniquely represented by the $l = n+1$ interpolation (points or doublets or) pairs (s_j, b_j) $j = 1, \cdots, l$.

The polynomial matrix interpolation theory deals with this interpolation problem in the matrix case. Let $S(s) \triangleq blk\ diag\ \{[1, s, \cdots, s^{d_i}]^T\}$ where d_i $i = 1, \cdots, m$ are non-negative integers; let $a_j \neq 0$ and b_j denote $(m \times 1)$ and $(p \times 1)$ complex vectors respectively and s_j complex scalars.

Theorem 1: Given interpolation (points) triplets (s_j, a_j, b_j) $j = 1, \cdots, l$ and nonnegative integers d_i with $l = \Sigma d_i + m$ such that the $(\Sigma d_i + m) \times l$ matrix

$$S_l \triangleq [S(s_1)a_1, \cdots, S(s_l)a_l] \tag{2}$$

has full rank, there exists a unique $(p \times m)$ polynomial matrix $Q(s)$, with ith column degree equal to $d_i, i = 1, \cdots, m$ for which

$$Q(s_j)a_j = b_j \;\; j = 1, \cdots, l \tag{3}$$

Proof. Since the column degrees of $Q(s)$ are d_i, $Q(s)$ can be written as

$$Q(s) = QS(s) \tag{4}$$

where $Q(p \times (\Sigma d_i + m))$ contains the coefficients of the polynomial entries. Substituting in (3), Q must satisfy

$$QS_l = B_l \tag{5}$$

where $B_l \triangleq [b_1, \cdots, b_l]$. Since S_l is nonsingular, Q and therefore $Q(s)$ are uniquely determined. □

It should be noted that when $p = m = 1$ and $d_1 = l - 1 = n$ this theorem reduces to the polynomial interpolation theorem; if $a_j = 1$, then S_l is a Vandermonde matrix.

Rational Matrix Interpolation

Similarly to the polynomial matrix case, the problem here is to represent a $(p \times m)$ rational matrix $H(s)$ by interpolation tripolets or points $(s_j, a_j, b_j) j = 1, \cdots, l$ which satisfy

$$H(s_j)a_j = b_j \quad 1, \cdots, l \tag{6}$$

where s_j are complex scalars and $a_j \neq 0, b_j$ complex $(m \times 1), (p \times 1)$ vectors respectively.

It can be shown that the rational matrix interpolation problem reduces to a special case of polynomial matrix interpolation. To see this:

Write $H(s) = \tilde{D}^{-1}(s)\tilde{N}(s)$ where $\tilde{D}(s)$ and $\tilde{N}(s)$ are $(p \times p)$ and $(p \times m)$ polynomial matrices respectively. Then (6) can be written as $\tilde{N}(s_j)a_j = \tilde{D}(s_j)b_j$ or as

$$[\tilde{N}(s_j), -\tilde{D}(s_j)] \begin{bmatrix} a_j \\ b_j \end{bmatrix} = Q(s_j)c_j = 0 \quad j = 1, \cdots, l \tag{7}$$

That is the rational matrix interpolation problem for a $p \times m$ rational matrix $H(s)$ can be seen as a polynomial interpolation problem for a $p \times (p + m)$ polynomial matrix $Q(s) \triangleq [\tilde{N}(s), -\tilde{D}(s)]$ with interpolation points $(s_j, c_j, 0) = (s_j, [a_j^T, b_j^T]^T, 0)$ $j = 1, \cdots, l$. There is also the additional constraint that $\tilde{D}^{-1}(s)$ exists.

Solution of Matrix Equations

In this section polynomial matrix equations of the form $M(s)L(s) = Q(s)$ are studied. The main result is Theorem 2 where it is shown that all solutions $M(s)$ of degree r can be derived by solving equation (16). In this way, all solutions of degree r of the polynomial equation, if they exist, are parameterized. It is also shown that Theorem 2 can be applied to solve rational matrix equations of the form $M(s)L(s) = Q(s)$.

Consider the equation

$$M(s)L(s) = Q(s) \tag{8}$$

where $L(s)(t \times m)$ and $Q(s)(k \times m)$ are given polynomial matrices. Determine the polynomial matrix solutions $M(s)(k \times t)$ when they exist.

First consider the left hand side of equation (8). Let

$$M(s) \triangleq M_0 + \cdots + M_r s^r \tag{9}$$

and $d_i \triangleq \deg_{ci}[L(s)]$ $i = 1, \cdots, m$ that is the column degrees of $L(s)$. If

$$\hat{Q}(s) \triangleq M(s)L(s) \tag{10}$$

then $\deg_{ci}[\hat{Q}(s)] = d_i + r$ for $i = 1, \cdots, m$. According to the polynomial matrix interpolation Theorem 1, the matrix $\hat{Q}(s)$ can be uniquely specified using $\Sigma_{i=1}^m (d_i + r) + m = \Sigma_{i=1}^m d_i + m)(r + 1)$ interpolation points. Therefore consider l interpolation points (s_j, a_j, b_j) $j = 1, \cdots, l$ where

$$l = \Sigma d_i + m(r + 1) \tag{11}$$

Let $S_r(s) \triangleq blk\ diag\{[1, s, \cdots, s^{d_i + r}]^T\}$ and assume that the $(\Sigma d_i + m(r + 1)) \times l$ matrix

$$S_{rl} \triangleq [S_r(s_1)a_1, \cdots, S_r(s_l)a_l] \tag{12}$$

has full rank; that is the assumptions in Theorem 1 are satisfied. Note that for distinct s_j, S_{rl} will have full column rank for almost any set of nonzero a_j. Now in view of Theorem 1 the matrix $\hat{Q}(s)$ which satisfies

$$\hat{Q}(s_j)a_j = b_j \quad j = 1, \cdots, l \tag{13}$$

is uniquely specified given these l interpolation points (s_j, a_j, b_j). To solve (8) these interpolation points must be appropriately chosen so that the equation $\hat{Q}(s)(= M(s)L(s)) = Q(s)$ is satisfied:

Write (8) as

$$ML_r(s) = Q(s) \tag{14}$$

where

$$M \triangleq [M_0, \cdots, M_r] \quad (k \times t(r + 1))$$
$$L_r(s) \triangleq [L^T(s), \cdots, s^r L^T(s)]^T \quad (t(r + 1) \times m).$$

Let $s = s_j$ and postmultiply by a_j $j = 1, \cdots, l$; note that s_j and a_j $j = 1, \cdots, l$ must be so that S_{rl} above has full rank. Define

$$b_j \triangleq Q(s_j)a_j \ j = 1, \cdots, l \tag{15}$$

and combine the equations to obtain

$$ML_{rl} = B_l \tag{16}$$

where

$L_{rl} \triangleq [L_r(s_1)a_1, \cdots, L_r(s_l)a_l]$ $(t(r+1) \times l)$ and

$B_l \triangleq [b_1, \cdots, b_l]$ $(k \times l)$.

Theorem 2 Given $L(s), Q(s)$ in (8), let $d_i \triangleq \deg_{ci}[L(s)]$ $i = 1, \cdots, m$ and select r to satisfy

$$\deg_{ci}[Q(s)] \leq d_i + r \quad i = 1, \cdots, m \tag{17}$$

Then a solution $M(s)$ of degree r exists if and only if a solution M of (4.16) does exist; $M(s) = M[I, sI, \cdots, s^r I]^T$. □

It is not difficult to show that solving (16) is equivalent to solving

$$M(s_j)c_j = b_j \quad j = 1, \cdots, l \tag{18}$$

where

$$c_j \triangleq L(s_j)a_j, b_j \triangleq Q(s_j)a_j \quad j = 1, \cdots, l \tag{19}$$

$M(s)$ that satisfy (18) are obtained by solving

$$MS_{rl} = B_l \tag{20}$$

where $S_{rl} \triangleq [S_r(s_1)c_1, \cdots, S_r(s_l)c_l]$ $(t(r+1) \times l)$, with $S_r(s) \triangleq [I, sI, \cdots, s^r I]^T (t(r+1) \times t)$ and $B_l \triangleq [b_1, \cdots b_l]$ $(k \times l)$. Solving (20) is an alternative to solving (16).

Constraints on Solutions: When there are more unknowns $(t(r+1))$ than equations $(l = \Sigma d_i + m(r+1))$ in (16), the additional freedom can be exploited so that $M(s)$ satisfies additional constraints. In particular, $k \triangleq t(r+1) - l$ additional linear constraints, expressed in terms of the coefficients of $M(s)$ (in M), can be satisfied in general. The equations describing the constraints can be used to augment the equations in (4.16). In this case the equations to be solved become

$$M[L_{rl}, C] = [B_l, D] \tag{21}$$

where $MC = D$ represents the k linear constraints imposed on the coefficients M; C and D are matrices (real or complex) with k columns each.

The Diophantine Equation

An important case of (8) is the *Diophantine equation*:

$$X(s)D(s) + Y(s)N(s) = Q(s) \tag{22}$$

where the polynomial matrices $D(s), N(s)$ and $Q(s)$ are given and $X(s), Y(s)$ are to be found. Note that if

$$M(s) = [X(s), Y(s)], L(s) = \begin{bmatrix} D(s) \\ N(s) \end{bmatrix} \tag{23}$$

it is immediately clear that the Diophantine equation is a polynomial equation of the form (8) and all the previous results do apply. That is, Theorem 2 guarantees that all solutions of (22) of degree r are found by solving (16). In the theory of Systems and Control the Diophantine equation used involves a matrix $L(s) = [D^T(s), N^T(s)]^T$ which has rather specific properties. These are exploited to solve the Diophantine equation and to derive conditions for existence of solutions of (22) of degree r. It can be shown that:

Theorem 3: Let r satisfy

$$\deg_{ci}[Q(s)] \leq d_i + r \quad i = 1, \cdots, m \text{ and } r \geq \nu - 1. \tag{24}$$

where ν is the observability index of the system $\{D, I, N, 0\}$. Then the Diophantine equation (22) has solutions of degree r which can be found by solving (16) (or (20)).

Solving Rational Matrix Equations

Now let us consider the rational matrix equation:

$$M(s)L(s) = Q(s) \tag{25}$$

where $L(s)(t \times m)$ and $Q(s)(k \times m)$ are given rational matrices. The polynomial matrix interpolation theory developed above can be used to solve this equation and determine the rational matrix solutions $M(s)(k \times t)$. Let $M(s) = \tilde{D}^{-1}(s)\tilde{N}(s)$, a polynomial fraction form of $M(s)$ to be determined. Then (24) can be written as:

$$[\tilde{N}(s), -\tilde{D}(s)] \begin{bmatrix} L(s) \\ Q(s) \end{bmatrix} = 0 \tag{26}$$

Note that one could equivalently solve

$$[\tilde{N}(s), -\tilde{D}(s)] \begin{bmatrix} L_p(s) \\ Q_p(s) \end{bmatrix} = 0 \tag{27}$$

where $[L_p(s)^T, Q_p(s)^T]^T = [L(s)^T, Q(s)^T]^T \phi(s)$ a polynomial matrix with $\phi(s)$ the least common denominator of all entries of $L(s)$ and $Q(s)$; in general, $\phi(s)$ could be any matrix denominator in a right fractional representation of $[L(s)^T, Q(s)^T]^T$. The problem to be solved is now (8), a polynomial matrix equation, where $L(s) = [L_p(s)^T, Q_p(s)^T]^T$ and $Q(s) = 0$. Therefore, all solutions $[\tilde{N}(s), -\tilde{D}(s)]$ of degree r can be determined by solving (16) or (20). Let $s = s_j$ and postmultiply (27) by a_j $j = 1, \cdots, l$ with a_j and l chosen properly. Define

$$c_j \triangleq \begin{bmatrix} L_p(s) \\ Q_p(s) \end{bmatrix} a_j \quad j = 1, \cdots, l \tag{28}$$

The problem now is to find a polynomial matrix $[\tilde{N}(s), -\tilde{D}(s)]$ which satisfies

$$[\tilde{N}(s_j), -\tilde{D}(s_j)]c_j = 0 \quad j = 1, \cdots, l \qquad (29)$$

Note that restrictions on the solutions can be easily imposed to guarantee that $\tilde{D}^{-1}(s)$ exists and/or that $M(s) = \tilde{D}^{-1}(s)\tilde{N}(s)$ is proper. Additional constraints can be added so the solution satisfies additional specifications; see (21).

Pole Placement

Output Feedback. All proper output controllers of degree r (of order mr) that assign all the closed loop eigenvalues to arbitrary locations are characterized in a convenient way using interpolation results.

Given $N(s)D^{-1}(s)$ proper, we are interested in solutions $[X(s), Y(s)](m \times (p+m))$ of the Diophantine equation where only the roots of $|Q(s)|$ are specified; furthermore $X^{-1}(s)Y(s)$ should exist and be proper. The equation to be solved is

$$(X(s_j)D(s_j) + Y(s_j)N(s_j))a_j = 0 \; j = 1, \cdots, l \qquad (30)$$

or $ML_{rl} = 0 (l = \Sigma d_i + mr)$; that is the $\Sigma d_i + mr$ roots of $|X(s)D(s) + Y(s)N(s)|$ are to be assigned the values $s_j \quad j = 1, \cdots l$. Note the difference between the problem studied above where $Q(s)$ is known, and the problem studied here where only the roots of $|Q(s)|$ (or $|Q(s)|$ within multiplication by some nonzero real scalar) are given. Here the vectors a_j can be seen as design parameters and they can be selected almost arbitrarily to satisfy requirements in addition to pole assignment.

Theorem 4 Let $r \geq \nu - 1$. Then $(X(s), Y(s))$ exists such that all the $n + mr$ zeros of $|X(s)D(s) + Y(s)N(s)|$ are arbitrarily assigned and $X^{-1}(s)Y(s)$ is proper. □

Example: Let $D(s) = \begin{bmatrix} s-2 & 0 \\ 0 & s+1 \end{bmatrix}, N(s) = \begin{bmatrix} s-1 & 0 \\ 1 & 1 \end{bmatrix}$ with $n = \deg|D(s)| = 2$. Here there are $\deg|X(s)D(s) + Y(s)N(s)| = n + mr = 2 + 2r$ closed-loop poles to be assigned. Note that $r \geq \nu - 1 = 1 - 1 = 0$.

i) For $r = 0$ and $\{(s_j, a_j), j = 1, 2\} = \{(-1, [1\ 0]^T), (-2, [0\ 1]^T)\}$, a solution of $ML_{rl} = 0$ is

$$M = \begin{bmatrix} 2 & 0 & -3 & 0 \\ 0 & 2 & 1 & 2 \end{bmatrix}.$$

For this case, $M = M(s) = [X(s), Y(s)]$.

ii) For $r = 1$, and $\{(s_j, a_j), j = 1, \cdots, 4\} = \{(-1, [1\ 0]^T), (-2, [0\ 1]^T, (-3, [-1\ 0]^T), (-4, [0\ -1]^T)\}$ a solution of $ML_{rl} = 0$ is $[X(s), Y(s)]$ $\begin{bmatrix} s-7 & -1 & 12 & s+1 \\ 5 & s+4 & -6 & s+4 \end{bmatrix}$. Note that here $X(s)^{-1}Y(s)$ exists and it is proper. □

State Feedback: Let A, B, F be $n \times n, n \times m, m \times n$ real matrices respectively. Note that $|sI - (A + BF)| = |sI - A| \cdot |I_n - (sI - A)^{-1}BF| = |sI - A| \cdot |I_m - F(sI - A)^{-1}B|$. If now the desired closed-loop eigenvalues s_j are different from the eigenvalues of A, then F will assign all n desired closed loop eigenvalues s_j if and only if

$$F[(s_jI - A)^{-1}Ba_j] = a_j \quad j = 1, \cdots, n \qquad (31)$$

The $m \times 1$ vectors a_j are selected so that $(s_jI - A)^{-1}Ba_j \quad j = 1, \cdots, n$ are linearly independent vectors. Alternatively one could approach the problem as follows: let $M(s)(n \times m), D(s)(m \times m)$ be right coprime polynomial matrices such that $(sI - A)^{-1}B = M(s)D^{-1}(s)$. An internal representation equivalent to $\dot{x} = Ax + Bu$ in polynomial matrix form is $Dz = u$ with $x = Mz$. The eigenvalue assignment problem is then to assign all the roots of $|D(s) - FM(s)|$; or to determine F so that

$$FM(s_j)a_j = D(s_j)a_j \quad j = 1, \cdots, n \qquad (32)$$

The $m \times 1$ vectors a_j are selected so that $M(s_j)a_j \quad j = 1, \cdots, n$ are independent. Note that when s_j are distinct, a_j can almost be arbitrarily selected. For all the appropriate choices of a_j $(M(s_j)a_j \quad j = 1, \cdots, n$ linearly independent), the n eigenvalues of the closed-loop system will be at the desired locations $s_j \quad j = 1, \cdots, n$. Different a_j correspond to different F that produce, in general, different closed loop behavior.

The exact relation of the eigenvectors to the a_j can be found as follows: $[s_jI - (A + BF]M(s_j)a_j = (s_jI - A)M(s_j)a_j - BFM(s_j)a_j = BD(s_j)a_j - BD(s_j)a_j = 0$. Therefore, $M(s_j)a_j = v_j$ are the closed-loop eigenvectors corresponding to s_j.

Conclusions

Interpolation is a very general and flexible way to deal with problems involving polynomial and rational matrices. Results presented provide an appropriate theoretical setting and algorithms to deal with such problems. Matrix interpolation has been used successfully in a number of control problems to determine polynomial and rational matrix solutions. It has also been shown to have excellent numerical properties in deriving solutions of polynomial matrix equations. There is need to explore the full potential of the approach, to identify strengths and weaknesses and to develop a full range of efficient computer algorithms to meet the many needs of control design problems.

References

1. P.J. Antsaklis and Z. Gao, "Polynomial and Rational Matrix Interpolation: Theory and Control Applications", *International Journal of Control*, vol 58, no. 2, 349-404, August 1993.

2. P.J. Antsaklis and W. A. Wolovich, "Arbitrary Pole Placement Using Linear Output Feedback Compensation", *Intern. J. Control*, Vol. 25, pp. 915-925, 1977.

3. P.J. Antsaklis, Some New Matrix Methods Applied to Multivariable System Analysis and Design, Ph.D. Dissertation, Brown University, 1977.

4. P.J. Antsaklis, "Polynomial Matrix Characterization Using Characteristic Values and Vectors", Publ. No. 80/18, Dept. of Electr. Engr., Imperial College, London, July 1980.

5. P.J. Antsaklis, "Polynomial Matrix Interpolation in Control Equations: The Diophantine Equation", *Proc. of the 1983 Conf. on Info. Sciences and Systems*, pp. 873-878, The Johns Hopkins University, March 1983.

6. P.J. Antsaklis and R.L. Lopez, "Control Design Using Polynomial Matrix Interpolation", *Proc. of the 23rd IEEE Conf. on Decision and Control*, pp 621-622, Las Vegas, Nevada, Dec. 12-14, 1984.

7. R. Lopez, Multivariable Control Design via Polynomial Interpolation Methods, M.S.E.E. Thesis, Dept. of Electr. Engr., University of Notre Dame, May 1984.

8. P.J. Antsaklis and Z. Gao, "On the Theory of Polynomial Matrix Interpolation and its Role in Systems and Control", Twenty-Eighth Annual Allerton Conference on Communication, Control and Computing, Univ. of Illinois at Urbana -Champaign, Sept. 28 - Oct. 1, 1990.

9. P.J. Antsaklis and Z. Gao, "Polynomial and Rational Matrix Interpolation: Systems and Control Applications", *Proc.of the 1993 American Control Conf.*, pp. 640-644, San Francisco, CA, June 2-4, 1993.

10. P.J. Antsaklis and Zhiqiang Gao, "On Polynomial and Rational Matrix Interpolation", *Proc.IEEE Mediterranean Symposium on New Directions in Control Theory and Applications*, Chania, Greece, June 21-23, 1993.

11. Z. Gao and P.J. Antsaklis, "New Methods for Control System Design Using Matrix Interpolation", *Proc.of the 33rd IEEE Conference on Decision and Control*, pp 2506-2511, Lake Buena Vista, FL, Dec 14-16, 1994.

12. Zhiqiang Gao, Bruce Tabachnik, and Razvan Savescu, "A New Approach to Multi-Input and Multi-Output System Identification in Frequency Domain", *Journal of the Franklin Institute*, vol. 331B, pp.435-448, 1994.

13. Brian Boulter and Zhiqiang Gao, "A Real-Time Self-Tuning Web Tension Regulation Scheme", to appear in the Special Issue on "Engineering of Complex Real-Time Computer Control Systems" in the *Journal of Real-Time Systems*.

14. Zhiqiang Gao, "An Algorithmic Approach to Loop Shaping with Applications to Self-Tuning Control Systems", to appear in *Journal of the Franklin Institute*.

15. Brian Boulter and Zhiqiang Gao, "A Novel Approach for Efficient Self-tuning Web Tension Regulation", *Proc. of the 4th IEEE Conference on Control Applications*, August 1995.

16. Brian Boulter and Zhiqiang Gao, "Matrix Interpolation Based Self-Tuning Web Tension Regulation", *Proc of the 3rd International Web Handling Conference*, Oklahoma State University, Stillwater, OK, June 19 - 21, 1995.

17. Razvan V. Savescu, *System Identification from Frequency Response Using Matrix Interpolation Theory*, MS Thesis, Department of Electrical Engineering, Cleveland State University, Oct. 1992.

18. Bruce Tabachnik, *A Computer Algorithm for Multi-input multi-output system identification Using Chebychev Polynomials*, MS Thesis, Department of Electrical Engineering, Cleveland State University, May 1994.

19. Brian Boulter, *Matrix Interpolation Based Self-Tuning Web Tension Regulation*, MS Thesis, Department of Electrical Engineering, Cleveland State University, October 1995.

20. I.K. Konstantopoulos, *Optimal Controller Design and Reconfigurable Control Under Stability Robustness*, Ph.D. Dissertation, Dept. of Electrical Engr, Univ of Notre Dame, January 1996.

21. M.Sebek, S. Pejchova, F. Kraffer, "Numerical Properties Of Linear Polynomial Operations Or What Do Experiments Reveal?", *IEEE Mediterranean Symposium on New Directions in Control Theory and Applications*, Chania, Crete, Greece, June 21-23, 1993.

22. M.Sebek, S. Pejchova, F. Kraffer, "Testing Algorithms For Linear Polynomial Equations", *Proc of the 33rd IEEE Conference on Decision and Control*, pp 2518-2520, Lake Buena Vista, FL, Dec 14-16, 1994.

23. F. Kraffer, S. Pejchova and M. Sebek, "Numerical Performance In Matrix Diophantine Equation", *Kybernetika (Prague)*, vol.30, 6, pp. 745-753, 1994.

24. M. Sebek, F. Kraus, S. Pejchova and H. Kwakernaak, "A Comparison Of Algorithms For Determinant And Zeros Of Polynomial Matrices", *Proc.of 4th IEEE Mediterranean Symposium on Control and Automation*, Chania, Crete, Greece, June 10-14, 1996.

25. H. Kwakernaak, M. Sebek, "Polynomial J-Spectral Factorization", IEEE Trans. AC, vol 29, pp.315-328, Feb. 1994.

26. T. Kaczorek, "Polynomial and Rational Matrix Interpolations for 2-D Systems", *Proc.IMACS-IEEE/SMC Symposium on Signal Processing, Robotics and Neural Networks*, pp.162-165, Lille, April 25-27, 1994.

27. T. Kaczorek, "Polynomial and Rational Matrix Interpolations for 2-D Systems", *Bull. Pol. Acad. Techn. Sci.*, vol.42, No.1, pp.45-51, 1994.

28. T. Kaczorek, "Neural Networks and Matrix Interpolation in 1-D and 2-D Control Systems", *Proc. Conf.on Neural Networks and their Applications*, Szczyrk, Poland, April 30 - May 4, 1996.

29. A.N. Andry, E.Y. Shapiro and J.C. Chung, "Eigenstructure Assignment for Linear Systems", *IEEE Trans Aerosp. Electr. Systems*, Vol.19, No.5, pp. 711-729, 1983.

30. K.M. Sobel, E.Y. Shapiro and A.N. Andry, "Eigenstructure Assignment", *Intern. Journal of Control*, Vol.59, No.1, pp. 13-37, 1994.

31. J. Jiang, "Design of Reconfigurable Control Systems Using Eigenstructure Assignments", *Intern. Journal of Control*, Vol.59, No.2, pp. 395-410, 1994.

32. H. Kimura, "Directional Interpolation Approach to H Optimization and Robust Stabilization", *IEEE Trans on Automatic Control*, Vol AC-32, pp. 1085-1093, 1987.

33. D.J.N. Limebeer and B.D.O. Anderson, "An Interpolation Theory Approach to H -Controller Degree Bounds", *Linear Algebra and Its Applications*, 98, pp. 347-386, 1998.

34. J. Kamali, T. Boros, T. Kailath and G. Franklin, "Q-Parametrization for Unstable Plants: An Interpolation Approach", *Proc. of the American Control Conference*, pp. 4401-4402, Seattle, Washington, June 1995.

35. J.A. Ball and J. Kang, "Matrix Polynomial Solutions of Tangential Lagrange-Sylvester Interpolation Conditions of Low McMillan Degree", *Linear Algebra and Its Applications*, 137/138, pp. 699-746, 1990.

36. A.C. Antoulas, J.A. Ball, J. Kang, and J.C. Willems, "On the Solution of the Minimal Rational Interpolation Problem", *Linear Algebra and Its Applications*, 137/138, pp. 511-573, 1990.