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Optimisation approach to robust eigenstructure assignment

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Abstract: A systematic optimisation approach to robust eigenstructure assignment for control systems with output feedback is presented. The proposed scheme assigns the maximum allowable number of closed-loop eigenvalues to desired locations, and determines the corresponding closed-loop eigenvectors as close to desired ones as possible. Additionally, the stability of the remaining closed-loop eigenvalues is guaranteed by the satisfaction of an appropriate Lyapunov equation. The overall design is robust with respect to time-varying parameter perturbations. The approach is applied to a literature example, where it is shown to capture the shape of the desired transient response.

1 Introduction

Eigenstructure assignment is a powerful technique that has developed considerably over the last 15 years or so; see for instance the seminal paper of [1], the review papers of [2–4] and the recent book of [5]. The technique is concerned with the placing of eigenvalues and their associated eigenvectors, via feedback control laws, to meet closed-loop design specifications. Specifically, the method allows the designer to satisfy damping, settling time and mode decoupling specifications directly by selecting appropriate closed-loop eigenvalues and eigenvectors. Note that comprehensive lists of papers dealing with eigenstructure assignment can be found in [4–6].

Eigenstructure assignment methodologies for the robust control of linear uncertain systems have appeared in [3, 7, 8]. In [7], an algorithm for robust eigenstructure assignment, which utilises a sufficient condition for robust stability, is proposed; constraints are placed on the desired modes. In [3, 8], constrained optimisation procedures are presented. The objective functions to be minimised are based on the transient response; constraints are placed on the desired modes and some sufficient conditions for robust stability. The interest here is in a formulation similar to that presented in [6], where an eigenstructure-assignment procedure for the state feedback case, which minimises the difference between the actual and desired closed-loop eigenvalues and eigenvectors, is presented. A robustness term is included in the objective function enlarge the class of nondestabilising perturbations. However, the inclusion of this term may result in closed-loop eigenvalues and eigenvectors considerably away from the desired ones.

In this paper, a systematic optimisation approach is proposed for the more general case of output feedback. This approach assigns the maximum allowable number of closed-loop eigenvalues to desired locations, and determines the corresponding closed-loop eigenvectors as close as possible to those desired. The structure of the achievable closed-loop eigenvectors is taken into consideration. The stability of the remaining closed-loop eigenvalues is maintained by the inclusion of a closed-loop Lyapunov equation in the objective function. As in [6], a robustness term is also included in the objective function enhance the robustness of the control scheme. Unlike in [6], though, the inclusion of this term does not affect the determined eigenstructure, since the actual eigenvalues can still be placed at the desired locations. Note that, for the state feed-case, as in [6], the proposed robust-eigenstructure-assignment approach here places all closed-loop eigenvalues at the desired locations.

2 Eigenstructure assignment using output feedback

Consider the linear multivariable continuous system with the state-space description

$$\dot{x}(t) = A\, x(t) + B\, u(t) \tag{1}$$

$$y(t) = C\, x(t) \tag{2}$$

where $x \in \Re^n$ is the state vector, $u \in \Re^r$ is the input vector, and $y \in \Re^q$ is the output vector; $A \in \Re^{n\times n}$, $B \in \Re^{n\times r}$, $C \in \Re^{q\times n}$ are the system matrices. The above system is assumed to be both controllable and observable, i.e.

$$\text{rank}\,[B\ AB \cdots A^{n-1}B] = n \tag{3}$$

$$\text{rank}\,[C^T\ A^TC^T \cdots (A^T)^{n-1}C^T]^T = n \tag{4}$$

It is assumed that the input and output matrices are of full rank, i.e. rank $(B) = r$ and rank $(C) = q$. Also, as is usually

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the case in aircraft problems, it is assumed that $r < q < n$. Consider static output feedback of the form

$$u(t) = K\, y(t) = KCx(t) \tag{5}$$

The freedom which characterises the placing of the closed-loop poles using output feedback has been studied extensively; see for instance [9, 10]. For the additional freedom that characterises the selection of the associated closed-loop eigenvectors, the following theorem has been proven in [10].

Theorem 2.1 Consider the controllable and observable system of eqns. 1 and 2 with the output feedback law of eqn. 5 and the assumption that the matrices B and C are of full rank. Then, there exists a matrix $K \in \Re^{r\times q}$ such that

(i) max(r, q) closed-loop eigenvalues can be assigned;
(ii) max(r, q) eigenvectors can be partially assigned with min(r, q) entries in each arbitrarily chosen vector.

Note that the above theorem also applies to the general case where the closed-loop eigenvalues can be repeated or in complex-conjugate pairs. Note that eigenvalue assignment for the state feedback case has been investigated thoroughly as well; see for instance [11, 12]. For controllable systems with the state feedback law $u(t) = Kx(t)$, it has been shown in [12] that:

(a) All n closed-loop eigenvalues and a maximum of nr eigenvector entries can be arbitrarily assigned, and

(b) No more than r entries of any one eigenvector can be chosen arbitrarily.

In other words, a maximum of r entries in each of the n closed-loop eigenvectors can be chosen arbitrarily. It is apparent that state feedback offers a greater flexibility with regard to eigenstructure assignment than does output feedback. Note, however, that, from a practical point of view, state feedback is undesirable, since for large systems it requires measurement and feedback of all states of the system. This can be expensive; in addition, several states are usually not available for measurement. This is the reason why it is usually preferable to feed back only the measured states, which makes output feedback very attractive. Note that an extensive discussion of eigenstructure assignment with respect to both state and output feedback can be found in [1].

3 Robust eigenstructure assignment

3.1 Problem formulation

For a linear multivariable system with the state-space description of eqns. 1 and 2, and the conditions of eqns. 3 and 4, an output feedback control gain (eqn. 5) needs to be determined such that:

(i) the set of closed-loop eigenvalues includes a subset of q closed-loop eigenvalues located at $\{\lambda_i,\ i = 1, \ldots, q\}$, and
(ii) their associated eigenvectors should be given by the set $\{v_i,\ i = 1, \ldots, q\}$.

Note that the desired q modes, as specified by the desired q pairs of eigenvalues and eigenvectors, can either be the most significant ones (such as the roll or Dutch roll modes in an aircraft-control problem), or the q most dominant ones in a control-reconfiguration scheme [13].

As is seen next, the desired eigenvectors are not always achievable. Therefore, the control design needs to determine closed-loop eigenvectors as close to the desired ones as possible. Therefore, if the actual closed-loop eigenvalues/eigenvectors are denoted by $\{(\lambda_i^a,\ v_i^a),\ i = 1, \ldots, n\}$, the above objectives are translated into

$$\lambda_i^a(A + BKC) = \lambda_i \qquad i = 1, \ldots, q \tag{6}$$

$$\min\left(\sum_{i=1}^{q} \|v_i^a - v_i\|^2\right) \tag{7}$$

For reasons explained in [1, 14], consider the state-transformation matrix $T = (B\ S)$, where S is selected such that rank $(T) = n$. In the new state co-ordinates specified by T above, the system is described by the matrices $(\tilde{A}, \tilde{B}, \tilde{C})$, with

$$\tilde{B} = T^{-1}B = \begin{pmatrix} I_r \\ O_{n-r,r} \end{pmatrix} \tag{8}$$

where $O_{n-r,r}$ is defined as an $[(n-r)\times r]$ zero matrix. Note the special structure of the input matrix $\tilde{B}$. Note also that the state transformation does not affect the output-feedback matrix. This is also true for the eigenvalues of the transformed system, which remain the same as the eigenvalues of the original system. However, the desired closed-loop eigenvectors $\{v_i,\ i = 1, \ldots, q\}$, together with the actual closed-loop eigenvectors $\{v_i^a,\ i = 1, \ldots, q\}$, need to be transformed to the new state co-ordinates. Define

$$\tilde{v}_i = T^{-1}v_i \tag{9}$$

$$\tilde{v}_i^a = T^{-1}v_i^a \tag{10}$$

as the desired and actual closed-loop eigenvectors for the state-transformed system, respectively. From now on, the discussion will continue by considering the system in the new state co-ordinates specified above. Therefore, the objective of eqn. 7 for the transformed system is given by

$$\min\left(\sum_{i=1}^{q} \|\tilde{v}_i^a - \tilde{v}_i\|^2\right) \tag{11}$$

As discussed in [1], all achievable eigenvectors $\tilde{v}_i^a$ which correspond to the closed-loop eigenvalue $\lambda_i^a = \lambda_i$ must lie in the subspace spanned by the columns of $(\lambda_i I_n - \tilde{A})^{-1}\tilde{B}$. Define

$$\tilde{\Pi}_i = (\lambda_i I_n - \tilde{A})^{-1}\tilde{B} \tag{12}$$

All achievable closed-loop eigenvectors that correspond to the eigenvalue λ_i should be of the form

$$\tilde{v}_i^a = \tilde{\Pi}_i g_i \tag{13}$$

where g_i is an $(r \times 1)$ vector. Note that g_i is a real vector if λ_i is a real eigenvalue, or a complex vector if λ_i is a complex eigenvalue. In view of eqn. 13, the objective of eqn. 11 is rewritten as

$$\min\left(\sum_{i=1}^{q} \|\tilde{\Pi}_i g_i - \tilde{v}_i\|^2\right) \tag{14}$$

and the minimising quantity is defined as

$$J_1' = \mathrm{Tr}\left\{\sum_{i=1}^{q} (\tilde{\Pi}_i g_i - \tilde{v}_i)^H (\tilde{\Pi}_i g_i - \tilde{v}_i)\right\} \tag{15}$$

where v^H denotes the complex-conjugate transpose of a vector v. It can easily be shown that each pair of closed-loop eigenvalues/eigenvectors should satisfy

$$(\tilde{A}_1 + K\tilde{C} - \lambda_i I_{r,n})\tilde{v}_i^a = 0$$

$$\Rightarrow\quad (\tilde{A}_1 + K\tilde{C} - \lambda_i I_{r,n})\tilde{\Pi}_i g_i = 0 \tag{16}$$

where $\tilde{A}_1$ contains the first r rows of $\tilde{A}$ and

$$I_{r,n} = (I_r \qquad O_{r,n-r}) \tag{17}$$

It can be seen that the vectors $\{g_i, i = 1, \ldots, q\}$ that minimise eqn. 15 also need to satisfy the eigenstructure condition of eqn. 16. Therefore, it is necessary to include this condition for the q eigenvectors of interest in the minimising quantity, which becomes

$$J_1 = \mathrm{Tr}\left[\sum_{i=1}^{q}(\tilde{\Pi}_i g_i - \tilde{v}_i)^H(\tilde{\Pi}_i g_i - \tilde{v}_i) + \sum_{i=1}^{q} M_i\left\{(\tilde{A}_1 + K\tilde{C} - \lambda_i I_{r,n})\tilde{\Pi}_i g_i\right\}\right] \tag{18}$$

where M_i, $i = 1, \ldots, q$ are $(1 \times r)$ Lagrange-multiplier vectors, which are real if they correspond to a real eigenvalue, or complex if they correspond to a complex eigenvalue. So far, this paper has concentrated on the q closed-loop eigenvalues that we want to assign with the procedure outlined above. However, it is necessary to guarantee that the remaining $(n - q)$ eigenvalues of the closed-loop system remain stable. Therefore, the output-feedback gain needs to be such that the closed-loop system $\tilde{A} + \tilde{B}K\tilde{C}$ is stable. In other words, it suffices to satisfy the Lyapunov equation

$$(\hat{A})^T P + P\hat{A} + Q = 0 \tag{19}$$

where

$$\hat{A} = \tilde{A} + \tilde{B}K\tilde{C} \tag{20}$$

As discussed in [6, 15, 16], it is also necessary to safeguard against uncertainties in the state-space matrices of the closed-loop system. It has been shown that, for the case of unstructured perturbations in the system matrix $\tilde{A}$, this can be done by including the term $\mathrm{Tr}(P^2)$ in the minimising quantity; the smaller this robustness term, the more robustly stable the closed-loop system will be to unstructured time-varying parameter perturbations. Therefore, the overall minimising quantity is given finally by

$$J = \mathrm{Tr}\left[\sum_{i=1}^{q}(\tilde{\Pi}_i g_i - \tilde{v}_i)^H(\tilde{\Pi}_i g_i - \tilde{v}_i) + L_1(\hat{A}^T P + P\hat{A} + Q) + \sum_{i=1}^{q} M_i\left\{(\tilde{A}_1 + K\tilde{C} - \lambda_i I_{r,n})\tilde{\Pi}_i g_i\right\} + P^2\right] \tag{21}$$

where $L_1 \in \Re^{n\times n}$ is another Lagrange-multiplier matrix. To summarise the approach outlined above, it should be stated that, with the minimisation of the quantity in eqn. 21 above, an output-feedback matrix K is sought such that

(*a*) A specified subset of q desired closed-loop eigenvalues belong to the set of eigenvalues of the closed-loop system $A + BKC$.
(*b*) The achieved eigenvectors are as close to the corresponding desired eigenvectors as possible.
(*c*) The remaining $(n - q)$ closed-loop eigenvalues are stable.
(*d*) Uncertainties in the state-space matrices, in the form of unstructured time-varying parameter perturbations, are taken care of by maximising the stability margin allowed to the closed-loop system.

3.2 Algorithmic approach

Without loss of generality, it is assumed that the set of q desired eigenvalues consists of a complex conjugate pair, i.e. $\lambda_1 = (\lambda_2)^* \in \mathbb{C}$, and $(q - 2)$ real eigenvalues, i.e. is $\{\lambda_i \in \Re, i = 3, \ldots, q\}$. Then, $\tilde{v}_1 = (\tilde{v}_2)^*$. The generalisation to the case of more complex conjugate pairs of eigenvalues is straightforward.

It is necessary to compute the partial derivatives of the minimising quantity of eqn. 21 with respect to all the matrix parameters entailed. These parameters are the Lagrange-multiplier vectors $\{M_i, i = 1, \ldots, q\}$, the Lagrange-multiplier matrix L_1, the positive-definite matrix P, the output-feedback matrix K and the vectors $\{g_i, i = 1, \ldots, q\}$ that specify the closed-loop eigenvectors. Using the properties of [17] one obtains

$$\frac{\partial J}{\partial M_i} = \Delta_{M_i} = [(\tilde{A}_1 + K\tilde{C} - \lambda_i I_{r,n})\tilde{\Pi}_i g_i]^T \qquad i = 1, \ldots, q \tag{22}$$

$$\frac{\partial J}{\partial L_1} = \Delta_{L_1} = \hat{A}^T P + P\hat{A} + Q \tag{23}$$

$$\frac{\partial J}{\partial P} = \Delta_P = \hat{A}L_1^T + L_1^T\hat{A}^T + 2P \tag{24}$$

$$\frac{\partial J}{\partial K} = \Delta_K = \tilde{B}^T P L_1 \tilde{C}^T + \tilde{B}^T P L_1^T \tilde{C}^T + \sum_{i=1}^{q} M_i^T g_i^T \tilde{\Pi}_i^T \tilde{C}^T \tag{25}$$

$$\frac{\partial J}{\partial g_1} = \Delta_{g_1} = 2\tilde{\Pi}_2^H\tilde{\Pi}_2 g_2 - 2\tilde{\Pi}_2^H\tilde{v}_2 + \tilde{\Pi}_1^T(\tilde{A}_1 + K\tilde{C} - \lambda_1 I_{r,n})^T M_1^T \tag{26}$$

$$= \left(\frac{\partial J}{\partial g_2}\right)^* \tag{27}$$

$$\frac{\partial J}{\partial g_i} = \Delta_{g_i} = 2\tilde{\Pi}_i^T\tilde{\Pi}_i g_i - 2\tilde{\Pi}_i^T\tilde{v}_i + \tilde{\Pi}_i^T(\tilde{A}_1 + K\tilde{C} - \lambda_i I_{r,n})^T M_i^T \quad i = 3, \ldots, q \tag{28}$$

The derivation of eqn. 26, and the equivalence of eqn. 27, can easily be shown [14]. To minimise eqn. 21 a version of the Broyden-Fletcher-Goldfarb-Shanno (BFGS) optimisation method of conjugate directions is used. Note that, in each algorithmic step, the gradients of eqns. 22–25 are set equal to zero, i.e. $\{\Delta_{M_i} = 0, i = 1, \ldots, q\}$, $\Delta_{L_1} = 0$, $\Delta_P = 0$, $\Delta_K = 0$, and solve for K, P, L_1, $\{M_i, i = 1, \ldots, q\}$, respectively, in that specific order; then a line search is performed to update the vectors $\{g_i, i = 1, \ldots, q\}$ using eqns. 26 and 28. The optimal K determined by the above approach is also the optimal gain for the system in the original state co-ordinates. The algorithm also determines the optimal vectors $\{g_i, i = 1, \ldots, q\}$ and therefore, in view of eqn. 13, the optimal eigenvectors $\{\tilde{v}_i^a, i = 1, \ldots, q\}$; note that these eigenvectors need to be transformed back to the original state co-ordinates using eqn. 10.

The proposed algorithm is presented in detail in [14], which is available via anonymous FTP. Note that there are significant changes compared with similar algorithms used in [16, 18]. This is due to the structure of the present problem, since the vectors $\{g_i, i = 1, \ldots, q\}$ are now updated instead of the output-feedback matrix. On the other hand, the existence of complex eigenvalues/eigenvectors imposes certain modifications to the algorithmic scheme.

There are certain advantages in using the algorithmic approach above, since it can easily be extended to include more terms in the minimising quantity. These terms could be associated with an LQR performance, as studied in [18], or with the robust stability of the closed-loop system with respect to time-varying unstructured/structured perturbations in the input, output matrices $\tilde{B}$, $\tilde{C}$ as well. A similar generalisation has appeared in [18]. For example, for the case of unstructured perturbations in both $\tilde{A}$ and $\tilde{B}$, it can easily be shown that it is only necessary to add the term $\mathrm{Tr}\{(K\tilde{C})^T\ (K\tilde{C})\}$ in the minimising quantity of eqn. 21, which results in the term $2K\tilde{C}\tilde{C}^T$ being added in eqn. 25.

In some practical applications, complete specification of the desired closed-loop eigenvectors is not needed [3]. In such cases, the interest is only on certain components of the eigenvectors that are related to design specifications such as mode decoupling, whereas the remaining eigenvector components can be varied freely. The proposed optimisation scheme here can easily accommodate the case of unspecified components in the desired closed-loop eigenvectors. This can be done by considering only the significant components of the difference vectors in the minimising quantity of eqn. 14; this results in obvious changes in only the first two terms of eqns. 26 and 28.

4 Illustrative example

The optimisation approach for robust eigenstructure assignment presented here is used in the control-reconfiguration scenario of [13]. For the following nominal system

$$A = \begin{bmatrix} -0.0582 & 0.0651 & 0 & -0.171 \\ -0.303 & -0.685 & 1.109 & 0 \\ -0.0715 & -0.658 & -0.947 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 \\ -0.0541 & 0 \\ -1.11 & 0 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{29}$$

with the static output-feedback law of eqn. 5, the controller that assigns the closed-loop eigenvalues at $\{-0.5973, -1.5\pm j2, -2\}$ and their corresponding eigenvectors at

$$V = [v_1 \quad v_2 \quad v_3 \quad v_4]$$

$$= \begin{bmatrix} -0.1887 & 0.1465+j0.0958 & 0.1465-j0.0958 & 0.9680 \\ -0.9634 & 0.2257-j0.2492 & 0.2257+j0.2492 & 0.1441 \\ -0.0977 & 0.3790+j0.6047 & 0.3790-j0.6047 & 0.0905 \\ 0.1636 & 0.1025-j0.2664 & 0.1025+j0.2664 & -0.0453 \end{bmatrix} \tag{30}$$

is given in [13] by

$$K = \begin{bmatrix} -0.00031 & 4.77004 & 1.70457 \\ -2.01505 & -1.13002 & 0.02904 \end{bmatrix} \tag{31}$$

Next, assume that the system dynamics change due to system-component failures. The state-space matrices of the impaired model are given below [13]:

$$A_f = \begin{bmatrix} -0.0582 & 0.10 & 0.0 & -0.171 \\ -0.103 & -0.685 & 1.109 & 0 \\ -0.0715 & -0.658 & 1.98 & 0 \\ 0 & 0 & 1.5 & 0 \end{bmatrix}$$

$$B_f = \begin{bmatrix} 0 & 0.9 \\ -0.09 & 0.0 \\ -1.11 & 0.0 \\ 0 & 0.0 \end{bmatrix} \quad C_f = \begin{bmatrix} 0.9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{32}$$

The desired eigenstructure for the impaired system is that specified above for the nominal system; therefore, the objective (as specified in [13]) is to preserve the first three most dominant eigenvalues of the nominal closed-loop system i.e. $\{-0.5973, -1.5\pm j2\}$, and achieve closed-loop eigenvectors as close to the corresponding eigenvectors of eqn. 30 as possible. The algorithmic approach proposed here is used to find the optimal output-feedback matrix K_f that will maintain the nominal eigenstructure shown above; in other words, the algorithm determines the controller gain that minimises J of eqn. 21 for the impaired system.

First it is necessary to transform the impaired system (A_f, B_f, C_f) to new state co-ordinates. Select

$$T = \begin{bmatrix} 0 & 0.9 & 0 & 0 \\ -0.09 & 0 & 1 & 0 \\ -1.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{33}$$

The best results, with regard to the closeness of the closed-loop eigenvectors of the impaired system to the desired eigenvectors specified in eqn. 30, are obtained when a weight factor of 0.1 is assigned to the term $\{(\tilde{\Pi}_1 g_1 - \tilde{v}_1)^T (\tilde{\Pi}_1 g_1 - \tilde{v}_1)\}$ of the minimising quantity of eqn. 21. This is the term that corresponds to the real eigenvalue -0.5973. By assigning this weight, it is possible to emphasise the task of achieving optimal eigenvectors for the complex conjugate pair of eigenvalues, $(-1.5\pm j2)$. Note that this task is the most difficult to achieve due to the complex nature of the corresponding eigenvectors. The introduction of this weight factor affects only eqn. 28, whose first two terms $\{2\tilde{\Pi}_1^T \Pi_1 g_1 - 2\tilde{\Pi}_1^T \tilde{v}_1\}$ need to be multiplied by this weight factor. The algorithm yields

$$\|\tilde{v}_1^f - \tilde{v}_1\|^2 = 0.0242 \tag{34}$$

$$\|\tilde{v}_2^f - \tilde{v}_2\|^2 = \|\tilde{v}_3^f - \tilde{v}_3\|^2 = 0.0230 \tag{35}$$

$$\mathrm{Tr}\,(P^2) = 0.0788 \tag{36}$$

Note that the robustness term of eqn. 36 implies that the closed-loop system $\tilde{A}$ is robustly stable to unstructured time-varying parameter perturbation given by $\sigma_{max}(\Delta\tilde{A}) < 0.4037$. The output-feedback gain that achieves these results is

$$K_f = \begin{bmatrix} -4.42776 & 5.95419 & 5.59306 \\ -4.15014 & -0.71481 & 0.49365 \end{bmatrix} \tag{37}$$

With the above controller, the fourth closed-loop eigenvalue is placed at -4.7358. Note that the above results concern the impaired system in the new state co-ordinates specified by eqn. 33. However, the controller is the same, as discussed above. The eigenvectors obtained transformed back to the original co-ordinates of the impaired system are given by

$$V_f = [v_1^f \quad v_2^f \quad v_3^f]$$

$$= \begin{bmatrix} -0.0674 & 0.1424+j0.0945 & 0.1424-j0.0945 \\ -0.9950 & 0.1834-j0.1722 & 0.1834+j0.1722 \\ -0.0453 & 0.3519+j0.5667 & 0.3519-j0.5667 \\ 0.1136 & 0.1453-j0.3729 & 0.1453+j0.3929 \end{bmatrix} \tag{38}$$

where the first column is the eigenvector that corresponds to the real eigenvalue -0.5973, and the last two columns are the eigenvectors that correspond to the complex conjugate pair of eigenvalues $(-1.5\pm j2)$. As can be seen, the above eigenvectors are indeed very close to the desired eigenvectors of eqn. 30, as suggested by eqns. 34 and 35. This can also be shown by computing

$$\|v_1^f - v_1\|^2 = 0.0231 \tag{39}$$

$$\|v_2^f - v_2\|^2 = \|v_3^f - v_3\|^2 = 0.0210 \tag{40}$$

In Figs. 1 and 2, the state response of the nominal system of eqn. 29 is compared with the output-feedback matrix $\boldsymbol{K}$ of eqn. 31, and the state response of the impaired system of eqn. 32 is compared with the output feedback matrix $\boldsymbol{K}_f$ of eqn. 37. The initial condition vector is chosen as

$$V_{in} = (0.75 \quad 0.5 \quad 0.3 \quad 1)^T \tag{41}$$

As can be seen, the algorithm is capable of recovering the performance of the nominal system. This should be expected, since the eigenvectors of the impaired closed-loop system are assigned to be very close to the eigenvectors of the nominal closed-loop system, as shown in eqns. 39 and 40.

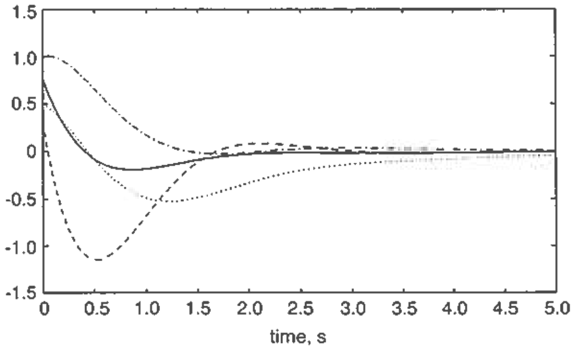

Fig. 1 *Nominal system*
Closed-loop state response for the initial condition vector V_{in}
—— x_1
····· x_2
– – – x_3
–·– x_4

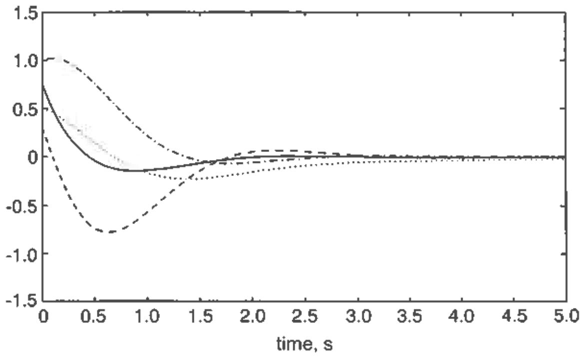

Fig. 2 *Impaired system*
Closed-loop state response for the initial condition vector V_{in}
—— x_1
····· x_2
– – – x_3
–·– x_4

5 Conclusions

An optimisation approach to robust eigenstructure assignment for systems with output feedback has been presented. The proposed algorithm assigns the maximum allowable number of closed-loop eigenvalues to desired locations, and determines the corresponding eigenvectors as close to the desired ones as possible. The overall design is robust with respect to unstructured time-varying parameter perturbations. The approach has been applied to a literature example, where it was shown to achieve the nominal/ideal eigenstructure.

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