Synthesis of Supervisors Enforcing General Linear Constraints in Petri Nets

Marian V. Iordache and Panos J. Antsaklis

***Abstract*—Efficient techniques exist for the design of supervisors enforcing constraints consisting of linear marking inequalities. This note shows that without losing the benefits of the prior techniques, the class of constraints can be generalized to linear constraints containing marking terms, firing vector terms, and Parikh vector terms. We show that this extended class of constraints is more expressive. Furthermore, we show that the extended constraints can describe any supervisor consisting of control places arbitrarily connected to the transitions of a plant Petri net (PN). The supervisor design procedure we propose is as follows. For PNs without uncontrollable and unobservable transitions, a direct method for the design of a PN supervisor that is least restrictive is given. For PNs with uncontrollable and/or unobservable transitions, we reduce the problem to the design of supervisors enforcing linear marking inequalities.**

***Index Terms*—Linear constraints, Petri nets (PNs), supervisory control.**

I. Introduction

Efficient methods have been proposed in [1]–[4] for the design of supervisors enforcing that the marking μ of a Petri net (PN) satisfies constraints of the form

$$L\mu \leq b \tag{1}$$

where $L \in \mathbb{Z}^{n_c \times m}$, $b \in \mathbb{Z}^{n_c}$, $\mathbb{Z}$ is the set of integers, m is the number of places, and n_c the number of constraints. The methods address both the fully controllable and observable PNs and the PNs which may have uncontrollable and unobservable transitions. Constraints of the form (1) can describe (generalized) mutual exclusion, deadlock prevention constraints, and others [3]. The constraints (1) have been extended in [3] and [4] to the form

$$L\mu + Hq \leq b \tag{2}$$

which adds a firing vector term, where $H \in \mathbb{N}^{n_c \times n}$ and n is the number of transitions. (Without loss of generality, H has been assumed to have nonnegative elements.) In such constraints, an element q_i of the firing vector q is set to 1 if the transition t_i is to be fired next from μ; else $q_i = 0$. The constraint is interpreted as follows. A supervisor enforcing (2) ensures that: 1) all markings μ must satisfy (1) and 2) if $\mu \xrightarrow{t_i} \mu'$, t_i is allowed to fire only if $L\mu + Hq \leq b$ and $L\mu' \leq b$. The form (2) describes constraints on the enabling of transitions [as opposed to the constraints on the state, naturally described by (1)] [3], [5]. In this note, we consider constraints which add to (2) a Parikh vector term

$$L\mu + Hq + Cv \leq b \tag{3}$$

where $C \in \mathbb{Z}^{n_c \times n}$. In (3), v is the Parikh vector, that is v_i, the ith element of v, counts how often the transition t_i has fired since system initialization. The constraint is interpreted as follows. A supervisor enforcing (3) ensures that: 1) all states (μ, v) satisfy $L\mu + Cv \leq b$ and 2) if q is the firing vector of a transition t_i, $\mu \xrightarrow{t_i} \mu'$, and $v' = v + q$, then $L\mu + Hq + Cv \leq b$ and $L\mu' + Cv' \leq b$. Note that the Parikh vector term may also be viewed as a marking term in a PN extended with sink places on transitions. Regardless of the viewpoint, whether we look at the constraints (3) as involving the Parikh vector or the markings of additional sink places, it is apparent that such constraints need to be considered, as they effectively increase the expressivity power of the constraints (2). In fact, we will show that (3) can represent any supervisor implemented by additional places (*control places*) connected to the transitions of a plant PN. This means that the operation of any Petri net can be entirely described by constraints (3), with a one-to-one correspondence between each place and each inequality of (3). We also show that (3) are as expressive as the constraints of the form

$$Hq + Cv \leq b. \tag{4}$$

While the marking term in (3) does not make (3) more expressive, in practice it may be more intuitive to write constraints that involve also the marking. This is one reason we consider constraints of the form (3) instead of just (4). Note that Parikh vector terms can be used to describe fairness requirements, such as the constraint that the difference between the number of firings of two transitions is limited by one.

The contribution of this note is as follows. Section II-A makes the observation that any place of a PN can be seen as a supervisor place enforcing a constraint of the form (4). This has been known for constraints of the form $Cv \leq b$ and PNs without self-loops [6]. A manufacturing illustration involving constraints of the form (3) is presented in Section II-B. The supervisor design for specifications (3) is presented in Section III-A for fully controllable and observable PNs, and in Section III-B for the PNs that may have uncontrollable and unobservable transitions. In the latter case, we reduce our problem to the design of supervisors enforcing constraints of the form (1), for which effective methods exist. Note that our supervisor design approach extends also the indirect method of [3] on enforcing constraints (2), as both coupled and uncoupled constraints can be considered. Note also that our approach can be naturally extended for the enforcement of constraints involving both conjunctions and disjunctions of linear inequalities.

Due to space limitations, we refer the reader to [7] for the proofs of the results.

II. On the Significance of the General Linear Constraints

A. Representing the Operation of PNs Via Generalized Linear Constraints (GLCs)

This section shows that the operation of any PN can be described by constraints of the form (4). Given a PN, let D^+ and D^- denote the input and output matrices, and $D = D^+ - D^-$ the incidence matrix. We denote by t_i the transition corresponding to the column i of D.

The common algebraic PN representation is via the state equation

$$\mu = \mu_0 + Dv \tag{5}$$

where μ_0 is the initial marking. From (5), we derive $(-D)v \leq \mu_0$. Let $C = -D$. For any PN without self-loops, the inequality $Cv \leq \mu_0$ determines the operation of the PN. Indeed, after firing from μ_0 a firing sequence σ of Parikh vector v, we have that 1) $Cv \leq \mu_0$ and 2) a transition t_i is enabled iff $C(v + q^{(i)}) \leq \mu_0$, where $q^{(i)}$ is the firing vector q such that $q_i = 1$ and $q_j = 0$ for all $j \neq i$. As the incidence matrix D is insufficient to determine whether a transition is enabled in a PN with self loops, the additional term Hq is introduced

$$Hq + Cv \leq \mu_0 \tag{6}$$

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where $H = D^-$. Note that $H_{i,j} \geq 0$ for all indexes i and j. The constraints (6) completely describe the operation of a PN, regardless of whether it has self loops or not. Indeed, after we fire from μ_0 a sequence σ of Parikh vector v, the transition t_i is enabled iff $Hq^{(i)} + Cv \leq \mu_0$ and $C(v + q^{(i)}) \leq \mu_0$. (Note that as $H = D^-$ and $C = -D$, we have that $Hq^{(i)} + Cv \leq \mu_0 \Rightarrow C(v + q^{(i)}) \leq \mu_0$.)

As an example, consider the PNs of Fig. 1(a)–(c). The PN in Fig. 1(a) is not restricted: the firings of t_1, t_2 and t_3 are free. Thus H and C are empty matrices. By adding the places p_1, p_2 and p_3 as in the PN (b), we obtain the following inequalities for (6): $v_1 \leq 3$, $v_2 - v_3 \leq 0$, and $-v_2 + v_3 \leq 1$, where the inequalities are generated, in this order, by p_1, p_2, and p_3. The inequalities of the PN in Fig. 1(c) are: $q_1 + v_2 \leq 3$, $v_2 - v_3 \leq 0$, and $-2v_1 - v_2 + v_3 \leq 1$.

Given a PN $\mathcal{N} = (P, T, D^-, D^+)$, let P, T, T_{uc} and T_{uo} denote the sets of places, transitions, uncontrollable transitions, and unobservable transitions, respectively. Let $P_o : \mathbb{N}^{|T|} \to \mathbb{N}^{|T \setminus T_{uo}|}$ be the projection excluding from v the entries corresponding to unobservable transitions. In this note, a **supervisor** is a map[1] $\Xi : \mathcal{M} \times \mathbb{N}^{|T\setminus T_{uo}|} \to 2^{T\setminus T_{uc}}$, where $\mathcal{M}$ is a set of initial markings. When Ξ supervises $(\mathcal{N}, \mu_0)$, a controllable transition t is enabled at the state (μ, v) (where $\mu = \mu_0 + Dv$) if $t \in \Xi(\mu_0, P_o(v))$ and $\mu_0 \in \mathcal{M}$. For simplicity, we also call *supervisor* the PN implementation of a supervisor Ξ. $\mathcal{N}$ is in **closed-loop** with Ξ when Ξ supervises the operation of $\mathcal{N}$. We denote by $(\mathcal{N}, \mu_0, \Xi)$ the PN $(\mathcal{N}, \mu_0)$ in closed-loop with Ξ, and by $\mathcal{R}(\mathcal{N}, \mu_0, \Xi)$ the set of all reachable states (μ, v) of $(\mathcal{N}, \mu_0, \Xi)$.

A place of the PN implementation of a supervisor is said to be a **control place**. For instance, in Fig. 1(d) the place C is a control place implementing a supervisor. The PN of Fig. 1(d) illustrates also the fact that the extended linear constraints (3) are more expressive than the marking constraints. Indeed, the closed-loop of Fig. 1(d) has no place invariants, and so the supervisor cannot be described by (2). However, it can be described as the supervisor enforcing $-v_1 + v_2 + v_3 \leq 1$. Note also that every place of a PN can be seen as a control place restricting the firings of the net transitions according to a constraint (3). Indeed, in view of (6), the constraint of each place p_i is $hq + cv \leq \mu_{0\,i}$, where h and c are the i'th rows of H and C. This proves the following.

Proposition 1: Every place of a PN can be seen as a control place enforcing a single inequality of the form (3).

B. Manufacturing Illustration

This section illustrates the use of the constraints (3). The application of the constraints (1) is illustrated in [2], [3], and applications of the constraints (2) can be found in [2], [3], and [5]. The PN of Fig. 2 models a manufacturing cell in which autonomous vehicles (AVs) can enter a restricted area (RA) from the left and from the right. The left AVs enter the RA via t_2 and exit via t_{13}; the right AVs enter via t_5 and exit via t_{14}. The number of AVs in the RA is limited to m. Moroever, left and right AVs should not be at the same time in the RA. These constraints can be written as

$$mq_2 \leq m - v_5 + v_{14} \quad (7)$$

$$mq_5 \leq m - v_2 + v_{13}. \quad (8)$$

The marking of p_1 (p_2) represents the number of left (right) AVs that wait to enter the RA. Such an AV may be rerouted to another RA via t_3 (t_6). The constraint that a left AV should stay in the line if there is no right AV in p_2 or in the RA can be written as

$$q_3 \leq \mu_2 + v_5 - v_{14}. \quad (9)$$

[1] $|X|$ denotes the number of elements of X.

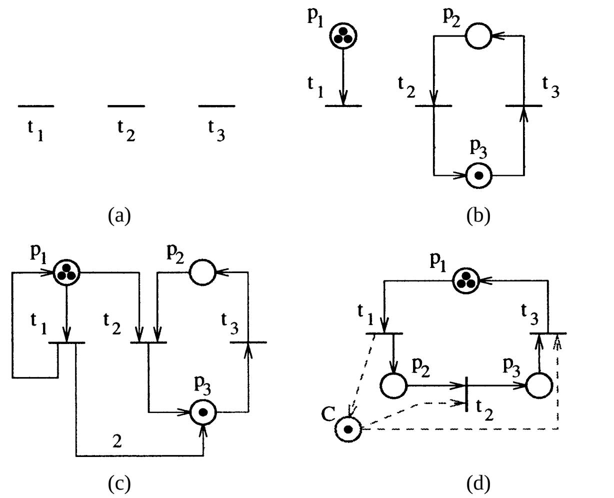

Fig. 1. PNs used in the examples of Section II-A. The markings displayed are initial markings.

Assuming we desire t_{11} and t_{12} to fire approximately the same number of times, we have the following fairness constraints (similar to those of the example of [6]):

$$v_{11} - v_{12} \leq n \quad (10)$$

$$v_{12} - v_{11} \leq n. \quad (11)$$

To restrict the firing of t_2 when $v_{11} - v_{12} \geq k$ (for $k < n$), we can write

$$(n - k)q_2 \leq n - (v_{11} - v_{12}). \quad (12)$$

Note that the places p_7 and p_8 can be introduced in the PN model to represent the fact that $v_{13} \leq v_2$ and $v_{14} \leq v_5$.

III. Enforcing General Linear Constraints

A. Supervisor Design for Fully Controllable and Observable PNs

A least restrictive supervisor can be constructed as follows. Let $D^+_{lc} = \max(0, -LD - C)$ and $D^-_{lc} = \max(0, LD + C)$, where the $\max$ operator is defined as follows. If X is a matrix, $Y = \max(0, X)$ is the matrix of elements $Y_{ij} = 0$ for $X_{ij} < 0$, and $Y_{ij} = X_{ij}$ for $Y_{ij} \geq 0$. For two matrices X and Y of the same size, $Z = \max(X, Y)$ is the matrix of elements $Z_{ij} = \max(X_{ij}, Y_{ij})$. Then, define

$$D^+_c = D^+_{lc} + \max(0, H - D^-_{lc}) \quad (13)$$

$$D^-_c = \max(D^-_{lc}, H). \quad (14)$$

The matrices D^+_c and D^-_c describe a PN structure with the same transitions as the plant. This PN structure represents the PN implementation of the supervisor. This means that the closed-loop is the PN of input and output matrices

$$D^+_o = \begin{bmatrix} D^+ \\ D^+_c \end{bmatrix} \text{ and } D^-_o = \begin{bmatrix} D^- \\ D^-_c \end{bmatrix}.$$

Let μ_{c0} and μ_0 be the initial markings of the supervisor and of the plant; μ_{c0} is set to

$$\mu_{c0} = b - L\mu_0. \quad (15)$$

Except for D^+_{lc} and D^-_{lc}, which add the C term, this construction is identical to that of [8] for constraints of the form (2).

Theorem 1: The supervisor defined by the incidence matrices D^+_c, D^-_c, and the initial marking μ_{c0} enforces (3) and is least restrictive.

B. Supervisor Design for Partially Controllable and Observable PNs

The approach we propose can be divided into the following steps. Given (3) and $\mathcal{N}$, a supervisor design problem for a specification (1) and a transformed net $\mathcal{N}_{HC}$ is solved first. Then, the solution to this problem is used to derive a solution to (3) and $\mathcal{N}$. The details follow next.

1) Admissibility and Transformations to Admissible Constraints: A set of constraints is *admissible* if the constraints can be enforced as in Section III-A, in spite of the inability to detect or control certain transitions. Formally, the following holds.

Definition 1: Given a set of constraints (3) on a PN $(\mathcal{N}, \mu_0)$, consider the construction of Section III-A. The set of constraints (3) is **admissible** if for all reachable states (μ, v) of the closed-loop net, the following are true.

1) If t is uncontrollable and $\mu|_{\mathcal{N}}$ enables[2] t in $\mathcal{N}$, then μ enables t in the closed-loop net.
2) If t is unobservable and μ enables t, then $D_c^+(\cdot, t) = D_c^-(\cdot, t)$.

Note that the condition 2) of the definition corresponds to the requirement that the firing of unobservable transitions should not change the marking of the control places. Obviously, a sufficient condition for admissibility that is easy to test is

$$D_c^-(\cdot, t) = 0 \; \forall\, t \in T_{uc} \text{ and } D_c^-(\cdot, t) = D_c^+(\cdot, t) \; \forall\, t \in T_{uo}. \quad (16)$$

We propose to use (16) to test whether the constraints can be enforced as in Section III-A. On the other hand, when (16) is not satisfied, we propose to transform (3) to

$$L_a \mu + H_a q + C_a v \le b_a \quad (17)$$

such that $L_a\mu + H_a q + C_a v \le b_a \Rightarrow L\mu + Hq + Cv \le b$ and (17) is admissible. Then, we can enforce (17) as in Section III-A, while the supervisor enforcing (17) is guaranteed to enforce (3) also. In the remaining part of this note, we propose an approach that reduces the transformation of (3) to (17) to the transformation to admissible constraints of constraints (1), for which several methods are already available in the literature. The reduction technique uses the PN transformations defined next.

2) C-Transformation and H-Transformation: It is desired to transform PNs such that the constraints (3) map into constraints (1). Parikh vector terms can be easily transformed to marking terms by adding sink places to transitions. For instance, in Fig. 3, the constraint $\mu_1 + q_1 + v_2 - v_3 \le 3$ on the PN in Fig.1(a) is equivalent to $\mu_1 + q_1 + \mu_4 - \mu_5 \le 3$ on the PN (b). The inverse transformation is also possible: $\mu_1 - 3\mu_4 + 2\mu_5 + q_1 \le 5$ on the PN in Fig.1(b) can be mapped into $\mu_1 + q_1 - 3v_2 + 2v_3 \le 5$ in the PN in Fig.1(a). The direct transformation is called the **C-transformation**, and the inverse the **C^{-1}-transformation**. Note that the input of the C-transformation is a set of constraints (3) and a PN $(\mathcal{N}, \mu_0, T_{uc}, T_{uo})$, while the output is a set of constraints $L_C \mu_C + Hq \le b$ and a PN $(\mathcal{N}_C, \mu_0, T_{uc}, T_{uo})$. On the other hand, the input of the C^{-1}-transformation is $(\mathcal{N}, \mu_0, T_{uc}, T_{uo})$, $\mathcal{N}_C$, and $L_C\mu_C + Hq \le b$, while the output is a set of constraints (3).

The H-transformation removes the firing vector terms. It improves the indirect method for enforcing firing vector constraints in [2]. As an illustration, consider the constraint $\mu_1 + \mu_2 + 2\mu_3 + q_3 \le 5$ on the PN of Fig. 3(c). The H-transformation is the PN (d). The transformation adds a place and a transition which correspond to the factor q_3. The transformed constraint is $\mu_1 + \mu_2 + 2\,\mu_3 + 4\,\mu_5 \le 5$, where the term $4\,\mu_5$ is obtained as follows. Consider firing t_3 in the transformed net. If $\mu \xrightarrow{t_3} \mu'$ and a is the coefficient of μ_5, we desire $a + \mu_1' + \mu_2' + 2\,\mu_3' = 1 + \mu_1 + \mu_2 + 2\,\mu_3$, where the factor 1 is the coefficient of q_3. Thus, we obtain $a = 4$. The H- and H^{-1}-transformations are defined next:

H-Transformation

[2]We denote by $\mu|_{\mathcal{N}}$ the restriction of μ to the places of $\mathcal{N}$.

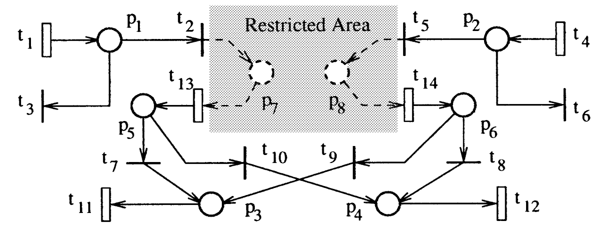

Fig. 2. PN model of a manufacturing cell.

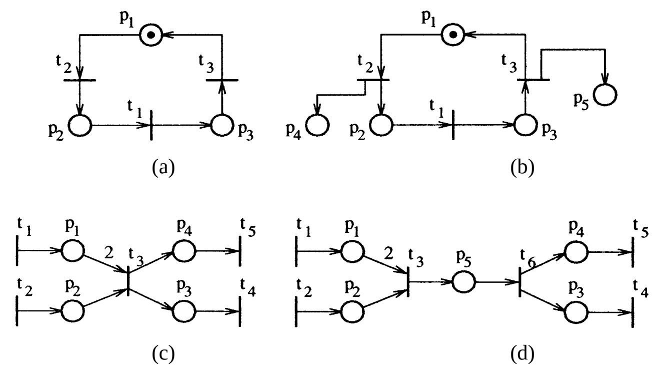

Fig. 3. Illustration of the C-transformation and of the H-transformation.

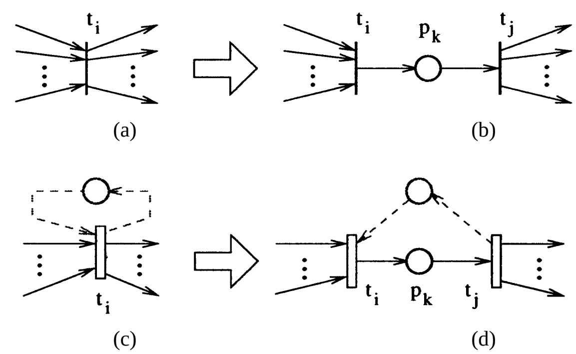

Fig. 4. Illustration of the transition split operation.

Input: The PN $(\mathcal{N}, T_{uc}, T_{uo})$ with $\mathcal{N} = (P, T, D^-, D^+)$, the constraints $L\mu + Hq \le b$, and optionally the initial marking μ_0.

Output: The H-transformed PN $(\mathcal{N}_H, T_{H,uc}, T_{H,uo})$ with $\mathcal{N}_H = (P_H, T_H, D_H^-, D_H^+)$, the H-transformed constraint $L_H \mu_H \le b$, and the initial marking μ_{0H} of $\mathcal{N}_H$.

1) Initialize $(\mathcal{N}_H, T_{H,uc}, T_{H,uo})$ to $(\mathcal{N}, T_{uc}, T_{uo})$, L_H to L, μ_{0H} to μ_0.
2) For all $t_i \in T$ such that $H(\cdot, t_i)$ is not zero, perform the following.
   a) Add a new place p_k and a new transition t_j to $\mathcal{N}_H$ as in Fig. 4(a)-(b), and include t_j in $T_{H,uc}$ $(T_{H,uo})$ if t_i is in T_{uc} (T_{uo}).
   b) Set $L_H(\cdot, p_k) = H(\cdot, t_i) + LD^-(\cdot, t_i)$ and $\mu_{0H}(p_k) = 0$.

The H^{-1}-Transformation

Input: The PN $\mathcal{N} = (P, T, D^-, D^+)$, the H-transformed net $\mathcal{N}_H = (P_H, T_H, D_H^-, D_H^+)$, and a set of constraints $L_H\mu_H \le b$ on $\mathcal{N}_H$.

Output: The H^{-1}-transformed constraint $L\mu + Hq \le b$.

1) Set $L(\cdot, p) = L_H(\cdot, p)\; \forall p \in P$ and H to the null matrix.
2) For all $p_k \in P_H \setminus P$

a) Let $t_i = \bullet p_k$; set $H(\cdot, t_i) = L_H(\cdot, p_k) - L_H D_H^-(\cdot, t_i)$.

Note that the H-transformation preserves the controllability/observability attributes of the transitions of $\mathcal{N}$. Thus, $T_{H,uc} \cap T = T_{uc}$ and $T_{H,uo} \cap T = T_{uo}$. Further, a new transition that results from a split has the same controllability/observability attributes as the transition that is split. For instance, in Fig. 4(a)-(b) t_j is controllable/observable iff t_i is controllable/observable.

3) Transformation to Admissible Constraints: Given a PN $(\mathcal{N}, T_{uc}, T_{uo})$, the constraints $L\mu + Hq + Cv \leq b$, and optionally[3] the initial marking μ_0, the following algorithm can be used for the supervisor design.

1) Apply the C-transformation and then the H- transformation. Let $(\mathcal{N}_{HC}, T_{HC,uc}, T_{HC,uo})$, $L_{HC}\mu_{HC} \leq b$, and μ_{HC0} be the transformed net, constraints, and initial marking.
2) Find admissible constraints $L_{HCa}\mu_{HC} \leq b_a$ such that $\forall \mu_{HC}$: $L_{HCa}\mu_{HC} \leq b_a \Rightarrow L_{HC}\mu_{HC} \leq b$. If such admissible constraints could not be found, declare failure and exit.
3) Apply to $L_{HCa}\mu_{HC} \leq b_a$ the H^{-1}-transformation and then the C^{-1}-transformation. Let $L_a\mu + H_a q + C_a v \leq b_a$ be the result.

Theorem 2: The set of constraints $L_a\mu + H_a q + C_a v \leq b_a$ is admissible, and any supervisor enforcing it enforces also $L\mu + Hq + Cv \leq b$.

In view of Theorem 2, a supervisor enforcing $L\mu + Hq + Cv \leq b$ is the supervisor of $L_a\mu + H_a q + C_a v \leq b_a$ constructed as in Section III-A. Note that at the step 2) approaches generating disjunctive constraints can also be used, by applying the step 3) to each component of the disjunction. In fact, any method of transformation to admissible constraints can be used. However, it is most natural to use at the step 2) the methods that test the admissibility of a set of constraints $L\mu \leq b$ with the sufficient condition $LD_{uc} \leq 0$ and $LD_{uo} = 0$, such as some of the methods of [2], [3], and [9]. (D_{uc} and D_{uo} are the restrictions of the incidence matrix to the uncontrollable and unobservable transitions, respectively.) Note also that this condition can be customized to $\mathcal{N}_{HC}$ as follows. To allow the situation of Fig. 4(c)-(d), in which a control place can be connected through a self-loop to an unobservable but controllable transition t_i, we can replace $LD_{HC}(\cdot, t_i) = 0$ and $LD_{HC}(\cdot, t_j) = 0$ with the less restrictive $LD_{HC}(\cdot, t_i) + LD_{HC}(\cdot, t_j) = 0$ for all $t_i \in T_{uo} \setminus T_{uc}$ with $H(\cdot, t_i) \neq 0$. Let $LA \leq 0$ and $LB = 0$ be the constraints $LD_{uc} \leq 0$ and $LD_{uo} = 0$ after performing this substitution. Then it can be seen that (3) satisfies the admissibility condition (16) iff the H- and C-transformed constraint $L_{HC}\mu \leq b$ satisfies $L_{HC}A \leq 0$ and $L_{HC}B = 0$.

Note the tradeoff of our approach. The benefits are that the supervisor design can be done in a computationally efficient manner and independently of the initial marking. (Of course, the designed supervisors still depend on the initial marking.) The drawback is that the solution may be suboptimal, in the sense that less restrictive solutions may be possible. Suboptimality may arise from three sources. First, from the method applied at the step 2), especially if the step 2) is restricted to generate conjunctions rather than disjunctions of inequalities. Second, from the fact that the condition $LD_{uc} \leq 0$ and $LD_{uo} = 0$ is only sufficient for admissibility. Third, from the H-transformation. However, note that the third possibility can be excluded with some enhancements detailed in [10].

As an example, this approach can be applied to the constraints (7)–(12). Assuming the uncontrollable transitions to be t_1, t_4, t_{11}, t_{12}, t_{13}, and t_{14}, and the unobservable transitions t_{13} and t_{14}, the following admissible constraints can be obtained: a) $mq_2 + \mu_6 \leq m - v_5 + v_{14}$; b) $mq_5 + \mu_5 \leq m - v_2 + v_{13}$; c) $q_3 \leq \mu_2$; d) $\mu_3 + v_{11} - v_{12} \leq n$; e) $\mu_4 - v_{11} + v_{12} \leq n$; and f) $\mu_3 + (n-k)q_2 \leq n - v_{11} + v_{12}$. The control places corresponding to these constraints can be found out using the construction of Section III-A. Fig. 5 represents the control places C_1, C_2, and C_3 corresponding to the constraints b), c), and e).

[3]It is possible to carry out the algorithm independently of the initial marking.

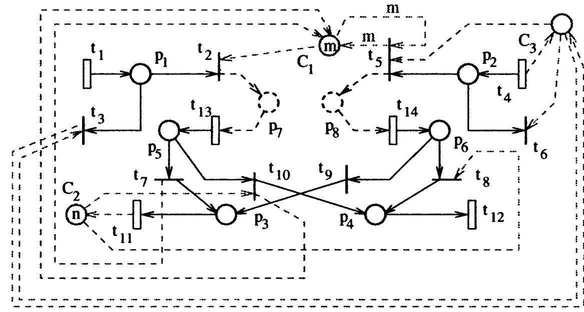

Fig. 5. Plant PN with three control places.

IV. Conclusion

Enforcing linear marking and firing vector constraints can be done effectively in Petri nets. This note has extended this class of constraints to include Parikh vector constraints. Then, we have shown how these more expressive constraints can be enforced as effectively as linear marking constraints. Our approach has also enhanced a previous technique for enforcing firing vector constraints in the presence of uncontrollable and unobservable transitions.

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