Control with Intermittent Sensor Measurements: A New Look at Feedback Control.
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Abstract
In many control systems, including networked control systems, feedback information is not necessarily continuous or instantaneous, but intermittent, where the loop is closed for finite time intervals. Intermittent feedback is not uncommon in applications, but it has not been adequately studied in control theory. The aim of this work is to explore theoretically the advantages (and disadvantages) of intermittent feedback.

In this paper, we apply the concept of Intermittent Feedback to a class of networked control systems known as Model-Based Networked Control Systems (MB-NCS). We first introduce the basic architecture for model-based control with intermittent feedback, then address the cases with output feedback (through the use of a state observer) and with delays in the network, providing a full description of the state response of the system, as well as a necessary and sufficient condition for stability in each case. Extensions of our results to cases with nonlinear plants are also presented. Finally, we propose future research directions.

I. INTRODUCTION
In this paper, we deal with control systems where sensor measurements are available intermittently. We refer to this concept as intermittent feedback. Intermittent feedback is displayed in nature and has been applied in a variety of fields for a long time, but its application to control systems and, in particular, to networked control systems is novel.

The basic idea of intermittent feedback is simple: rather than using closed loop control all the time, apply closed loop control only for a certain interval, then go back to open loop. After a certain period of time, apply closed loop control again, and so on. Essentially, the goal is to only apply closed loop control when it is needed, and thus reduce the overall control effort. Its application to control is highly intuitive and, in fact, it presents in biological systems. Take, for example, the kind of control one performs when driving a car. When in the presence of a straight road, less attention or control effort is required; but when we anticipate a curve, we focus on the road and apply closed loop control. When the curve has been passed and we are once again in a straightaway, we can change to an “open loop” variety of control. Parting from this biologically-inspired concept, then, the transition to control systems applications is intuitive and natural. Fig 1 shows a control system whose feedback loop contains an interface, which could be a network, for example. Fig 2 provides a look at the closed loop and open loop time intervals in an intermittent feedback setup. This will be explained in more detail in Section 2.

The concept of intermittent feedback has been applied in other areas of study. For example, in applications to chemical engineering processes, intermittent feedback is rather prominent. See, for example [23], where intermittent feedback is used to address turbulence. Oldroyd [24] addresses the issue of “intermittent distillation”, using intermittent feedback to address product removal. The concept also arises when dealing with product treatment, such as chlorine disinfection or combined sewer overflows, in that the problem is in itself of an intermittent nature [25]. Most of the articles in this area are very application-oriented and focus on processes such as manufacturing.

In the field of psychology, the use of intermittent feedback is widespread. The corresponding term often used in psychology papers is “intermittent reinforcement” and often arises in the literature regarding education, learning, and child-rearing. The main idea is that human behavior, in itself, follows this intermittent nature. This does not just apply to physical processes such as motor control, but to psychological pulls to practices, such as work, gambling [26]. The learning process is another area where intermittent feedback arises very naturally, and where methods based on it have proven to be very effective. [27] Intermittent feedback is also used in regards to motor control,

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such as controlling seizures, epilepsy, etc. [28], [29] The main idea in terms of psychological aspects of motion control is that while, initially, continuous control is applied, as the human being learns, and there is growth in cognitive and associative skills, there is a shift to intermittent feedback and a more automatic nature of motion control.

Other researchers have considered how these concepts may apply to other, more complex areas, such as speech development [30]. All these ideas are very interesting to us in that they provide an initial justification as to why intermittent feedback makes so much sense in nature, and why, then, it would make sense to seek to apply them in other contexts.

Intermittent feedback has also been used to some extent in robotics and mechanical engineering. This makes sense due to the fact that the visual component of robots is often designed to follow a biologically inspired analogous process. Thus, intermittent feedback arises naturally. For example, in [31] Ronco, et al use intermittent feedback to address a conceptual, and practical difficulty in robotics: by replacing the continuously moving horizon by an intermittently moving horizon, they solve a continuous-time generalized predictive controller. [32] Koay and Bugmann also address intermittent visual feedback in robotics and study how to compensate for the effects of delays [33], while Leonard and Krishnaprasad [34] use intermittent feedback in dealing with motion control of robots, leading to nonlinear control with fewer state variables. Also, because the concept of "learning" in robots is closely tied in with the learning process in human beings, the application of intermittent feedback here makes sense as well, as has been dealt with in [35] and [36].

Finally, while intermittent feedback, per se, has not been prominently featured in electrical engineering research, or in systems and control theory, in particular, similar concepts do arise in the literature. Consider, for example, [5], [18]. Also, in the field of information theory, Kramer uses intermittent feedback in [37], by employing a feedback communication scheme where the feedback channel is only used to inform the transmitter, at specified times, which message the receiver considers most likely - that is, the information is used to modify the transmission according to a rule known by the receiver.

In particular, the potential of intermittent feedback in networked control systems is of special interest to us. The concept is extremely appealing in that it effectively addresses one of the key concerns, that of saving bandwidth by reducing the use of network as much as possible. Yet, the benefits of intermittent feedback may not limit themselves to this. As discussed above, intermittent feedback is closely associated with the learning process, and, when adapting these ideas to control, we can begin to see that significant improvements may be possible. For
example, by combining intermittent feedback with the model-based architecture, we can gradually improve the parameters of the model -in a way, the system is "learning" or "adapting"- so that as time elapses, the control performance increases and the required use of network decreases. Additionally, the fact that in an intermittent feedback setting there are times when the loop is closed and closed-loop control is being applied suggests that results from classical, continuous-time control theory can be useful here and may be able to give us additional insight into the nature of networked control systems, as well. Finally, when dealing in networks, the notion that when one has access to the network, one will send all the information possible (as opposed to just one packet) is intuitively appealing and consistent with the notion of intermittent feedback.

Throughout this paper, we apply intermittent feedback to a particular class of networked control systems known as Model-Based Networked Control Systems (MB-NCS) and obtain the corresponding state responses under different setups. We obtain stability conditions in each case as well. We focus on MB-NCS because this architecture has proven to work well and makes sense in this context; also, the use of the model allows us to derive concrete, useful results. Our main goal is to take advantage of the concept of intermittent feedback as applied to MB-NCS to bridge the gap between instantaneous feedback and closed-loop control, thus providing a new look at feedback control.

The rest of the paper is organized as follows. In Section 2, we provide the initial setup for model-based control with intermittent feedback and provide a complete state response of the system, as well as necessary and sufficient condition for stability. In Sections 3 and 4, we do the same for the cases with output feedback (using a state observer) and with delays in the network, respectively. We extend our results to nonlinear plants in Section 5. Finally, in Section 6, we briefly discuss our ongoing work.

II. MB-NCS WITH INTERMITTENT FEEDBACK: BASIC SETUP

Let us start by introducing model-based control with intermittent feedback, in its simplest setup. The problem formulation is as follows.

The basic setup for MB-NCS with intermittent feedback is essentially the same as that proposed in the literature for traditional MB-NCS. Please see references [10], [11], [12] for more results on MB-NCS.

Consider the control of a continuous linear plant where the state sensor is connected to a linear controller/actuator via a network. In this case, the controller uses an explicit model of the plant that approximates the plant dynamics and makes possible the stabilization of the plant even under slow network conditions.

![Fig. 3. MB-NCS with intermittent feedback - basic architecture](image)

The main idea here is to perform the feedback by updating the model’s state using the actual state of the plant that is provided by the sensor. The rest of the time the control action is based on a plant model that is incorporated in the controller/actuator and is running open loop for a period of $h$ seconds.

As mentioned before, the main difference between model-based networked control systems as have been studied previously, and the case with intermittent feedback is that in the literature, the loop is closed instantaneously, and the rest of the time the system is running with input based on the model of the plant. Here, we part from the same basic idea, but the loop will remain closed for intervals of time which are different from zero. Intuitively, we should be able to achieve much better results the longer the loop is closed, as the level of degradation of the information
increases the longer the system is running open loop, so intermittent feedback should yield better results than those for traditional MB-NCS.

In dealing with intermittent feedback, we have two key time parameters: how frequently we want to close the loop, which we shall denote by $h$, and how long we wish the loop to remain closed, which we shall denote by $\tau$. Naturally, in the more general cases both $h$ and $\tau$ can be time-varying. For the purposes of this paper, however, we will deal initially with the case where both $h$ and $\tau$ are fixed.

We consider then a system such that the loop is closed periodically, every $h$ seconds, and where each time the loop is closed, it remains so for a time of $\tau$ seconds. The loop is closed at times $t_k$, for $k = 1, 2, \ldots$. Thus, there are two very clear modes of operation: closed loop and open loop. The system will be operating in closed loop mode for the intervals $[t_k, t_k + \tau)$ and in open loop for the intervals $[t_k + \tau, t_{k+1})$. When the loop is closed, the control decision is based directly on the information of the state of the plant, but we will keep track of the error nonetheless.

The plant is given by $\dot{x} = Ax + Bu$, the plant model by $\hat{\dot{x}} = \hat{A}\hat{x} + \hat{B}u$, and the controller by $u = K\hat{x}$.

The state error is defined as $e(t) = x(t) - \hat{x}(t)$ and the vector $z = [x \ e]^T$.

The state response of the system can be summarized in the following proposition.

**Proposition 1:** The system described above with initial conditions $z(t_0) = \begin{bmatrix} x(t_0) \\ 0 \end{bmatrix} = z_0$ has the following response:

$$
\begin{align*}
    z(t) &= \begin{cases}
        e^{A_\tau(t-t_k)} \left( \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \right)^k z_0, & t \in [t_k, t_k + \tau) \\
        e^{A_\tau(t-(t_k+\tau))} e^{A_\tau(\tau)} \left( \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \right)^k z_0, & t \in [t_k + \tau, t_{k+1})
    \end{cases}
\end{align*}
$$

(1)

where $\Sigma = e^{A_\tau(h-\tau)} e^{A_\tau(\tau)}$, $A_\phi = \begin{bmatrix} A + BK & -BK \\ \hat{A} + \hat{B} & \hat{B}K \end{bmatrix}$, $A_\psi = \begin{bmatrix} A + BK & -BK \\ 0 & 0 \end{bmatrix}$, and $t_{k+1} - t_k = h$.

We now present a necessary and sufficient condition for stability.

**Theorem 2:** The system described above is globally exponentially stable around the solution $z = \begin{bmatrix} x \\ e \end{bmatrix}$ if and only if the eigenvalues of $\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ are strictly inside the unit circle, where $\Sigma = e^{A_\tau(h-\tau)} e^{A_\tau(\tau)}$.

### III. MB-NCS with Intermittent Feedback: Output Feedback Case (State Observer)

In the previous section we considered plants where the full vector of the state was available at the output. When the state is not directly measurable, we must resort to a state observer. In this section we extend our results to this situation.

As in the architecture used in [12] for instantaneous model-based feedback, we assume that the state observer is collocated with the sensor. We use the plant model to design the state observer. Our configuration is based on the analogous setup for model-based control with output feedback, proposed by Montestruque.

In [12] it is justified the sensor carry the computational load of an observer by the fact that, typically, sensors that can be connected to a network have an embedded processor (usually in charge of performing the sampling, filtering, etc.) inside.

The observer has the form of a standard state observer with gain $L$. It makes use of the plant model.

In summary, the system equations are the following:

- **Plant:** $\dot{x} = Ax + Bu$, $y = Cx + Du$
- **Model:** $\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u$, $y = \hat{C}\hat{x} + \hat{D}u$
- **Controller:** $u = K\hat{x}$
- **Observer:** $\dot{\hat{x}} = (\hat{A} - L\hat{C})\hat{x} + [\hat{B} - L\hat{D}] \begin{bmatrix} u \\ y \end{bmatrix}$
- **Controller model state:** $\hat{x}$
- **Observer’s estimate:** $\hat{x}$
Fig. 4. MB-NCS with intermittent feedback - state observer

When loop is closed: $e = 0$

Error matrices: $\tilde{A} = A - \tilde{A}$, $\tilde{B} = B - \tilde{B}$, $\tilde{C} = C - \tilde{C}$, $\tilde{D} = D - \tilde{D}$

The state response of the system is summarized in the following proposition.

**Proposition 3:** The system described above has a state response:

$$z(t) = \begin{cases} e^{\Lambda_c(t-t_k)\Sigma^k}z_0, & t \in [t_k, t_k + \tau) \\ e^{\Lambda_o(t-(t_k+\tau))}e^{\Lambda_c(\tau)}\Sigma^k z_0, & t \in [t_k + \tau, t_{k+1}) \end{cases}$$  \hspace{1cm} (2)

where $\Sigma = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and

$$\Lambda_o = \begin{bmatrix} A & BK \\ LC & \tilde{A} - LC + \tilde{B}K + L\tilde{D}K \\ 0 & 0 \end{bmatrix}, \quad \Lambda_c = \begin{bmatrix} A & BK \\ LC & \tilde{A} - LC + \tilde{B}K + L\tilde{D}K \\ 0 & 0 \end{bmatrix}.$$

The following gives a necessary and sufficient condition for stability.

**Theorem 4:** The system described above is globally exponentially stable around the solution $z = \begin{bmatrix} x \\ \hat{x} \end{bmatrix} = 0$ if and only if the eigenvalues of $\Sigma$ are strictly inside the unit circle, where $\Sigma = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} e^{\Lambda_o(\tau)} e^{\Lambda_c(\tau)}$, and $\Lambda_o$, $\Lambda_c$ as before.

**IV. MB-NCS with Intermittent Feedback: Case with Delays**

In the previous sections, we have assumed that the delays in the network are negligible. However, in reality, this is usually not the case. We now consider the case where delays in the network are present. Although in real-life
plants delays might be variable, for the sake of analysis we will consider the case where delays are constant and known.

Consider the following setup:

The corresponding equations are as follows:

- **Plant:** \( \dot{x} = Ax + Bu \)
- **Model:** \( \dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u \)
- **Controller:** \( u = K\hat{x}, \quad t \in [t_k, t_{k+1}) \)
- **Propagation unit:** \( \dot{\hat{x}}' = \hat{A}\hat{x} + \hat{B}u, \quad t \in [t_{k+1} - \tau_d, t_{k+1}] \)
- **Update law:** \( \hat{x} \leftarrow x, \quad t = t_{k+1} - \tau_d; \quad \hat{x} \leftarrow \hat{x}, \quad t = t_k \)

This setup follows the original one proposed by Montestruque for the case with instantaneous feedback. See [13] for more details.

The state response of the system is given by the following proposition.

**Proposition 5:** The system described above has a state response:

For \( t \in [t_k, t_{k+1} + \tau) \)

\[
 z(t) = e^{A_c(t-t_k)}\Sigma^k z_0, \quad t \in [t_k, t_{k+1} + \tau) \tag{3}
\]

For \( t \in [t_{k+1} + \tau, t_{k+1} + \tau_d) \)

\[
 z(t) = e^{A_c((t-(t_k+\tau))} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} e^{A_c \tau \Sigma^k} z_0, \quad t \in [t_{k+1} + \tau, t_{k+1} + \tau_d) \tag{4}
\]

For \( t \in [t_{k+1} - \tau_d, t_{k+1}) \)

\[
 z(t) = e^{A_c((t-(t_k+\tau))} \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & I \end{bmatrix} e^{A_c \tau_d} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} e^{A_c \tau \Sigma^k} z_0, \tag{5}
\]

where

\[
 \Sigma = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} e^{A_c \tau_d} \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & I \end{bmatrix} e^{A_c (h-\tau_d-\tau)} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} e^{A_c \tau} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{6}
\]
We also provide a necessary and sufficient condition for stability.

**Theorem 6:** The system described above is globally exponentially stable around the solution \( z = \begin{bmatrix} x \\ \dot{e} \end{bmatrix} = 0 \) if and only if the eigenvalues of \( \Sigma \) are strictly inside the unit circle, where \( \Sigma \) is defined by (6) and \( \Lambda_o, \Lambda_c \) are defined as before.

### V. Nonlinear Plants

In the previous sections we have restricted our study to the cases where the plant is linear. Let us now lift this restriction and seek to find the corresponding stability properties for nonlinear plants with intermittent feedback.

The setup and procedure that follows closely mirrors that proposed by Montestruque [10] for traditional MB-NCS. The sufficient conditions obtained relate the stability of the nonlinear MB-NCS with the value of a function that depends on the Lipschitz constants of the plant and model as well as the stability properties of the compensated non-networked model. The results are obtained by studying the worst-case behavior of the norm of the plant state and the error, thus leading to conservative results.

Let the plant be given by:

\[
\dot{x} = f(x) + g(u) \tag{7}
\]

We use a model on the actuator side of the plant to estimate the actual state of the plant. The controller will be assumed to be a nonlinear state feedback controller. The control signal \( u \) is generated by taking into account the plant model state. The plant state sensor will send through the network the real value of the plant state to the plant model dynamics are given by:

\[
\hat{\dot{x}} = \hat{f}(x) + \hat{g}(u) \tag{8}
\]

And the controller has the following form:

\[
u = \hat{h}(\hat{x}) \tag{9}\]

We define as the error between the plant state and the plant model state, \( e = x - \hat{x} \). Combining the above, we obtain:

\[
\dot{x} = f(x) + g(\hat{h}(\hat{x})) = f(x) + m(\hat{x})
\]

\[
\hat{x} = \hat{f}(x) + \hat{g}(\hat{h}(\hat{x})) = f(x) + \hat{m}(\hat{x})
\]

Assume also that the plant model dynamics differ from the actual plant dynamics in an additive fashion:

\[
\hat{f}(\zeta) = f(\zeta) + \delta_f(\zeta) \tag{11}
\]

\[
\hat{m}(\zeta) = m(\zeta) + \delta_m(\zeta)
\]

Thus:

\[
\dot{x} = f(x) + m(\hat{x}) \tag{12}
\]

\[
\hat{x} = \hat{f}(x) + \hat{m}(\hat{x}) + \delta_f(\hat{x}) + \delta_m(\hat{x})
\]

Assume that \( f \) and \( \delta \) satisfy the following local Lipschitz conditions for with \( x, y \in B_L \), a ball centered on the origin:

\[
\|f(x) - f(y)\| \leq K_f \|x - y\| \tag{13}
\]

\[
\|\delta(x) - \delta(y)\| \leq K_\delta \|x - y\|
\]

It is to be noted that if the plant model is accurate the Lipschitz constant \( K_\delta \) will be small.

Assume that the non-networked compensated plant model is exponentially stable when \( \hat{x}(t_0) \in B_S \), \( \dot{x}(t) \in B_r \), for \( t \in [t_0, t_0 + \tau] \) with \( B_S \) and \( B_r \) balls centered on the origin.

\[
\|\hat{x}(t)\| \leq \alpha \|\hat{x}(t_0)\| e^{-\beta(t-t_0)} \text{ with } \alpha, \beta > 0. \tag{14}
\]
The non-linear MB-NCS with dynamics described above, and that satisfies the Lipschitz conditions described and with exponentially stable compensated plant model satisfying is asymptotically stable if:

\[
1 - \alpha \left( e^{-\beta(h-\tau)} + \left( e^{K_f(h-\tau)} - e^{-\beta(h-\tau)} \right) \left( \frac{K_f}{K_f + \delta} \right) \right) > 0
\]  

(15)

VI. ONGOING WORK

In addition to the previous results, there is ongoing work pertaining to model-based control and intermittent feedback. We will complement the above results by investigating stability properties for cases with time-varying \(\tau\) and \(h\) and with discrete-time plants.

Another aspect we are addressing is performance. Through simulations, we have observed that intermittent feedback yields excellent benefits in performance, when compared to instantaneous feedback; in particular, the benefits are especially significant in the cases when the model is of poor quality, that is, its values are very different from those of the plant. While these simulations give us initial insight into the effect of intermittent feedback on performance, we are also addressing this issue from a systematic, analytical perspective.

Closely tied to this are the issues of optimal control and robustness. We hope to obtain results on controller design meeting robustness or optimality demands consistent with the intermittent feedback setup.

Another potential benefit of intermittent feedback is that, as time may pass, the model may be updated during the closed loop intervals -through system identification techniques, for example- so that, as time elapses, the system needs progressively less feedback to achieve satisfactory stability and performance margins. We are currently investigating this issue as well.

Throughout this research, we keep the aim of bridging the gap between instantaneous feedback and full feedback control.

REFERENCES


