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A UNIFYING APPROACH TO DECOUPLING

T. W. C. Williams
EECS
Kingston Polytechnic
Kingston upon Thames KT12EE
ENGLAND

P. J. Antsaklis
Dept. of Electrical Engineering
University of Notre Dame
Notre Dame, Indiana 46556
USA

INTRODUCTION

The problem of decoupling has been extensively studied in the past [1-3], with particular attention being paid to the special case of diagonal decoupling [4-7]. Both state-space and polynomial matrix methods have been used to study the diagonal decoupling problem, while most of the block decoupling results have been obtained using the geometric approach [3].

A unifying approach to the decoupling problem (diagonal, triangular, block diagonal decoupling) is introduced here. The key element is the interactor, introduced in [8], which is a matrix of unique structure associated with the transfer matrix of the system. The results obtained are conceptually simple; in essence, the (block) structure of the interactor specifies the decoupled system structure attainable via state feedback while, if input dynamics are to be used in addition to state feedback, the interactor is essential in determining the minimal orders of the appropriate input dynamics and of the decoupled system. The use of the interactor makes the mechanism of decoupling transparent, it helps to completely resolve the stability questions in the block decoupling problem, as well as the question of minimal order when input dynamics in addition to state feedback are used, and it unifies and expands existing results [9]. In this paper, as an introduction to this approach, we shall mainly concentrate on the conditions for block decoupling via linear state feedback.

PRELIMINARIES

Consider an nth order m-input/p-output controllable system with state space representation $\{\dot{x}=Ax+Bu,\ y=Cx+Eu\}$ and polynomial matrix representation $\{Pz=u,\ y=Rz\}$ where P is column proper [7] with column degrees $\{d_i\ i=1,2,\dots,m\}$, the controllability indices. Let $T(s)=C(sI-A)^{-1}B+E=R(s)P^{-1}(s)$ be the pxm transfer matrix. Define the linear state feedback (lsf) control law by $u=Fx+Gv$ where v is an ℓx1 external input ($\ell=\min(p,m)$). It is known [7] that the transfer matrix of the lsf compensated system can be written as

$$T_{F,G}(s) = R(s)P_F^{-1}(s)G \qquad (1)$$

with $P_F(s)\overset{\Delta}{=}P(s)-F(s)$ where the column degrees of F(s) are strictly less than d_i; there is a unique correspondence between F(s) and F.

PROBLEM STATEMENT

We are interested in decoupling the system using lsf or lsf with input dynamics. A system is called diagonally decoupled if its transfer matrix is diagonal and of full rank. In general, a system is said to be $\{(p_i x \ell_i)\}$ -block decoupled if its (pxℓ) transfer matrix has full rank and it is of the form blk diag $\{T_i(s)\}$ with $T_i(s)p_i x\ell_i$ $i=1,2,\dots,r(\sum_i^r p_i=p,\ \sum_i^r \ell_i=\ell)$; if $p_i=\ell_i$ for all i, it is said to be $\{p_i\}$ -block decoupled. When $p\leq m$, we are interested in pxp ($\ell=p$) compensated systems and in the $\{p_i\}$ -block decoupling problem. When $p>m$ we are interested in pxm ($\ell=m$) systems and in the $\{(p_i x m_i)\}$ -block decoupling problem ($\sum_i^r m_i=m$). This latter case ($p>m$) reduces to decoupling a square system [9] so we will concentrate here on the $\{p_i\}$ -block decoupling problem. It is assumed in the following that rank $T(s)=p(\leq m)$.

The Interactor

The interactor $X_T(s)$ of $T(s)$, defined in [8], is a (unique) pxp polynomial matrix such that

$$\lim_{s\to\infty}\{X_T(s)T(s)\}=K_T \qquad (2)$$

with rank $K_T=p$ and of the form

$$X_T(s)=H(s)\Delta(s) \qquad (3)$$

where $\Delta(s)=\text{diag}\{s^{\bar{f}_i}\}$ and H(s) lower triangular, with 1s on the diagonal and the nonzero off diagonal elements divisible by s . $X_T(s)$ can be obtained from T(s)[8], from a state-space representation {A,B,C, E}[10], or from the numerator R(s) and the controllability indices d_i[9]. The interactor is closely related to the Inversion/Structure algorithm of Silverman [2][11]; this state-space method is equivalent to finding a polynomial matrix X(s) satisfying (2) but not necessarily of the special form (3). Note that it is the special structure (3) of the interactor $X_T(s)$ which makes the study of a variety of decoupling questions (involving stability, minimal order) possible.

Lemma 1 $X_{T_{F,G}}(s) = X_T(s)$ under all lsf pairs (F,G) for which rank $K_TG=p$.

Proof By (1), $T_{F,G}(s)=(RP^{-1})(PP_F^{-1}G)=T[P_F+F]P_F^{-1}G=TG+TFP_F^{-1}G$. By (2), $X_T(s)T(s)=K_T+V(s)$ for some strictly proper V(s). Combining, $X_T(s)T_{F,G}(s) = K_TG$ + (strictly proper) since FP_F^{-1} is strictly proper. This implies that $X_{T_{F,G}}(s)=X_T(s)$ because of the uniqueness of the interactor. QED

Lemma 2 There exists lsf (F,G) such that $T_{F,G}(s)=X_T^{-1}$.

Proof Choose F (or F(s)) to satisfy $X_T(s)R(s)=K_T[P(s)-F(s)](=K_TP_F(s))$ and G such that $K_TG=I$. This is always possible since column degrees of $X_TR=d_i=$ column degrees of P_F and rank $K_T=p$ (see also [12] [10]). QED

Conditions for Decoupling

Lemma 2 and the fact that X_T is triangular, clearly shows that in this case it is always possible to triangularly decouple the system via lsf, a well known result [13]. For diagonally decoupling via lsf, the rank of a

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matrix B*[5] is important. The relation between B* and the interactor is as follows:

Given T(s), there exists a unique set of integers $\{f_i\ i=1,2,\ldots,p\}$, its decoupling indices, for which [6] $D(s) \triangleq \mathrm{diag}\{s^{f_i}\}$ yields

$$\lim_{s\to\infty} D(s)T(s)\} = B^* \qquad (4)$$

with B* finite with no zero rows. The following result follows from (4) and the definition of the interactor

Lemma 3 $X_T(s)$ is diagonal iff B* is of full rank. In this case, $X_T(s)=D(s)$ and $K_T=B^*$.

When p=m, diagonal decoupling via lsf is possible iff rank B*=p[5], that is, iff $X_T(s)$ is diagonal in view of Lemma 3. When p<m, the condition is only sufficient. These results are special cases of the following block decoupling results:

Theorem 4 An invertible system can be $\{p_i\}$ -block decoupled via lsf, iff its interactor is $\{p_i\}$ -block diagonal.

Proof Sufficiency: If $X_T(s)$ is $\{p_i\}$ -block diagonal, so is the closed loop transfer matrix $X_T^{-1}(s)$ produced by lsf (F,G) as in Lemma 2.

Necessity: If $T_{F,G}(s)=\mathrm{blkdiag}\{T_i(s)\}\{p_i\}$ -block diagonal and nonsingular, for some (F,G), then so is its interactor (as $\lim_{s\to\infty}[\mathrm{blkdiag}\{X_{T_i}(s)\}\ \mathrm{blkdiag}\{T_i(s)\}] = \mathrm{blkdiag}\{K_{T_i}\}$, finite and nonsingular). But by Lemma 1, $X_T(s)=X_{T_{F,G}}(s) = \mathrm{blkdiag}\{X_{T_i}(s)\}$ (rank G=p=m), that is $X_T(s)$ is $\{p_i\}$ -block diagonal. QED

When p<m, the sufficiency proof still holds but that of necessity does not since $X_{T_{F,G}}(s)$ is not necessarily equal to $X_T(s)$ (unless rank $K_TG=p$). Similarly limited results, sufficient for p≤m but necessary as well only for p=m, have appeared in [2], and in [1][3] using the geometric approach. The next result is a tighter sufficient condition for p<m.

Theorem 5 A right-invertible system (rank T(s)=p) can be $\{p_i\}$ -block decoupled via lsf if there exists some (mxp) real $\tilde{G}$ such that $\tilde{T}(s)=T(s)\tilde{G}$ has $\{p_i\}$ -block diagonal interactor.

Proof If such $\tilde{G}$ exists, then by Theorem 4, $\tilde{T}(s)$ can be $\{p_i\}$ -decoupled by some lsf(F,G). Then $(\tilde{G}F,\tilde{G}G)$ lsf, $\{p_i\}$ -block decouples T(s). QED
Algorithms to calculate such $\tilde{G}$, if one exists, are derived in [9].

A brief note on stability. Appropriate (F,G) which decouple the system are given in Lemma 2. It has now been shown in [12] that among the n roots of $|P_F(s)|$ (closed loop eigenvalues), q will be exactly equal to the q invariant zeros of the open loop system and $d_T(=\mathrm{degr}|X_T(s)|)$ will be equal to the poles of $X_T^{-1}(s)(n=q+d_T$ for p=m); when p<m there are $k(=n-(d_T+q))$ more arbitrarily assignable eigenvalues. The d_T eigenvalues can be arbitrarily assigned by defining the generalized interactor $\bar{X}_T(s) = \bar{H}(s)\bar{\Delta}(s)$ where $\bar{\Delta}(s)=\mathrm{diag}\{\delta_i(s)\}$ with $\delta_i(s)$ any nomic polynomial of degree $\bar{f}_i$[9] and then using Lemma 2; $\bar{X}_T(s)$ has the same block structure as $X_T(s)$ and it satisfies Lemmas 2 and 3 and therefore Theorems 4 and 5. Finally, it can be shown that the only fixed zeros of $|P_F(s)|$ are the $\{p_i\}$ -block coupling zeros, a subset of the q invariant zeros, which must lie in the left half s-plane for stability.

CONCLUDING REMARKS

The use of the interactor in the study of the decoupling problem unifies and expands existing results involving conditions for triangular, diagonal and block diagonal decoupling and helps resolve the stability questions which arise. Here, as an introduction to this approach, conditions for block diago nal decoupling via lsf were presented.

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