P.J. Antsaklis, "Quasi-Determinism in the Supervisory Control of Hybrid Systems," Proceedings of the ARO Workshop 2000 on Intelligent Systems, Section 7, Australian National University, Canberra, Australia, December 8-9, 2000.

QUASI-DETERMINISM IN THE SUPERVISORY CONTROL OF HYBRID SYSTEMS

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EXTENDED ABSTRACT

Hybrid systems are dynamical systems the behavior of which is determined by interacting continuous and discrete dynamics. When the continuous and discrete dynamics coexist and interact with each other it is important to develop models that accurately describe the dynamic behavior of such hybrid systems. In a manufacturing process, for example, parts may be processed in a particular machine, but only the arrival of a part triggers the process. That is, the manufacturing process is composed of the event-driven dynamics of the parts moving among different machines and the time-driven dynamics of the processes within particular machines. Frequently in hybrid systems in the past, the event-driven dynamics were studied separately from the time-driven dynamics, the former via automata or Petri net models (also via PLC, logic expressions etc.) and the latter via differential or difference equations. To fully understand the system's behavior and meet high performance specifications one needs to model all dynamics together with their interactions. Only then problems such as optimization of the whole process may be addressed in a more meaningful manner. There are of course cases where the time-driven and event-driven dynamics are not tightly coupled and/or the demands on the system performance are not difficult to meet, and in those cases considering simpler separate models for the distinct phenomena may be adequate. However hybrid models must be used when there is significant interaction between the continuous and discrete parts and high performance specifications are to be met by the system. For a rather comprehensive description of hybrid systems research, see [1] and the references therein.

Here, a novel, promising approach to hybrid control controller synthesis that has been recently introduced will be described. Details may be found in [2]; earlier versions have appeared in [3-5]. The methodology offers promise in successfully addressing some of the open issues in the hybrid control literature, such as robustness issues.

It is a supervisory control approach based on discrete abstractions. It emphasizes control synthesis, contrary to other approaches that emphasize primarily the analysis of properties such as stability and reachability. In this approach hybrid systems are represented in a general framework that makes possible to describe explicitly what control actions are available and to define open loop and closed loop connections between the plant and the controller, so to exploit the advantages of feedback. The regulator problem for hybrid systems is formulated and a feedback architecture is used to modify and improve the hybrid system properties so the desired specifications can be met.

The approach uses discrete-time models for the systems to be controlled, focuses on piecewise linear systems and develops strong theoretical results and efficient algorithms. Such piecewise linear hybrid systems arise when the state set and/or the input set are partitioned into regions described by linear

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equalities and inequalities and the dynamics in each region are described by linear (or affine) state transitions. Output and measurement maps can be defined also in a similar way. The class of piecewise-linear systems is quite general as it includes linear systems, finite state machines, and their interconnections. They can be used also in many instances as approximations of more general systems. Piecewise linear hybrid dynamical systems have an efficient representation for modeling and simulation. Furthermore, current modeling tools such as Matlab, Simulink, and Stateflow offer the necessary flexibility for modeling and simulation of this class of systems.

The approach first considers the design of the continuous-to-discrete dynamics interface and explicitly uses control actions and considers disturbances. It uses the concept of quasi-determinism to decide on a appropriate partitioning of the state space. Although the notion of quasi-determinism was developed for refinement of the state space so to be able to control the system, it connects quite nicely to the work on verification in hybrid automata, where abstractions are used to develop algorithms for verification of properties such as safety and reachability.

In more detail, in order to analyze hybrid systems and design control algorithms, it is desirable to induce dynamical systems in finite quotient spaces that preserve the properties of interest and then study the simplified models. The main characteristic of the approach is that the available control inputs are taken into consideration in order to simplify the system. The main mathematical tool to be used is the predecessor operator applied recursively to subsets of the hybrid state space. The application of the predecessor operator corresponds to partition refinement into finer partitions that allow the formulation of conditions that guarantee the existence of appropriate controls to satisfy the control specifications. Control specifications are formulated with respect to a partition of the state space of the system. Examples include safety problems, where the controller guarantees that the plant will not enter an unsafe region for example guaranteeing that two interacting robots will not collide. Also reachability problems where the controller drives the plant from an initial operating region or state to a desired one; this is the case for example in the startup procedure of a chemical plant. In order to study safety specifications for piecewise hybrid dynamical systems, we introduce the notion of quasi-determinism. Quasi-determinism represents the case when the future behavior only for the next time interval of the original system can be uniquely determined by the current state of the induced system. Based on the desired control specifications a primary partition is first defined and it is appropriately refined to a final partition. The safety conditions can be tested using efficient linear programming techniques. We also present an algorithm for the computation of the maximal safe set. Reachability conditions are also formulated. Our approach is based on conditions that guarantee that the state can be forced to reach a desirable region of the state space by selecting appropriate controls.

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