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Hybrid State Feedback Stabilization with l_2 Performance for Discrete-Time Switched Linear Systems

Hai Lin and Panos J. Antsaklis

Abstract

In this paper, the co-design of continuous-variable controllers and discrete-event switching logics, both in state feedback form, is investigated for a class of discrete-time switched linear control systems. It is assumed that none of the subsystems is stabilized through a continuous state feedback alone. However, it is possible to stabilize the whole switched system via carefully designing both the continuous controllers and the switching logics. Sufficient synthesis conditions for this co-design problem are proposed here in the form of bilinear matrix inequalities, which is based on the argument of multiple Lyapunov functions. The closed-loop switched system forms a special class of linear hybrid system, and is shown to be asymptotically stable with a finite l_2 induced gain.

Index Terms

Switched systems, controller synthesis, l_2 induced gain, Lyapunov methods.

I. Introduction

A remarkable feature of a switched system is that even when all its subsystems are unstable it is still possible to be stabilized by properly designed switching laws [16], [8]. As one of the benchmark problems [16], [19], the synthesis of stabilizing switching signals for a given collection of dynamical systems, especially linear systems, has attracted a lot of attentions recently; see for example the survey papers [16], [20], [8], [27], [17], the recent books [15], [28] and the references cited therein.

Early efforts along this direction were focused on quadratic stabilization for certain classes of systems. For example, a quadratic stabilization switching law between two linear time invariant (LTI) systems was considered

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in [30], and it was shown that the existence of a stable convex combination of the two subsystem matrices implies the existence of a quadratic Lyapunov function and a state-dependent switching rule that (quadratically) stabilizes the switched system. A generalization to more than two LTI subsystems was suggested in [24] by using a “min-projection strategy”. In [11], it was shown that the stable convex combination condition is also necessary for the quadratic stabilizability of two mode switched LTI system. However, it is only sufficient for switched LTI systems with more than two modes. A necessary and sufficient condition for quadratic stabilizability of switched controller systems was derived in [26]. Some extensions of [30] to the output-dependent switching and discrete-time cases were reported in [16], [32]. For robust stabilization, a quadratic stabilizing switching law was designed for polytopic uncertain switched linear systems based on linear matrix inequality (LMI) techniques in [32]. All of these methods guarantee stability by using a common quadratic Lyapunov function, which is conservative in the sense that there are switched systems that can be asymptotically (or exponentially) stabilized without using a common quadratic Lyapunov function.

More recent efforts were based on multiple Lyapunov functions, especially piecewise quadratic Lyapunov functions, to construct stabilizing switching signals. For example, in [29], piecewise quadratic Lyapunov functions were employed for two mode switched LTI systems. Exponential stabilization for continuous-time switched LTI systems was considered in [22] also based on piecewise quadratic Lyapunov functions, and the synthesis problem was formulated as a bilinear matrix inequality (BMI) problem. In [14], a probabilistic algorithm was proposed for the synthesis of an asymptotically stabilizing switching law for switched LTI systems along with a piecewise quadratic Lyapunov function. Stabilization for switched nonlinear systems was considered in [9] based on multiple Lyapunov functions. Additionally, exponentially stabilizing switching laws were designed based on solving extended LQR optimal problems in [6].

In this paper, we consider a collection of discrete-time LTI systems described by the difference equations

$$\left\{\begin{array}{rcl} x(t+1) & = & A_i x(t) + B_i u(t) + B_i^w w(t) \\ z(t) & = & C_i x(t) + D_i u(t) + D_i^w w(t) \end{array}\right. \tag{1}$$

where $t \in \mathbb{Z}^+$, $x \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control, $w \in \mathbb{R}^r$ is the disturbance, and $z \in \mathbb{R}^p$ is the output. It is assumed that the disturbance $w(t)$ is with finite l_2 norm. Denote the finite set $\mathfrak{I} = \{1, \cdots, N\}$, which stands for the collection of these finite discrete modes. For any subsystem $i \in \mathfrak{I}$, the state matrices A_i, B_i, B_i^w, C_i, D_i and D_i^w are constant matrices of appropriate dimensions.

The problem being investigated here is not only to design static state feedback gains K_i for each subsystem but also to design switching signals in static state feedback form, i.e., $\sigma(x) : x \mapsto i$, such that the closed-loop switched system

$$\begin{cases} x(t+1) = (A_{\sigma(x)} + B_{\sigma(x)}K_{\sigma(x)})x(t) + B^w_{\sigma(x)}w(t) \\ z(t) = (C_{\sigma(x)} + D_{\sigma(x)}K_{\sigma(x)})x(t) + D^w_{\sigma(x)}w(t) \end{cases} \quad (2)$$

is asymptotically stable with a bounded l_2 induced gain from w to z. To make the problem nontrivial, it is assumed that none of the subsystems (1) is stabilizable. In general, the design of continuous-variable control laws and switching signals are coupled together, and the co-design of K_i and $\sigma(x)$ as formulated above is a challenging task.

There are some related works on designing the continuous-variable control laws for switched systems under certain classes of switching signals, e.g., [7], [5], [25]. However, the switching signals are usually assumed to be given or restricted *a priori*, instead of being dealt with as free design variables. For example, state-feedback and output feedback gains for each subsystem were designed so as to stabilize the switched system under arbitrary switching [7], [10], under given switching signals (e.g. slow switching [12], [5]), or under autonomous switching due to the partition of the state space for piecewise affine systems [1], [21], [2], [25]. Note that, in this paper, the switching signals (induced from the partition of $\mathbb{R}^n$ through the switching surfaces) is to be designed instead of being given *a priori*. It is clear that the design of continuous-variable control laws will affect the selection of switching surfaces, and vice versa. Therefore, the design of continuous-variable control laws and switching logics are coupled together. Hence, the main concern here is how to co-design the continuous control and switching law such that the closed-loop system, which forms a hybrid system, is stable. The co-design problem for a class of continuous-time switched LTI systems was considered in [23], where a BMI synthesis condition is developed for exponential stabilization.

The first part of this current paper can be seen as an extension of [23] to the discrete-time counterpart. However, the extension is nontrivial due to some distinctive features of the discrete-time switched systems. First, to guarantee stability, the value of a piecewise quadratic Lyapunov function is required to be non-increasing at each switching instant. For the continuous-time case, the switching happen exactly when the state trajectory hits a switching surface. Even without knowledge of the direction that the state trajectories will follow when crossing the switching surface, one still can fulfil the above non-increasing requirement by simply forcing the two quadratic Lyapunov functions' values to agree at the switching surface [22]. However, the situation becomes complicated in the discrete-time case, since the switching for the discrete-time system does not happen exactly on the switching surface. To guarantee the non-increasing requirement at the switching instants for the discrete-time case, one need to include more constraints involving state transitions, which make the problem more challenging. In the second part of the paper, we studied the switching controller synthesis problem to guarantee that the l_2 induced gain is below certain bound. To the authors' knowledge, most of the existing results on the robust performances of switched systems are usually on the analysis part [31], [13] or on the continuous feedback controllers design [21], [25], while the co-design of switching laws and state feedback gains to guarantee robust performance are rare.

Some preliminary results of this paper appeared in [18], where BMI based switching stabilization synthesis conditions were formulated. Our focus here is the co-design of switching signals and state feedback gains. The rest of the paper is organized as follows. First, Section II characterizes the stabilizing switching signals based on the MLF theorem. The stabilization co-design problem is investigated in Section II-F, while the co-design problem to achieve finite l_2 induced gain is studied in Section III, which is based on an extension of the MLF theorem. Sufficient conditions for controller synthesis are proposed in the form of BMIs. Finally, concluding remarks are presented.

Notation: The relation $A > B$ ($A < B$) means that the matrix $A - B$ is positive (negative) definite. The superscript T stands for matrix transposition and the matrix I stands for identity matrix of proper dimension. l_2 is the Lebesgue space consisting of all discrete-time vector-valued function that are square-summable over $\mathbb{Z}^+$. $\|z\|_2$ denotes the l_2 norm of a discrete-time signal z, which is defined as $\|z\|_2^2 = \sum_0^{+\infty} z^T(t)z(t)$.

II. Switching Stabilization

This section aims to characterize switching signals in static state feedback form, i.e., $\sigma(x) : x \mapsto i$, such that the following autonomous switched system

$$x(t+1) = A_{\sigma(x)}x(t) \tag{3}$$

is exponentially stable to the origin. Notice that for all the subsystems in the form of (3), the origin is the common equilibrium.

To be precise, the exponential stability of the switched system (3) is defined as follows.

Definition 1: The origin of the system (3) is exponentially stable if all trajectories satisfy

$$\|x(t)\| \leq \kappa\xi^t\|x_0\| \tag{4}$$

for some $\kappa > 0$ and $0 < \xi < 1$. Here $\|\cdot\|$ stands for standard Euclidian norm in $\mathbb{R}^n$. □

First, we recall a well-known approach in switched systems literature to guarantee exponentially stability using multiple Lyapunov functions.

A. Multiple Lyapunov Function Theorem

Since we assume that none of the subsystems is stabilizable, there does not exist a Lyapunov function for the subsystems, $x(t+1) = A_ix(t)$, in a classical sense. However, it is still possible to restrict ourselves in a certain region of the state space, say $\Omega_i \subset \mathbb{R}^n$, so that the abstracted energy of the i-th subsystem is decreasing along the trajectories inside this region (there is no requirement on the trajectories outside the region Ω_i). This idea is captured by the concept of Lyapunov-like function.

Definition 2 (Lyapunov-like function): By saying that a subsystem has an associated Lyapunov-like function $V_i : \mathbb{R}^n \to \mathbb{R}$ in a region Ω_i, we mean that

1) There exist constant scalars $\beta_i \geq \alpha_i > 0$ such that

$$\alpha_i \|x(t)\|^2 \leq V_i(x(t)) \leq \beta_i \|x(t)\|^2$$

hold for any $x(t) \in \Omega_i$;

2) For all $x(t) \in \Omega_i$ and $x(t) \neq 0$,

$$V_i(x(t+1)) \leq \xi_i V_i(x(t)),$$

holds for some positive scalar $\xi_i < 1$.

□

The first condition implies positiveness and radial unboundedness for $V_i(x)$ when $x \in \Omega_i$, while the second condition guarantees the decreasing of the value of $V_i(x)$ along trajectories of the i-th subsystem inside Ω_i. Notice that it is possible that $x(t) \in \Omega_i$ while $x(t+1) \notin \Omega_i$. Actually, the Lyapunov-like function $V_i(x(t))$ is defined for all $x(t) \in \mathbb{R}^n$, while the positiveness, radial unboundedness and decreasing properties are required only for $x(t) \in \Omega_i$.

Suppose that the union of all these regions Ω_i cover the whole state space. Then we obtain a set of Lyapunov-like functions. To study the global stability of the switched systems, one needs to concatenate these Lyapunov-like functions together and form a non-traditional Lyapunov function, called multiple Lyapunov function (MLF). The MLF is proved to be a powerful tool for studying the stability of switched systems, see e.g. [4], [20], [16], [8]. The basic idea of the MLF method can be described as follows. It is known that the MLF's value would decrease when every subsystem is active only in its corresponding region Ω_i. If we can also restrict the switching signals in such a way that, at every time we enter (switch into) a certain subsystem, its corresponding Lyapunov function value is smaller than its value at the previous entering time, then the switched system is asymptotically stable. In other words, for each subsystem the corresponding Lyapunov-like function's values at every entering instant form a monotonically decreasing sequence. Here, we require in addition that at every switching instant the MLF's value is also non-increasing. On one hand this will simplify our controller design, and on the other, it will deduce stronger property, i.e., exponential stability. In summary, we could present this result as a theorem [8], [22].

Theorem 1: Suppose that each subsystem has an associated Lyapunov-like function V_i in its active region Ω_i, and that $\bigcup_i \Omega_i = \mathbb{R}^n$. Let $\mathcal{S}$ be a class of switching sequences such that σ can take value i only if $x(t) \in \Omega_i$, and in addition

$$V_j(x(t_{i,j})) \leq V_i(x(t_{i,j}))$$

where $t_{i,j}$ denotes the time point that the switching from subsystem i to subsystem j occurs, i.e., $x(t_{i,j} - 1) \in \Omega_i$ while $x(t_{i,j}) \in \Omega_j$. Then, the switched linear system (3) is exponentially stable under the switching signals $\sigma \in \mathcal{S}$. □

Proof: First, denote $\xi = \max_i\{\xi_i\}$, $\alpha = \min_i\{\alpha_i\}$, and $\beta = \min_i\{\beta_i\}$. It is clear that $0 < \xi < 1$, $\alpha > 0$ and $\beta > 0$

Without loss of generality, it is assumed that there are totally N switchings occurring within the period $[0, t)$ for a switching sequence $\sigma \in \mathcal{S}$, and the sequence of active modes can be represented as $\{i_0, i_1, \cdots, i_{N-1}\}$. Let us denote the switching time from mode i_k to i_{k+1} as $t_{i_k, i_{k+1}}$. Then, it is obvious that $0 < t_{i_1,i_2} < \cdots < t_{i_{N+1},i_N} < t$.

By assumption, the current active mode is $\sigma(t) = i_N$, which implies that the current state $x(t) \in \Omega_{i_N}$. Then, it holds that

$$\begin{aligned}
\alpha\|x(t)\|^2 &\leq \alpha_{i_N}\|x(t)\|^2 \leq V_{i_N}(x(t)) \leq \xi_{i_N}^{t-t_{i_{N-1},i_N}} V_{i_N}(x(t_{i_{N-1},i_N})) \\
&\leq \xi_{i_N}^{t-t_{i_{N-1},i_N}} V_{i_{N-1}}(x(t_{i_{N-1},i_N})) \leq \xi^{t-t_{i_{N-1},i_N}} V_{i_{N-1}}(x(t_{i_{N-1},i_N})) \\
&\leq \cdots\cdots \\
&\leq \xi^{t-t_{i_0,i_1}} V_{i_0}(x(t_{i_0,i_1})) \leq \xi^t V_{i_0}(x(0)) \leq \xi^t \beta \|x(0)\|^2. \qquad (5)
\end{aligned}$$

Therefore,

$$\|x(t)\| \leq \sqrt{\frac{\beta}{\alpha}} (\sqrt{\xi})^t \|x(0)\|,$$

where $\sqrt{\frac{\beta}{\alpha}} > 0$ and $0 < \sqrt{\xi} < 1$. By definition, the switched systems is exponentially stable under the switching signals $\sigma \in \mathcal{S}$. ■

The above MLF theorem characterizes a class of switching signals for which the stability of the switched systems is achieved. It is worth pointing out that there exists a various of different versions of MLF theorem [20], [16], [8], which could be less conservative than the version we presented above. However, the basic idea is the same, and we choose this form on purpose to simplify the design process.

In the sequel, we will restrict our attention to quadratic Lyapunov-like functions and corresponding multiple quadratic Lyapunov function. Before that, we need to represent partitions of the state space in order to facilitate the search of quadratic Lyapunov-like functions.

B. Partition of the state space

In the above MLF theorem, it is critical to select Ω_i to divide the whole state space $\mathbb{R}^n$, so as to facilitate the identification of the Lyapunov-like functions $V_i(x)$ for each subsystem within a certain region. For this purpose, it is necessary to require that the union of all these regions Ω_i cover the whole state space, i.e., $\Omega_1 \bigcup \Omega_2 \cdots \bigcup \Omega_N = \mathbb{R}^n$, which is called the covering property.

Since we will restrict our attention to quadratic Lyapunov-like functions, we consider regions given (or approximated) by a quadratic form

$$\Omega_i = \{x \in \mathbb{R}^n : \ x^T Q_i x \geq 0\},$$

where $Q_i \in \mathbb{R}^{n \times n}$ are symmetric matrices, and $i \in \{1, \cdots, N\}$. The following lemma gives a sufficient condition for the covering property for regions given by quadratic forms [22].

Lemma 1: [22] If for every $x \in \mathbb{R}^n$

$$\sum_{i=1}^{N} \theta_i x^T Q_i x \geq 0 \tag{6}$$

where $\theta_i \geq 0$, $i \in \mathfrak{I}$, then $\bigcup_{i=1}^{N} \Omega_i = \mathbb{R}^n$. □

It should be pointed out that these regions Ω_i are not necessary to be mutual exclusive. Hence, it is possible that $\Omega_i \bigcap \Omega_j \neq \emptyset$, for two different regions Ω_i and Ω_j. To make the selection of active modes non-ambiguous, we follow the *largest region function strategy*, i.e.,

$$\sigma(x) = \arg\left(\max_{i \in \mathfrak{I}} x^T Q_i x\right) \tag{7}$$

This switching strategy was previously introduced in [22] for continuous-time switched linear systems, and the name comes from the fact that the selection of subsystems (at state x) corresponds to the largest value of the region function $x^T Q_i x$. If (6) is satisfied, then the largest region function strategy will guarantee that σ can take value i only if $x(t) \in \Omega_i$. This can be easily seen from (6), since $\sum_{i=1}^{N} \theta_i x^T Q_i x \geq 0$ for some positive coefficient θ_i and $\sigma(x) = \arg\left(\max_{i \in \mathfrak{I}} x^T Q_i x\right)$. Hence, $\sigma(x) = i$ implies $x^T Q_i x \geq 0$. So, $x \in \Omega_i$ by definition of Ω_i.

C. Quadratic Lyapunov-like Functions

In this subsection, we derive conditions expressed as matrix inequalities for the existence of a quadratic Lyapunov-like function, $V_i(x) = x^T P_i x$, assigned to each region Ω_i. By definition, the Lyapunov-like function $V_i(x) = x^T P_i x$ needs to satisfy the following two conditions:

1) Condition 1: There exist constant scalars $\beta_i \geq \alpha_i > 0$ such that

$$\alpha_i \|x(t)\|^2 \leq V_i(x(t)) \leq \beta_i \|x(t)\|^2$$

holds for any $x(t) \in \Omega_i$.

For a quadratic Lyapunov-like function candidate $V_i(x(t)) = x(t)^T P_i x(t)$, this means

$$\alpha_i x(t)^T I x(t) \leq x(t)^T P_i x(t) \leq \beta_i x(t)^T I x(t),$$

holds for $x(t)^T Q_i x(t) \geq 0$. That is

$$\begin{cases} x(t)^T(\alpha_i I - P_i)x(t) \leq 0 \\ x(t)^T(P_i - \beta_i I)x(t) \leq 0 \end{cases}$$

holds for $x(t)^T Q_i x(t) \geq 0$. Applying the $\mathcal{S}$-procedure [3], the above constrained inequalities follow from the LMIs

$$\begin{cases} \alpha_i I - P_i + \eta_i Q_i \leq 0 \\ P_i - \beta_i I + \rho_i Q_i \leq 0 \end{cases} \tag{8}$$

where $\eta_i \geq 0$ and $\rho_i \geq 0$ are unknown scalars. Define two scalars, $\alpha = \min_{i \in \mathfrak{I}}\{\alpha_i\}$ and $\beta = \max_{i \in \mathfrak{I}}\{\beta_i\}$. Notice that $0 < \alpha \leq \beta$. While normalizing $\beta = 1$ by resetting α as $\frac{\alpha}{\beta}$, η_i as $\frac{\eta_i}{\beta}$, and ρ_i as $\frac{\rho_i}{\beta}$, we obtain

$$\alpha I + \eta_i Q_i \leq P_i \leq I - \rho_i Q_i \tag{9}$$

2) Condition 2: For all $x(t) \in \Omega_i$, $x(t) \neq 0$,

$$V_i(x(t+1)) \leq \xi_i V_i(x(t)),$$

where $x(t+1) = A_i x(t)$.

This is equivalent to

$$x(t)^T[A_i^T P_i A_i - \xi_i P_i]x(t) \leq 0 \tag{10}$$

for $x(t) \in \Omega_i$.

In order to transform the above constrained matrix inequality into equivalent unconstrained form, let’s recall the Finsler’s Lemma [3], which has been used previously in the control literature mainly for eliminating design variables in matrix inequalities.

Lemma 2 (Finsler’s Lemma): Let $\zeta \in \mathbb{R}^n$, $P = P^T \in \mathbb{R}^{n \times n}$, and $H \in \mathbb{R}^{m \times n}$ such that $rank(H) = r < n$. The following statements are equivalent:

1) $\zeta^T P \zeta < 0$, for all $\zeta \neq 0$, $H\zeta = 0$;
2) $\exists X \in \mathbb{R}^{n \times m}$ such that $P + XH + H^T X^T < 0$. □

Applying the Finsler's Lemma to a strict inequality version of (10), with

$$P = \begin{bmatrix} -\xi_i P_i & 0 \\ 0 & P_i \end{bmatrix}, \ \zeta = \begin{bmatrix} x(t) \\ x(t+1) \end{bmatrix},$$

$X = \begin{bmatrix} F_i \\ G_i \end{bmatrix}$, and $H = \begin{bmatrix} A_i & -I \end{bmatrix}$, then (10) is implied by

$$\zeta^T \begin{bmatrix} A_i^T F_i^T + F_i A_i - \xi_i P_i & A_i^T G_i^T - F_i \\ G_i A_i - F_i^T & P_i - G_i - G_i^T \end{bmatrix} \zeta < 0 \tag{11}$$

for $\zeta^T \begin{bmatrix} Q_i & 0 \\ 0 & 0 \end{bmatrix} \zeta \geq 0$. Here $F_i,\, G_i \in \mathbb{R}^{n\times n}$ are unknown matrices.

Applying the $\mathcal{S}$-procedure [3], the above constrained stability condition is implied by the following unconstrained condition for unknown matrices $P_i = P_i^T$, $Q_i = Q_i^T$, F_i, $G_i \in \mathbb{R}^{n\times n}$, and scalars $\mu_i \geq 0$,

$$\begin{bmatrix} A_i^T F_i^T + F_i A_i - \xi_i P_i + \mu_i Q_i & A_i^T G_i^T - F_i \\ G_i A_i - F_i^T & P_i - G_i - G_i^T \end{bmatrix} < 0 \tag{12}$$

Combining (9) and (12), we obtain methods to find a quadratic Lyapunov-like function for each subsystem within certain regions in the state space.

D. Switching Condition

Following Theorem 1, in order to guarantee the exponential stability we also need to make sure that

1) subsystem i is active only when $x(t) \in \Omega_i$, and that
2) the Lyapunov function values are not increasing when switching occurs.

The first condition has been verified for the the largest region function strategy (7) provided that the covering condition (6) holds, i.e., $\sum_{i=1}^{N} \theta_i x^T Q_i x \geq 0$ for some $\theta_i \geq 0$, $i \in \mathfrak{I}$.

Regarding the second condition, assume that a switching, $i \to j$, occurs at time instant t, i.e., $x(t) \in \Omega_j$ while $x(t-1) \in \Omega_i$ for $i \neq j \in \mathfrak{I}$, it is required that $V_j(x(t)) \leq V_i(x(t))$.

This means that

$$x(t)^T [P_j - P_i] x(t) \leq 0 \tag{13}$$

and $x(t-1) \in \Omega_i$, $x(t) = A_i x(t-1) \in \Omega_j$.

Applying the Finsler's Lemma with

$$P = \begin{bmatrix} 0 & 0 \\ 0 & P_j - P_i \end{bmatrix}, \ \zeta = \begin{bmatrix} x(t-1) \\ x(t) \end{bmatrix},$$

$X = \begin{bmatrix} F_{ij} \\ G_{ij} \end{bmatrix}$, and $H = \begin{bmatrix} A_i & -I \end{bmatrix}$, (13) is implied by

$$\zeta^T \begin{bmatrix} A_i^T F_{ij}^T + F_{ij} A_i & A_i^T G_{ij}^T - F_{ij} \\ G_{ij} A_i - F_{ij}^T & P_j - P_i - G_{ij} - G_{ij}^T \end{bmatrix} \zeta < 0$$

for $\zeta^T \begin{bmatrix} Q_i & 0 \\ 0 & Q_j \end{bmatrix} \zeta \geq 0$. Here F_{ij}, $G_{ij} \in \mathbb{R}^{n \times n}$ are unknown matrices.

Applying the $\mathcal{S}$-procedure [3], the above constrained stability condition is implied by the following: there exist unknown matrices $P_i = P_i^T$, $Q_i = Q_i^T$, F_{ij}, $G_{ij} \in \mathbb{R}^{n \times n}$, and scalars $\mu_{ij} \geq 0$, such that the matrix

$$\begin{bmatrix} A_i^T F_{ij}^T + F_{ij} A_i + \mu_{ij} Q_i & A_i^T G_{ij}^T - F_{ij} \\ G_{ij} A_i - F_{ij}^T & P_j - P_i - G_{ij} - G_{ij}^T + \mu_{ij} Q_j \end{bmatrix} \tag{14}$$

is negative definite.

E. Synthesis Condition

In summary, the above discussion can be presented as the following sufficient condition for the discrete-time linear system (3) to be exponentially stabilized.

Theorem 2: If there exist matrices P_i ($P_i = P_i^T$), Q_i ($Q_i = Q_i^T$), F_i, G_i, F_{ij}, G_{ij}, and scalars $0 < \xi < 1$, $\alpha > 0$, $\eta_i \geq 0$, $\rho_i \geq 0$, $\mu_i \geq 0$, $\mu_{ij} \geq 0$, $\theta_i \geq 0$, solving the optimization problem

$$\begin{aligned} & \min \xi \\ s.t. & \begin{cases} \alpha I + \eta_i Q_i \leq P_i \leq I - \rho_i Q_i \\ \begin{bmatrix} A_i^T F_i^T + F_i A_i - \xi P_i + \mu_i Q_i & A_i^T G_i^T - F_i \\ G_i A_i - F_i^T & P_i - G_i - G_i^T \end{bmatrix} < 0, \\ \begin{bmatrix} A_i^T F_{ij}^T + F_{ij} A_i + \mu_{ij} Q_i & A_i^T G_{ij}^T - F_{ij} \\ G_{ij} A_i - F_{ij}^T & P_j - P_i - G_{ij} - G_{ij}^T + \mu_{ij} Q_j \end{bmatrix} < 0 \\ \theta_1 Q_1 + \cdots + \theta_N Q_N \geq 0 \end{cases} \end{aligned} \tag{15}$$

for all $i, j \in \{1, \cdots, N\}$, $i \neq j$, then the largest region function strategy (7) implies that the origin of the switched linear system (3) is exponentially stable with decay rate $\sqrt{\xi}$. □

Proof: The first two conditions guarantee the existence of a quadratic Lyapunov-like function, $V_i(x) = x^T P_i x$, for the i-th subsystem within the region $\Omega_i = \{x \in \mathbb{R}^n : \; x^T Q_i x \geq 0\}$. Due to the condition that $\theta_1 Q_1 + \cdots + \theta_N Q_N \geq 0$, it follows that $\bigcup_{i \in \mathfrak{I}} \Omega_i = \mathbb{R}^n$.

For any time $t \geq 0$, assume $\sigma(t) = i$ which implies that $x(t) \in \Omega_i$ according to the largest region function strategy (7).

If there is no switching at time t, then the second inequality implies that

$$V_i(x(t+1)) \leq \xi V_i(x(t)),$$

when there is no switching occur at t.

On the other hand, if there is a switching occur at $t \geq 1$, i.e., $x(t-1) \in \Omega_j$ while $x(t) \in \Omega_i$ $(i \neq j)$, then the third inequality implies that

$$V_i(x(t)) \leq V_j(x(t)).$$

Therefore, according to Theorem 1, it implies that the switched linear system (3) is exponentially stable with decay rate $\sqrt{\xi}$. ∎

Some remarks are in order. First, the optimization problem above is a Bilinear Matrix Inequality (BMI) problem, due to the product of unknown scalars and matrices. BMI problems are NP-hard, and not computationally efficient. However, practical algorithms for optimization problems over BMIs exist and typically involve approximations, heuristics, branch-and-bound, or local search. As suggested in [22] for the continuous-time case, one possible way to compute the BMI problem is to grid up the unknown scalars, and then solve a set of LMIs for fixed values of these parameters. It is argued that the gridding of the unknown scalars can be made quite sparsely [22]. The numerical complexity associated with these LMIs can be estimated in terms of the number of scalar variables (K) and number of LMI rows (L). As discussed in [3], the number of floating point operation or the time required to test the feasibility of the set of LMIs is proportional to K^3L. In our case, the value of K, L can be expressed as a function of the number of subsystems N and the dimension of the subsystem n. In particular, $K = 2N\frac{n(n+1)}{2} + 2Nn^2 + 2N^2n^2$ and $L = 4Nn + 2N^2n + n$ for (15).

It can be shown that the introduction of multiplier matrices, like F_i, G_i etc., gives a lot of flexibility, and several known stability conditions in the literature can be reduced to a special selection of these multiplier matrices, see e.g. [10]. In addition, these multiplier matrices would make the co-design of continuous feedback controllers and switching laws trackable. This is explored in the next section.

F. Switched State Feedback

This section focuses on the co-design of static state feedback gains K_i, and switching laws $\sigma(x)$ so that the closed-loop switched linear system

$$x(t+1) = (A_{\sigma(x)} + B_{\sigma(x)}K_{\sigma(x)})x(t) \tag{16}$$

is exponentially stable to the origin. An important aspect of the matrix inequality conditions in Theorem 2 is that there is no cross product between two unknown matrices, which makes it possible to represent a sufficient condition for this co-design problem as follows.

Theorem 3: If there exist matrices P_i ($P_i = P_i^T$), Q_i ($Q_i = Q_i^T$), R_i, G_i, and scalars $0 < \xi < 1$, $\alpha > 0$, $\eta_i \geq 0$, $\rho_i \geq 0$, $\mu_i \geq 0$, $\mu_{ij} \geq 0$, $\theta_i \geq 0$ that solve the optimization problem

$$\begin{aligned} &\min \xi \\ s.t.\ &\begin{cases} \alpha I + \eta_i Q_i \leq P_i \leq I - \rho_i Q_i \\ \begin{bmatrix} -\xi P_i + \mu_i Q_i & A_i G_i^T + B_i R_i \\ G_i A_i^T + R_i^T B_i^T & P_i - G_i - G_i^T \end{bmatrix} < 0, \\ \begin{bmatrix} \mu_{ij} Q_i & A_i G_i^T + B_i R_i \\ G_i A_i^T + R_i^T B_i^T & P_j - P_i - G_i - G_i^T + \mu_{ij} Q_j \end{bmatrix} < 0 \\ \theta_1 Q_1 + \cdots + \theta_N Q_N \geq 0 \end{cases} \end{aligned} \tag{17}$$

for all $i, j \in \Im$, $i \neq j$, then the state feedback gains given by the solution of

$$K_i G_i^T = R_i, \quad i \in \Im, \tag{18}$$

along with the largest region function switching strategy (7), exponentially stabilize the switched system (16) with decay rate $\sqrt{\xi}$. □

Proof: The above conditions lead to

$$\begin{cases} \alpha_i I + \eta_i Q_i \leq P_i \leq \beta_i I - \rho_i Q_i \\ \begin{bmatrix} -\xi P_i + \mu_i Q_i & (A_i + B_i K_i) G_i^T \\ G_i (A_i + B_i K_i)^T & P_i - G_i - G_i^T \end{bmatrix} < 0, \\ \begin{bmatrix} \mu_{ij} Q_i & (A_i + B_i K_i) G_i^T \\ G_i (A_i + B_i K_i)^T & P_j - P_i - G_i - G_i^T + \mu_{ij} Q_j \end{bmatrix} < 0 \\ \theta_1 Q_1 + \cdots + \theta_N Q_N \geq 0 \end{cases}$$

which implies the transposed version of conditions in Theorem 2 with $F_i = 0$, $F_{ij} = 0$, and $G_{ij} = G_i$. Hence, the exponential stability of the switched control system (16) follows. ■

III. l_2 Induced Gain Performance

Consider the discrete-time systems (1) with l_2-norm bounded disturbance w. The goal of this section is to guarantee that the l_2 induced gain from the disturbance w to the output z is below certain desirable bound.

Here the l_2 induced gain is defined in a standard way, i.e., the l_2 gain of the system is the quantity,

$$\sup_{\|w\|_2 \neq 0} \frac{\|z\|_2}{\|w\|_2},$$

where the sup is taken over all nonzero trajectories of the system, starting from $x(0) = 0$.

A. MLF Theorem for Performance

First, consider the switched autonomous systems

$$\begin{cases} x(t+1) &= A_{\sigma(x)}x(t) + B^w_{\sigma(x)}w(t) \\ z(t) &= C_{\sigma(x)}x(t) + D^w_{\sigma(x)}w(t) \end{cases} \tag{19}$$

and extend Theorem 1.

Proposition 1: Suppose that each subsystem has an associated Lyapunov-like function $V_i : \mathbb{R}^n \to \mathbb{R}$ in its active region Ω_i with finite l_2 gain performance, each with equilibrium point $x = 0$. This means that

1) There exist constant scalars $\beta_i \geq \alpha_i > 0$ such that

$$\alpha_i \|x(t)\|^2 \leq V_i(x(t)) \leq \beta_i \|x(t)\|^2$$

hold for any $x(t) \in \Omega_i$;

2) For all $x(t) \in \Omega_i$ and $x(t) \neq 0$,

$$\Delta V_i(x(t)) = V_i(x(t+1)) - V_i(x(t)) < \gamma^2 w(t)^T w(t) - z(t)^T z(t).$$

holds for a positive scalar γ. Also, suppose that $\bigcup_i \Omega_i = \mathbb{R}^n$. Let $\mathcal{S}$ be a class of piecewise-constant switching sequences such that σ can take value i only if $x(t) \in \Omega_i$, and in addition

$$V_j(x(t_{i,j})) \leq V_i(x(t_{i,j}))$$

where $t_{i,j}$ denotes the time point that the switching from subsystem i to subsystem j occurs, i.e., $x(t_{i,j} - 1) \in \Omega_i$ while $x(t_{i,j}) \in \Omega_j$. Then, the switched linear system (2) is asymptotically stable under the switching signals $\sigma \in \mathcal{S}$, and its l_2 induced gain is less than γ. □

Proof: First, define the Multiple-Lyapunov-Function candidate as

$$V(x(t), t) = V_i(x(t)), \quad \text{for } \sigma(t) = i.$$

It is straightforward to verify the positiveness, radially unboundedness for $V(x(t), t)$. It remains to show the negativeness of its difference.

For any time instant $t \geq 1$, assume that $\sigma(t) = i$, which implies that $x(t) \in \Omega_i$.

If there is no switching at t, then $\sigma(t-1) = i$ and $x(t-1) \in \Omega_i$ as well. Therefore, we obtain

$$\begin{aligned} V(x(t), t) &= V_i(x(t)) < V_i(x(t-1)) - z(t-1)^T z(t-1) + \gamma^2 w(t-1)^T w(t-1) \\ &= V(x(t-1), t-1) - z(t-1)^T z(t-1) + \gamma^2 w(t-1)^T w(t-1). \end{aligned}$$

For the case of switching occurring at time t ($t > 1$), assume that $x(t-1) \in \Omega_j$ while $j \neq i$. Then,

$$\begin{aligned} V(x(t),t) &= V_i(x(t)) \leq V_j(x(t)) < V_j(x(t-1)) - z(t-1)^T z(t-1) + \gamma^2 w(t-1)^T w(t-1) \\ &= V(x(t-1),t-1) - z(t-1)^T z(t-1) + \gamma^2 w(t-1)^T w(t-1). \end{aligned}$$

The first inequality comes from the condition that the value of Lyapunov-like function does not increase at the switching instants, while the last inequality is due to that fact that $x(t-1) \in \Omega_j$ and the definition of a Lyapunov-like function.

Therefore, for all $t \geq 1$

$$V(x(t),t) < V(x(t-1),t-1) - z(t-1)^T z(t-1) + \gamma^2 w(t-1)^T w(t-1). \tag{20}$$

Sum up (20) from 1 to $T > 0$, with the initial condition $x(0) = 0$, to obtain

$$V(x(T),T) + \sum_{0}^{T-1} (z(t)^T z(t) - \gamma^2 w(t)^T w(t)) < 0$$

Since $V(x(T),T) \geq 0$, this implies

$$\sum_{0}^{T-1} z(t)^T z(t) < \gamma^2 \sum_{0}^{T-1} w(t)^T w(t) \tag{21}$$

for all T, so

$$\sup_{\|w\|_2 \neq 0} \frac{\|z\|_2}{\|w\|_2} \leq \gamma,$$

which means that the l_2 gain of the system is less than γ. ■

Note that the regions Ω_i provide a cover of $\mathbb{R}^n$, i.e., $\bigcup_i \Omega_i = \mathbb{R}^n$, but they may overlap in the sense that $\Omega_i \bigcap \Omega_j$ may be nonempty. It is possible that at the switching instant $t_{i,j}$, $x(t_{i,j})$ is still contained in Ω_i as well. For this reason, the Multiple-Lyapunov-Function (MLF) $V(x(t),t)$ is defined according to the current active mode $\sigma(t) = i$, which is well-defined (single valued) for all t.

B. Synthesis Condition for Performance

In order to synthesize switching signals such that the switched system is asymptotically stable and achieve finite l_2 induced gain, we need to

1) Partition the state space and associate each partition with a subsystem and a Lyapunov-like function;
2) Make sure that each subsystem is activated only when the state x is in its corresponding partition;
3) Guarantee that the value of Lyapunov-like function is non-increasing when a switching is made.

In a parallel development to stabilization control in Section II, the partition of the state space can be achieved by solving (6) according to Lemma 1, while each region Ω_i is associated with the subsystem i. A nice property about this kind of partition is that we can employ the largest region function strategy (7) to guarantee that each subsystem is activated only when the state x is in its corresponding partition, i.e., $\sigma(x) = i$ implies $x \in \Omega_i$. Similar to the stabilization problem, we employ the quadratic form, $V_i(x) = x^T P_i x$, as a Lyapunov-like function for each subsystem. The non-increasing requirement at switching instants can follow exactly the same arguments for the stabilization design, and can be guaranteed by (14) as well. Therefore, the developments so far are quite similar to the stabilization case, and the details are omitted due to space limit. What remains is the condition that for all $x(t) \in \Omega_i$ and $x(t) \neq 0$,

$$\Delta V_i(x(t)) < \gamma_i^2 w(t)^T w(t) - z(t)^T z(t).$$

This means that

$$x^T(t)[A_i^T P_i A_i - P_i]x(t) + z(t)^T z(t) - \gamma_i^2 w(t)^T w(t) < 0, \tag{22}$$

for $x(t) \in \Omega_i$, and $z(t) = C_i x(t) + D_i^w w(t)$, $x(t+1) = A_i x(t) + B_i^w w(t)$. This can be transformed into a matrix inequality based on the Finsler's Lemma, with

$$P = \begin{bmatrix} -P_i & 0 & 0 & 0 \\ 0 & P_i & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}, \ \zeta = \begin{bmatrix} x(t) \\ x(t+1) \\ z(t) \\ w(t) \end{bmatrix},$$

$$X = \begin{bmatrix} F_{1i} & F_{2i} \\ G_{1i} & G_{2i} \\ H_{1i} & H_{2i} \\ J_{1i} & J_{2i} \end{bmatrix}, \ H = \begin{bmatrix} A_i & -I & 0 & B_i^w \\ C_i & 0 & -I & D_i^w \end{bmatrix}.$$

Then, (22) is equivalent to

$$P + XH + H^T X^T < 0, \tag{23}$$

for $x(t) \in \Omega_i$.

Analogously, applying the $\mathcal{S}$-procedure [3] to (23) and combining all other matrix inequalities, we can obtain the following sufficient conditions for the discrete-time switched linear system (19) to be asymptotically stabilized with l_2 induced gain less than γ. The proof is quite straightforward based on Proposition 1.

Theorem 4: If there exist matrices P_i ($P_i = P_i^T$), Q_i ($Q_i = Q_i^T$), F_{1i}, G_{1i}, H_{1i}, J_{1i}, F_{2i}, G_{2i}, H_{2i}, J_{2i}, L_{ij},

M_{ij}, and scalars $\alpha > 0$, $\eta_i \geq 0$, $\rho_i \geq 0$, $\gamma > 0$, $\mu_i \geq 0$, $\mu_{ij} \geq 0$, $\theta_i \geq 0$ that solve the optimization problem

$$\min \ \gamma$$
$$s.t. \begin{cases} \alpha I + \eta_i Q_i \leq P_i \leq I - \rho_i Q_i \\ \Lambda_i + U_i + U_i^T < 0 \\ \begin{bmatrix} A_i^T L_{ij}^T + L_{ij} A_i + \mu_{ij} Q_i & A_i^T M_{ij}^T - L_{ij} \\ M_{ij} A_i - L_{ij}^T & P_j - P_i - M_{ij} - M_{ij}^T + \mu_{ij} Q_j \end{bmatrix} < 0 \\ \theta_1 Q_1 + \cdots + \theta_N Q_N \geq 0 \end{cases} \tag{24}$$

where

$$\Lambda_i = \begin{bmatrix} -P_i + \mu_i Q_i & 0 & 0 & 0 \\ 0 & P_i & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$U_i = \begin{bmatrix} F_{1i} A_i + F_{2i} C_i & -F_{1i} & -F_{2i} & F_{1i} B_i^w + F_{2i} D_i^w \\ G_{1i} A_i + G_{2i} C_i & -G_{1i} & -G_{2i} & G_{1i} B_i^w + G_{2i} D_i^w \\ H_{1i} A_i + H_{2i} C_i & -H_{1i} & -H_{2i} & H_{1i} B_i^w + H_{2i} D_i^w \\ J_{1i} A_i + J_{2i} C_i & -J_{1i} & -J_{2i} & J_{1i} B_i^w + J_{2i} D_i^w \end{bmatrix},$$

for all $i, j \in \{1, \cdots, N\}$ $i \neq j$, then the switched system (19) can be asymptotically stabilized with l_2 induced gain less than γ by the largest region function strategy (7). □

Next, consider the following switched control system (2) and the aim is to find switching signal $\sigma(x)$ and static state feedback gains K_i, such that the closed-loop switched system is stable with finite l_2 induced gain. A sufficient condition can be expressed in the following theorem.

Theorem 5: If there exist matrices P_i ($P_i = P_i^T$), Q_i ($Q_i = Q_i^T$), R_i, G_{1i}, G_{2i}, F_{2i}, H_{2i}, J_{2i} and scalars $\alpha_i > 0$, $\beta_i > 0$, $\eta_i \geq 0$, $\rho_i \geq 0$, $\gamma > 0$, $\mu_i \geq 0$, $\mu_{ij} \geq 0$, $\theta_i \geq 0$ that solve the optimization problem

$$\min \ \gamma$$
$$s.t. \begin{cases} \alpha_i I + \eta_i Q_i \leq P_i \leq \beta_i I - \rho_i Q_i \\ \Lambda_i + U_i + U_i^T < 0 \\ \begin{bmatrix} \mu_{ij} Q_i & A_i G_{1i}^T + B_i R_i \\ G_{1i} A_i^T + R_i^T B_i^T & P_j - P_i - G_{1i} - G_{1i}^T + \mu_{ij} Q_j \end{bmatrix} < 0 \\ \theta_1 Q_1 + \cdots + \theta_N Q_N \geq 0 \end{cases} \tag{25}$$

where

$$\Lambda_i = \begin{bmatrix} -P_i + \mu_i Q_i & 0 & 0 & 0 \\ 0 & P_i & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$U_i = \begin{bmatrix} B_i^w F_{2i} & A_i G_{1i} + B_i R_i + B_i^w G_{2i} & B_i^w H_{2i} & B_i^w J_{2i} \\ 0 & -G_{1i} & 0 & 0 \\ -F_{2i} & -G_{2i} & -H_{2i} & -J_{2i} \\ D_i^w F_{2i} & C_i G_{1i} + D_i R_i + D_i^w G_{2i} & D_i^w H_{2i} & D_i^w J_{2i} \end{bmatrix}.$$

for all $i, j \in \{1, \cdots, N\}$ $i \neq j$, then (1) can be stabilized with l_2 gain less than γ by the state feedback gains

$$K_i G_{1i} = R_i, \quad i \in \mathfrak{I}$$

along with the largest region function strategy (7). □

To show this, use a transposed version of Theorem 4 with $F_{1i} = 0$, $H_{1i} = 0$ and $J_{1i} = 0$, $L_{ij} = 0$, $M_{ij} = G_{1i}$, and $R_i = K_i G_{1i}$.

Similar to the stabilization case, the optimization problem above is a Bilinear Matrix Inequality (BMI) problem. Since it only contains the product of unknown scalars and matrices, it is possible to solve the BMI problem via griding up these unknown scalars. After gridding up these unknown scalars, we get a bunch of LMIs with $K = 2N\frac{n(n+1)}{2} + Nn^2 + 2Nnr + Nr^2 + Npr + Nmn$ of unknown variables and $L = 2Nn + 2N^2 n + n + N(2n+p+r)$ rows. Here, N is the number of systems, and n, m, p, r are the dimensions of the state x, input u, output z and disturbance w respectively.

Example 1: We will now illustrate the synthesis procedure in this paper in the case of two unstable subsystems given by

$$A_1 = \begin{bmatrix} 1.1052 & -1.2234 \\ 0 & 1.3499 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.0572 \\ 0.1166 \end{bmatrix}.$$

$$A_2 = \begin{bmatrix} 1.3499 & 0 \\ 0.6117 & 1.1052 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.1166 \\ 0.0286 \end{bmatrix}.$$

First, to obtain stabilizing state feedback gains and switching signal, we solve the BMI in Theorem 3 through gridding up the unknown parameters, and obtain a solution

$$\xi = 0.7275$$

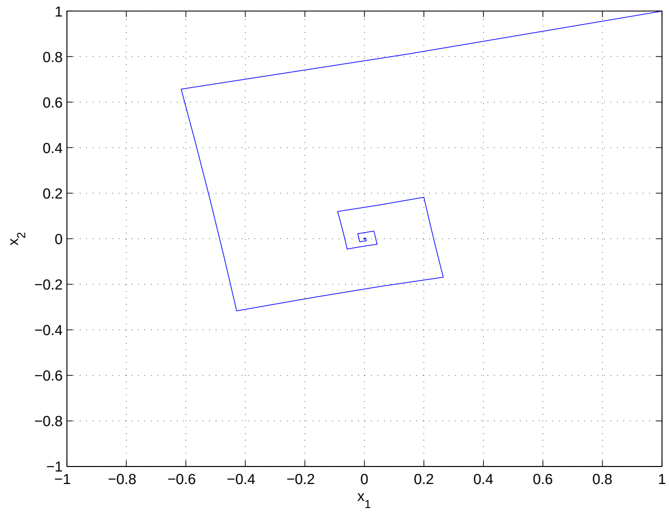

Fig. 1. The application of the design method in Theorem 3 results in a switched system that is exponentially stable.

and

$$K_1 = \begin{bmatrix} 0 & -4.6302 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -3.7864 & -0.0190 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 15.1065 & 17.9528 \\ 17.9528 & 77.1901 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 5.9394 & -5.0404 \\ -5.0404 & 5.0620 \end{bmatrix}. \tag{26}$$

$$Q_1 = \begin{bmatrix} -0.2037 & 0.1185 \\ 0.1185 & 0.2037 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0.2037 & -0.1185 \\ -0.1185 & -0.2037 \end{bmatrix}. \tag{27}$$

Hence, applying the largest region function strategy (7) results in an exponential stable closed-loop switched system. A trajectory simulation is shown in Fig 1.

Next, we consider the l_2 induced gain performance and introduce

$$B_1^w = B_2^w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D_1^w = D_2^w = 0$$

into the switched system. With the previous values of Q_1, Q_2, P_1, and P_2 as initial condition, we managed to solve the BMI in Theorem 5 with the following solution

$$\gamma = 1.6508$$

and

$$K_1 = \begin{bmatrix} 0.0013 & -12.6334 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -14.7844 & -0.0089 \end{bmatrix}. \tag{28}$$

$$P_1 = \begin{bmatrix} 1.3091 & -1.2093 \\ -1.2093 & 3.5540 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.8403 & -0.0475 \\ -0.0475 & 0.1614 \end{bmatrix}. \tag{29}$$

$$Q_1 = \begin{bmatrix} -0.3223 & 0.0185 \\ 0.0185 & 0.3223 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0.3223 & -0.0185 \\ -0.0185 & -0.3223 \end{bmatrix}. \tag{30}$$

Therefore, the closed-loop switched system along with the state feedback and the largest region function strategy is asymptotically stable and its the l_2 induced gain is less than 1.6508.

□

IV. Concluding Remarks

In this paper, the co-design of continuous-variable controllers and discrete-event switching logics, both in the state feedback form, is constructively shown for a class of discrete-time switched linear systems. The exponential stability and l_2 induced gain performance are investigated based on multiple quadratic Lyapunov-like functions. Sufficient synthesis conditions are proposed as an optimization problem with bilinear matrix inequality constraints, which can be dealt with as LMIs provided that certain associated parameters are selected in advance.

The contributions of the paper are threefold. First, switching control law synthesis methods based on multiple Lyapunov functions are extended to the discrete-time switched systems. These have significantly different features from the continuous-time case. Secondly, the MLF theorem is extended to guarantee l_2 induced gain performance, and this extension is applied to switching control law synthesis as well. Thirdly, an optimization based co-design approach for both state feedback gains and switching control laws is proposed for the discrete-time switched systems. The closed-loop switched systems form a class of linear hybrid systems, which is guaranteed to be stable with bounded l_2 induced gain.

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