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SUPERVISOR CONTROL DESIGN OF HYBRID SYSTEMS USING TIMED PETRI NETS BASED ON INVARIANT PROPERTIES [1]

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Abstract: In this paper, a class of timed Petri nets referred to as programmable timed Petri nets is used to model and control hybrid systems. The approach is illustrated using a gas storage example which has been proposed recently as a benchmark for hybrid modeling. Control algorithms are developed to satisfy safety specifications on the plant's discrete and continuous dynamics. The supervisor control design is based on the place invariants of the Petri net for the discrete specifications and on the natural invariants of the system for the continuous specifications.

Keywords: hybrid systems, Petri nets, supervisory control

1. INTRODUCTION

Hybrid control systems typically arise from the interaction of discrete planning algorithms and continuous processes, and as such, they frequently arise in the computer aided control of continuous processes in manufacturing, communication networks, and industrial process control among others. In hybrid systems the behavior of interest is governed by interacting continuous and discrete dynamic processes. Modeling, analysis, and synthesis of hybrid control systems are essential in designing intelligent control systems with a high degree of autonomy.

Petri nets have been used extensively as discrete-event system models (Murata, 1989; Holloway *et al.*, 1997). They provide an excellent tool for capturing the inherent concurrency of a complex system and the means of modeling conflict within the system. In this paper, Petri net are used to formulate discrete planning algorithms for hybrid systems by imposing additional logical constraints on their switching policy. In particular, a type of timed Petri nets, referred to as *programmable timed Petri nets* is used to represent the logical dependencies between mode switches among the continuous processes of a hybrid system.

Hybrid systems are used to model complex physical systems that require real-time control to ensure safety operation. The complex dynamics and the real-time requirements prohibit the design of detailed plans that take into consideration the continuous dynamics of the modes of the different system components. On the other hand, discrete planning algorithms cannot guarantee safety. A hybrid control architecture would be satisfactory if it provides the means for systematic and efficient design of discrete planning algorithms and simultaneously describes transparently the interconnection between the discrete and the continuous dynamics. The study of the continuous dynamics is essential to guarantee the safe op-

[1] The partial financial support of the National Science Foundation (ECS95-31485) and the Army Research Office (DAAG55-98-1-0199) is gratefully acknowledged.
[2] This author gratefully acknowledges the fellowship of the Center of Applied Mathematics at the University of Notre Dame for the academic year 1997-98.

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eration of the system. Stability and supervisory control of hybrid systems has been studied before in (Koutsoukos *et al.*, 1998).

In this paper, modeling and supervision of hybrid systems described by programmable timed Petri nets are illustrated using a gas storage example which has been proposed recently as benchmark for hybrid modeling (Champagnat *et al.*, 1998*a*). The paper is organized as follows. Section 2 presents the programmable timed Petri net model using the gas storage example. The supervision of hybrid systems for both the discrete and continuous dynamics is studied in Section 3.

2. PROGRAMMABLE TIMED PETRI NETS

In this section, a class of timed Petri nets named *programmable timed Petri nets* (Lemmon *et al.*, 1998) is used to model hybrid control systems. The main characteristic of the proposed modeling formalism is the introduction of a clock structure which consists of generalized local timers that evolve according to continuous-time vector dynamical equations. They can be seen as an extension of the approach taken in (Alur and Dill, 1994; Alur *et al.*, 1995) that provide a simple, but powerful way to annotate the Petri net graph with generalized timing constraints expressed by propositional logic formulae. In contrast to previous efforts to include continuous processes in the Petri net modeling framework (for example (Le Bail *et al.*, 1991; Giua and Usai, 1996; Flaus and Alla, 1997; Demongodin and Koussoulas, 1998)), the proposed model still consists of discrete places and transitions, and it preserves the simple structure of ordinary Petri nets. The information for the continuous dynamics of a hybrid system is embedded in the logical propositions that label the different elements of the Petri net graph.

Formally, a *programmable timed Petri net,(PTPN)* is denote by the ordered tuple $(\mathcal{N}, \mathcal{X}, \ell_P, \ell_T, \ell_I, \ell_O)$ where

- $\mathcal{N} = (P, T, I, O)$ is an ordinary Petri net (Murata, 1989) where P, T, I, and O denote the set of places, transitions, input arcs (from places to transitions) and output arcs respectively.
- $\mathcal{X}$ is a set of N *local clocks* which can be seen as a collection of continuous-time dynamical systems. The ith clock, $\mathcal{X}_i$ is described by $\dot{x}_i = f(x_i)$ where $x_i \in \Re^n$ is the continuous state (local time) and $f : \Re^n \to \Re^n$ is Lipschitz continuous automorphism over $\Re^n$ characterizing the local clock's rate $\dot{x}_i$.
- $\ell_P : P \to \mathcal{P}$, $\ell_T : T \to \mathcal{P}$, $\ell_I : I \to \mathcal{P}$, and $\ell_O : O \to \mathcal{P}$ are functions that label the places, transitions,input arcs, and output arcs (respectively) of the Petri net $\mathcal{N}$. $\mathcal{P}$ is the set of the logical formulas which are constucted by applying propositional connectives between *rate constraints* ($\dot{x}_i = f(x_i)$), *time constraints* ($h(x_i) < 0$ and/or $h(x_i) = 0$, $h : \Re^n \to \Re$) and *reset equations* ($x_i(\tau) = \bar{x}_0$).

For more details in PTPN modeling of hybrid systems see (Koutsoukos *et al.*, 1998).

A PTPN can be used to model a hybrid dynamical system in the following manner. The network, $\mathcal{N}$, is used to represent the logical dependencies between mode switches. The timers, $\mathcal{X}$, of the PTPN are the dynamical equations associated with the continuous-time dynamics of the system. The labels ℓ_P, ℓ_T, ℓ_I, and ℓ_O are chosen to represent conditions on the continuous state for mode switches as well as describing the various switching behaviour within the network. A gas storage example which has been proposed for hybrid modeling is used in the next section to illustrate modeling of hybrid systems based on PTPNs.

2.1 *Gas Storage Example*

Petri net based models have been introduced for this example in (Champagnat *et al.*, 1998*b*; Alla and Flaus, 1998; Valentin-Roubinet, 1998). See (Champagnat *et al.*, 1998*a*) for a detailed description of the benchmark problem. In this paper, the gas storage is modeled by PTPNs to facilitate supervisory control design as it has been proposed in (Koutsoukos *et al.*, 1998). First, the gas storage example is briefly presented.

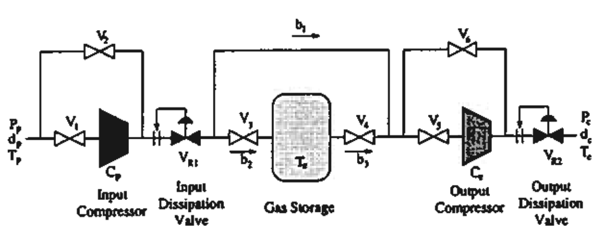

Fig. 1. The gas storage example

The gas storage example (Fig. 1) describes a buffer between gas production and gas customer consumption. The storage is fed by the production unit at a pressure P_p and a flow rate d_p and is drawn by the customer unit at a pressure P_c and a flow rate d_c. The system is composed of three main components. The first is a compressor between the production unit and the storage. The second component is the storage, and the third component is a compressor which is used to keep a flow rate between the tank and the customer unit.

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W_{c_p}	Required energy by the input compressor
W_{c_c}	Required energy by the output compressor
U_s	Number of moles of gas in the storage
V_s	Storage volume
P_s	Storage pressure
T_s	Temperature of the gas in the storage
R	Constant of the perfect gas
P_0	Reference pressure (54*bar*)
P_p	Production pressure
d_p	Production flow rate
T_p	Temperature of the gas in the production unit
P_c	Customer pressure
d_c	Customer flow rate
T_c	Temperature of the gas in the customer unit
P_2	Pressure after the input compressor
P_3	Pressure after V_{R_1}
P_4	Pressure before the output compressor
P_5	Pressure before V_{R_2}
T	Temperature of the gas through the unit

Table 1. Physical variables of the gas storage

The main constraint of the gas storage is that it cannot be both fed and drawn simultaneously. As a consequence, the storage can be in one of the following four configurations. In the first configuration, the customer flow rate is equal to the production flow rate ($d_p = d_c$) and the storage is by-passed. In the second configuration, the production flow rate is greater than the customer flow rate ($d_p > d_c$) and the surplus is stored in the storage unit. In the third one, the production flow rate is lower than the customer flow rate ($d_p < d_c$), the produced gas goes directly to the customer together with gas from the storage to compensate the low procution. The last configuration ($d_p = 0$) corresponds to the case when the production has stopped and the customer is served by the storage.

The physical variables that are used to describe the behavior of the gas storage are shown in table 1. The gas storage is a hybrid dynamical system consisting of three concurrent subsystems, the input compressor, the storage unit and the output compressor. The PTPN model for the gas storage is shown in fig. 2. The label functions for the places, transitions, input and output arcs are shown in table 2. The dashed lines represent arcs connecting controller places to existing transitions according to supervisory control based on place invariants as it is explained in the next section.

The PTPN model of the gas storage is well defined, since at any time instant the marking of the Petri net is uniquely defined and continuous dynamics are evolving according to the labels ℓ_p of the marked places. For safety, the gas storage has two breakdown states. The first one is when $d_p < d_c$ and the input pressure is less than the gas storage pressure. In this case, the gas will flow from the storage unit to the production unit. The second breakdown state is when $d_p = 0$ and the output pressure is greater than the storage pressure. In this case, the gas will flow from the customer unit to the storage.

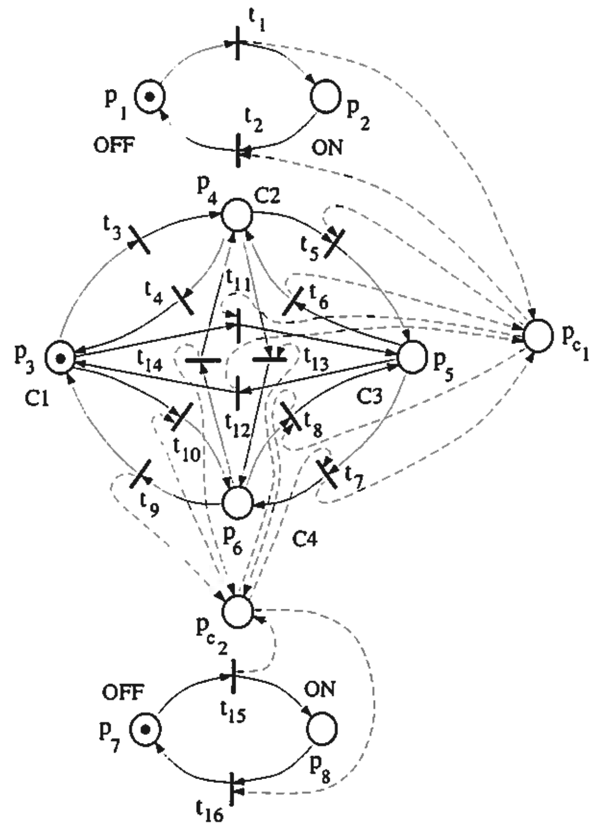

Fig. 2. PTPN model of the gas storage

The supervision of the hybrid system must guarantee that the operational constraints inherent in the system are respected, the safety constraints are never violated, and the performance of the system is satisfactory. The next section is concerned with supervisor control design for PTPNs based on invariant properties of both the discrete and continuous dynamics. The methodology is illustrated using the gas storage example.

3. SUPERVISION OF HYBRID SYSTEMS

Following a realistic scenario for the operation of a gas storage unit, it is assumed that a plan that describes the production policy and the customer demand for a time interval in the future (for example the next day) is available. The plan specifies the rate and pressure of the production unit, d_p and P_p, and the rate and pressure of the customer unit, d_c and P_c (the temperature is assumed to be constant $T = T_p = T_c = T_s$). It is assumed that these variables are piecewise constant functions of time and that they can change their values at regular time intervals (for example every hour). The design of such plans is usually based on production policies and costs,

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Input Compressor
$\ell_p(p_1) = (P_2 = P_p) \wedge (W_{c_p} = 0)$
$\ell_p(p_2) = (P_2 = 5P_p) \wedge (W_{c_p} = \frac{1}{\eta}\frac{\gamma RT}{\gamma - 1}((\frac{P_2}{P_p})^{\frac{\gamma-1}{\gamma}} - 1)$ $\gamma = 1.31,\ \eta = 0.8$
$\ell_T(t_1) = \ell_T(t_2) =$ tautology
$\ell_I(p_1, t_1) = \ell_O(t_2, p_1) = (P_p > P_3)$
$\ell_I(p_2, t_2) = \ell_O(t_1, p_2) = (P_p \leq P_3)$
Storage unit
$\ell_p(p_3) = (\frac{dU_s}{dt} = d_p - d_c = 0) \wedge (P_3 = P_4)$
$\ell_p(p_4) = (\frac{dU_s}{dt} = d_p - d_c) \wedge (P_3 = P_4) \wedge$ $(P_3 = P_s) \wedge (P_s V_s = U_s RT) \wedge (353.6 P_s (10(P_s - P_0))^{5/2})$
$\ell_p(p_5) = (\frac{dU_s}{dt} = d_p - d_c) \wedge (P_3 = P_4) \wedge$ $(P_4 = P_s) \wedge (P_s V_s = U_s RT) \wedge (353.6 P_s (10(P_s - P_0))^{5/2})$
$\ell_p(p_6) = (\frac{dU_s}{dt} = -d_c) \wedge$ $(P_4 = P_s) \wedge (P_s V_s = U_s RT) \wedge (353.6 P_s (10(P_s - P_0))^{5/2})$
$\ell_T(t_i) =$ tautology , $i = 3, 4, \ldots, 14$
$\ell_I(p_3, t_3) = \ell_I(p_3, t_{11}) = \ell_I(p_3, t_{10}) =$ $\ell_O(t_4, p_3) = \ell_O(t_{12}, p_3) = \ell_O(t_9, p_3) = (d_p = d_c)$
$\ell_I(p_4, t_4) = \ell_I(p_4, t_5) = \ell_I(p_4, t_{13}) =$ $\ell_O(t_3, p_4) = \ell_O(t_6, p_4) = \ell_O(t_{14}, p_4) =$ $(d_p > d_c) \wedge (5P_p \geq P_s) \wedge (P_s < 67)$
$\ell_I(p_5, t_6) = \ell_I(p_5, t_7) = \ell_I(p_5, t_{12}) =$ $\ell_O(t_5, p_5) = \ell_O(t_{11}, p_5) = \ell_O(t_8, p_5) =$ $(d_p < d_c) \wedge (5P_p \geq P_s)$
$\ell_I(p_6, t_8) = \ell_I(p_6, t_9) = \ell_I(p_6, t_{14}) =$ $\ell_O(t_7, p_6) = \ell_O(t_{10}, p_6) = \ell_O(t_{13}, p_6) =$ $((d_p > d_c) \wedge (5P_p \geq P_s) \wedge (P_s = 67))$ $\vee((d_p > d_c) \wedge (5P_p < P_s) \wedge (P_s \leq 67))$ $\vee((d_p = 0) \wedge (2P_s \geq P_c))$
Output Compressor
$\ell_p(p_7) = (P_5 = P_4) \wedge (W_{c_c} = 0)$
$\ell_p(p_8) = (P_5 = 2P_4) \wedge (W_{c_c} = \frac{1}{\eta}\frac{\gamma RT}{\gamma - 4}((\frac{P_5}{P_4})^{\frac{\gamma-1}{\gamma}} - 1)$ $\gamma = 1.31,\ \eta = 0.8$
$\ell_T(t_{15}) = \ell_T(t_{16}) =$ tautology
$\ell_I(p_7, t_{15}) = \ell_O(t_{16}, p_7) = (P_4 > P_c)$
$\ell_I(p_8, t_{16}) = \ell_O(t_{15}, p_8) = (P_4 \leq P_c)$

Table 2. Label functions for the PTPN model of the gas storage

techniques for energy savings, and policies for customer service.

In a hybrid control architecture, the supervisor control algorithms must guarantee the proper and safe operation of the system for a *large number* of different plans. Moreover, the algorithms should exhibit some capability to *react* to the perceived situation in order to handle unexpected events and uncertain plant behavior.

For the gas storage example, unexpected events can be failures in the production unit or changes in the customer demand that have not been incorporated in the plans. It is assumed that there is no uncertainty in the differential-algebraic equations that describe the continuous dynamics of the system (label functions $\ell_p(p_i)$ in table 2). Next, supervisor control algorithms based on invariant properties of the discrete and continuous dynamics are proposed. These algorithms are realized using state feedback control (discrete or continuous) and therefore they have some capability to react to the current state of the system.

3.1 *Supervisory Control of Petri Nets Based on Place Invariants*

A methodology for DES control based on Petri net place invariants has been proposed in (Yamalidou *et al.*, 1996; Moody, 1997). A feedback controller based on place invariants is implemented by adding control places and arcs to existing transitions in the Petri net structure. Although the method was developed for ordinary Petri nets, the introduction of time delays associated with each transition will not affect the controlled behavior of the Petri net with respect to the discrete specifications. The supervisor is used to enforce a set of linear constraints on the discrete state of the hybrid plant. These constraints can decribe a broad variety of problems including forbidden state problems, mutual exclusion problems, a class of logical predicates on plant behavior (Yamalidou and Kantor, 1991), conditions involving the concurrence of events, and the modeling of shared resources.

A *place invariant* of a Petri net is defined as every integer vector x which satisfies $x^T \mu = x^T \mu_0$ where μ_0 is the initial marking and μ any reachable subsequent marking. Place invariants characterize sets of places whose weighted sum of tokens remains constant at all reachable markings and is determined only by the initial marking. Consider linear constraints of the form $L\mu_p \leq b$ on the marking vector μ_p of the plant net. This inequality can be tranformed to the equality $L\mu_p + \mu_c = b$ by introducing an external Petri net controller whose places represent the "slack variables" μ_c. The incidence matrix of the controller is then computed by the equation $D_c = -LD_p$ and its initial marking is $\mu_{c_0} = b - L\mu_{p_0}$. The controller introduces place invariants in the closed loop system, that enforce the linear constraint $L\mu_p \leq b$. For more details on the place invariant method for controllable transitions see (Yamalidou *et al.*, 1996).

This control method is now applied in the gas storage example to disable any mode switches that lead to breakdown states. Consider the case when the system is in the third configuration ($d_p < d_c$). Then, there is the danger of a breakdown if the production pressure becomes less than the storage pressure. Therefore, it would be unsafe to turn off the input compressor while the storage is in the third configuration. Similarly for the second breakdown state, it would be unsafe to turn off the output compressor while the system is in the fourth configuration ($d_p = 0$). These constraints can be expressed as

$$\mu_1 + \mu_5 \leq 1$$
$$\mu_6 + \mu_7 \leq 1$$

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The controller is represented in Fig. 2 by the places p_{c_i}, $i = 1, 2$ and the arcs connecting these places to existing transitions (dashed lines). The label functions of the new places and arcs are chosen so that they do not affect the continuous dynamics and the operational constraints of the system (tautologies).

The Petri net supervisor designed above can guarantee that the system will not break down because of mode switching. To study the danger of breakdown while the system is in the third or fourth configuration it is necessary to take into consideration the continuous dynamics.

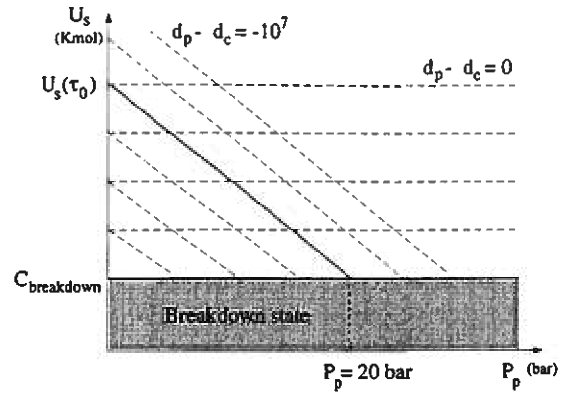

Fig. 3.

3.2 *Supervision of the Hybrid System Based on the Invariants of the Continuous Dynamics*

The purpose of this section is to describe a systematic procedure to add logical constraints that label the PTPN in order to guarantee the safety of the system. In the proposed methodology, the natural invariants of the system (explained in the following) are used to partition the state space into regions. The switching policy for the hybrid system is then derived by determining the region where the continuous state lies. The basic property of these regions is that their boundaries satisfy certain conditions that preclude the state trajectories from crossing them.

Consider a functional $h_i : \Re^n \to \Re$ that satisfies the condition $\nabla_\xi h_i(\xi) \neq 0$, $\forall \xi \in Null(h_i)$, which ensures that the null space of the functional $Null(h_i) = \{\xi \in \Re^n : h_i(\xi) = 0\}$ forms an $n-1$ dimensional manifold separating the state space. Then the smooth hypersurface $h_i = 0$ is invariant under the vector field f if $\nabla_\xi h_i(\xi) \cdot f(\xi) = 0$. Given the vector field that describes a dynamical system, there are different ways to compute the invariant hypersurfaces. When the dynamics are simple the invariants can be computed analytically. For complex dynamics, numerical methods can be used after the quantization of the state space. For details converning the invariant based design for hybrid control systems see (Stiver *et al.*, 1996). There are also interesting cases where it sufficient for the control objective to approximate the invariant hypersurfaces using Lyapunov functionals (Koutsoukos *et al.*, 1998).

In the following the invariant based approach for the design of hybrid control systems is illustrated using the gas storage example. Assume that the storage switches to the third configuration at time τ_0, and $d_p = 10^7 \frac{Kmol}{sec}, P_p = 20 bar, d_c = 2 \times 10^7 \frac{Kmol}{sec}$. Let $U_s(\tau_0)$ be the concentration of the gas at time τ_0. Suppose that because of some unexpected events (e.g. arrival of a customer not planned) these values are different than those of the plan. The objective is to find a switching policy that guarantees that the system will not reach the breakdown state. The concentration of the gas in the storage when the system breaks down is computed by solving the nonlinear algebraic equations for the third configuration (Table 2) to obtain $U_s = C_{breakdown}$. The hypersurface $U_s - C_{breakdown} = 0$ detects when the concentration falls below the critical value. The assumption that P_p is a constant is used only because it is easier to illustrate the results. In general, this threshold will be a function of P_p. When the critical value is detected the system must switch to another configuration so that the safety criterion will not be violated. For this purpose, the switching policy is chosen as follows. If $U_s \leq C_{breakdown}$ then set $d_c := d_p$ so that the system will switch to the first configuration. The natural invariant of the continuous dynamics for the first configuration $(d_p = d_c)$ ensures that the continuous state will not enter the breakdown region (Fig. 3).

The above condition can be embedded in the PTPN model of the gas storage by changing the label functions. In particular, this policy is implemented by updating the following label functions

$$\ell_I(p_5, t_6) = \ell_I(p_5, t_7) = \ell_I(p_5, t_{12}) =$$
$$\ell_O(t_5, p_5) = \ell_O(t_{11}, p_5) = \ell_O(t_8, p_5) =$$
$$(d_p < d_c) \wedge (5P_p \geq P_s) \wedge (U_s > C_{breakdown})$$

and

$$\ell_O(t_12, p_3) = (d_p = d_c) \vee (U_s \leq C_{breakdown})$$

If $U_s < C_{breadown}$ when the system switches to the second configuration, then t_{12} will fire instantaneously and the system switches safely to the first configuration. The breakdown state for the fourth configuration can be treated in a similar fashion and is not included due to length limitations.

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4. CONCLUSIONS

In this paper, supervisor control of hybrid systems is addressed using a class of timed Petri nets named programmable timed Petri nets. The methodology is applied to a gas storage example. The proposed approach exploits the advantages of modeling hybrid systems using Petri nets. For the gas storage the application of the proposed algorithms for supervisor control synthesis results in control policies that guarantee the proper and safe operation of the system.

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