

# Stability of Model-Based Networked Control Systems with Intermittent Feedback

Tomas Estrada\* Panos J. Antsaklis\*\*

\* *Department of Electrical Engineering, University of Notre Dame,  
email: testrada@nd.edu*

\*\* *Department of Electrical Engineering, University of Notre Dame,  
email: antsaklis.1@nd.edu*

---

**Abstract:** In this paper, we apply the concept of Intermittent Feedback to a class of networked control systems known as Model-Based Networked Control Systems (MB-NCS). We begin by introducing the basic architecture for model-based control with intermittent feedback, then address the case with output feedback (through the use of a state observer), providing a full description of the state response of the system, as well as a necessary and sufficient condition for stability in each case. Examples are provided to complement the theoretical results. Extensions of our results to cases with nonlinear plants are also presented. Finally, we investigate the situation where the update times  $\tau$  and  $h$  are time-varying, first addressing the case where they have upper and lower bounds, then moving on to the case where their distributions are i.i.d or driven by a Markov chain.

---

## 1. INTRODUCTION

A networked control system (NCS) is a control system in which a data network is used as feedback media. NCS is an important area in control, see for example the recent special issue Antsaklis [2007]. The use of networks as media to interconnect the different components of an industrial system is rapidly increasing. However, the use of NCSs poses some challenges. One of the main problems to be addressed when considering an NCS is the size of the bandwidth required by each subsystem. Clearly, the bandwidth required by the communication network is a major concern. Recently, modeling, analysis and control of networked control systems with limited communication capability has emerged as a topic of significant interest to control community, see for example Wong [1999], Walsh [1999], Brockett [2000], Elia [2001], Zhang [2001], Ishii [2002], Nair [2000], Took [2002], as well as recent survey papers such as Baillieul [2007] and Hespanha [2007]. An efficient way to address this is reducing the rate at which packets are transmitted.

A particular class of NCSs is model-based networked control systems (MB-NCS); Montestruque [2002]. The MB-NCS architecture makes explicit use of the knowledge of the plant dynamics to enhance the performance of the system, and it is an efficient way to address the issue of reducing packet rate. In this paper we extend the work done in MB-NCS by taking advantage of intermittent feedback. In the previous work done in MB-NCS, the state updates given to the model of the plant were for a time instant only, but with intermittent feedback the system remains in closed loop control for more extended intervals. This notion is also motivated by human motor control observation, see Schmidt [2005] and Ronco [1999]. For example, when driving a car, when approaching a curve

or hilly terrain, we pay close attention to the road for a longer time, which is equivalent to staying in closed-loop mode, and we only reduce our attention -switch to open loop control, with an occasional glance to provide instantaneous data values- when the road is once again straight. While intermittent control is a very intuitive notion, its combination with the MB-NCS architecture allows for obtaining important results and opening new paths in controlling NCSs effectively.

With the finite bit-rate constraints, quantization has to be taken into consideration in NCSs. Therefore, quantization and limited bit rate issues have attracted many researchers' attention with the aim to identify the minimum bit rate required to stabilize a NCS, see for example Delchamps [1990], Brockett [2000], Elia [2001], Tatikonda [2004], Nair [2000]. In Brockett [2000] Brockett and Liberzon proposed a dynamic quantization scheme, so called "zoom-in, zoom-out" approach, to asymptotically stabilize linear systems. The idea behind this scheme is to provide more detailed information when the states come closer to the origin through finer quantization (zoom-in), while only coarser quantization (zoom-out) is sufficient for states farther away from the origin. As an interesting observation of a person's response in face of changing environment, one usually tends to pay longer attention to objects of concern, instead of paying closer attention. This motivates us to use intermittent feedback in NCSs. Earlier results using this approach have appeared in Estrada [2006, 2007].

The rest of the paper is organized as follows. In Section 2, we describe the problem formulation in detail, as well as provide a full description of the system and necessary and sufficient conditions for stability. In Section 3, we extend our results to the output feedback case. In Section 4, we look into the case of nonlinear plants. The case for

time-varying updates is presented in Section 5. Finally, in Section 6, we provide conclusions and propose future work.

## 2. MB-NCS WITH INTERMITTENT FEEDBACK: SETUP AND FORMULATION

Let us start by introducing model-based control with intermittent feedback, in its simplest setup.

The basic setup for MB-NCS with intermittent feedback is essentially the same as that proposed in the literature for traditional MB-NCS; see references Montestruque [2002, 2003, 2004] for more results on MB-NCS.

Consider the control of a continuous linear plant where the state sensor is connected to a linear controller/actuator via a network. In this case, the controller uses an explicit model of the plant that approximates the plant dynamics and makes possible the stabilization of the plant even under slow network conditions.

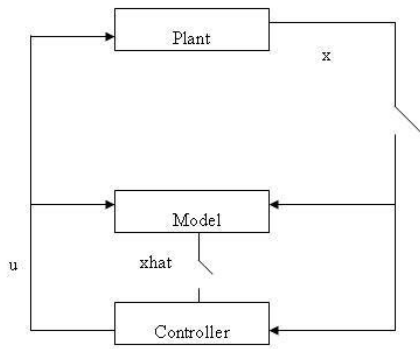


Fig. 1. MB-NCS with intermittent feedback - basic architecture

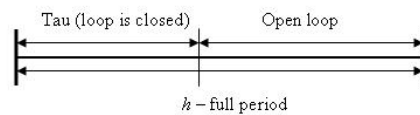


Fig. 2. Partition of the time interval into close and open loop intervals

The main idea here is to perform the feedback by updating the model's state using the actual state of the plant that is provided by the sensor. The rest of the time the control actions is based on a plant model that is incorporated in the controller/actuator and is running open loop for a period of  $h$  seconds.

As mentioned before, the main difference between model-based networked control systems as have been studied previously, and the case with intermittent feedback, which we are here discussing, is that in the literature, the loop is closed instantaneously, and the rest of the time the system is running open loop. Here, we part from the same basic idea, but the loop will remain closed for intervals of time which are different from zero. Intuitively, we should be able to achieve much better results the longer the loop

is closed, as the level of degradation of the information increases the longer the system is running open loop, so intermittent feedback should yield better results than those for traditional MB-NCS.

In dealing with intermittent feedback, we have two key time parameters: how frequently we want to close the loop, which we shall denote by  $h$ , and how long we wish the loop to remain closed, which we shall denote by  $\tau$ . Naturally, in the more general cases both  $h$  and  $\tau$  can be time-varying. For the purposes of this paper, however, we will deal only with the case where both  $h$  and  $\tau$  are fixed.

We consider then a system such that the loop is closed periodically, every  $h$  seconds, and where each time the loop is closed, it remains so for a time of  $\tau$  seconds. The loop is closed at times  $t_k$ , for  $k = 1, 2, \dots$ . Thus, there are two very clear modes of operation: closed loop and open loop. The system will be operating in closed loop mode for the intervals  $[t_k, t_k + \tau)$  and in open loop for the intervals  $[t_k + \tau, t_{k+1})$ . When the loop is closed, the control decision is based directly on the information of the state of the plant, but we will keep track of the error nonetheless.

The plant is given by  $\dot{x} = Ax + Bu$ , the plant model by  $\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u$ , and the controller by  $u = K\hat{x}$ . The state error is defined as  $e = x - \hat{x}$  and represents the difference between plant state and the model state. The modeling error matrices  $\tilde{A} = A - \hat{A}$  and  $\tilde{B} = B - \hat{B}$  represent the plant and the model. We also define the vector  $z = [x^T e^T]^T$ .

We derived the full state response of the system and a necessary and sufficient condition for stability in Estrada [2006]. For completeness, we summarize the results here.

*Proposition 1.* The system described above with initial conditions  $z(t_0) = \begin{bmatrix} x(t_0) \\ 0 \end{bmatrix} = z_0$  has the following response:

$$z(t) = \begin{cases} e^{\Lambda_c(t-t_k)} \left( \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \right)^k z_0 \\ \text{for } t \in [t_k, t_k + \tau) \\ e^{\Lambda_o(t-(t_k+\tau))} e^{\Lambda_c(\tau)} \left( \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \right)^k z_0 \\ \text{for } t \in [t_k + \tau, t_{k+1}) \end{cases} \quad (1)$$

where

$$\Sigma = e^{\Lambda_o(h-\tau)} e^{\Lambda_c(\tau)},$$

$$\Lambda_o = \begin{bmatrix} A + BK & -BK \\ \hat{A} + \hat{B}K & \hat{A} - \hat{B}K \end{bmatrix},$$

$$\Lambda_c = \begin{bmatrix} A + BK & -BK \\ 0 & 0 \end{bmatrix},$$

and  $t_{k+1} - t_k = h$ .

*Theorem 2.* The system described above is globally exponentially stable around the origin if and only if the eigenvalues of  $\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$  are strictly inside the unit circle, where  $\Sigma = e^{\Lambda_o(h-\tau)} e^{\Lambda_c(\tau)}$ .

While this theorem is restricted to the case where the time parameters remain constant and full information of the state is known, we believe it is a very valuable first step

in understanding more general situations. As we will see in the next section, the case with state observers is dealt with in a very similar fashion.

### 3. OUTPUT FEEDBACK CASE (STATE OBSERVER)

When the state of the plant is not directly measurable, we must resort to a state observer. In this section we derive results for this situation.

As in the architecture used in Montestruque [2002] for instantaneous model-based feedback, we assume that the state observer is collocated with the sensor. We use the plant model to design the state observer. See Figure 3. Our configuration is based on the analogous setup for model-based control with output feedback, proposed by Montestruque.

In Montestruque [2002] it is justified that the sensor carry the computational load of an observer by the fact that, typically, sensors that can be connected to a network have an embedded processor (usually in charge of performing the sampling, filtering, etc.) inside. The observer has as inputs the output and input of the plant. In the implementation, in order to acquire the input, which is at the other side of the communication link, the observer can have a version of the model and controller, and knowledge of the update times  $\tau$  and  $h$ . The controller and the observer are also synchronized.

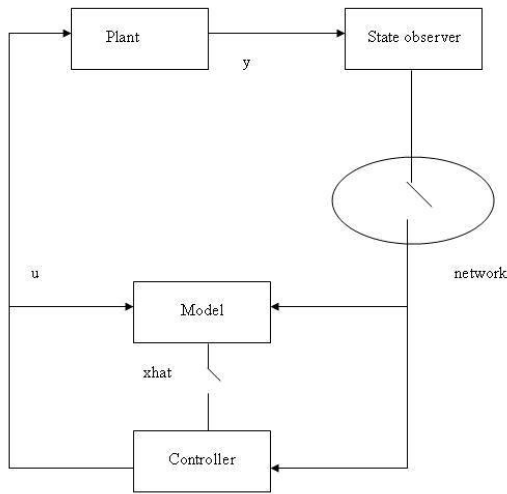


Fig. 3. MB-NCS with intermittent feedback - state observer

The observer has the form of a standard state observer with gain  $L$ . It makes use of the plant model.

In summary, the system equations are the following:

$$\text{Plant: } \dot{x} = Ax + Bu, y = Cx + Du$$

$$\text{Model: } \hat{\dot{x}} = \hat{A}\hat{x} + \hat{B}u, y = \hat{C}\hat{x} + \hat{D}u$$

$$\text{Controller: } u = K\hat{x}$$

$$\text{Observer: } \dot{\bar{x}} = (\hat{A} - L\hat{C})\bar{x} + [\hat{B} - L\hat{D} \ L] \begin{bmatrix} u \\ y \end{bmatrix}$$

Controller model state:  $\hat{x}$

Observer's estimate:  $\bar{x}$

When loop is closed:  $e = 0$

$$\text{Error matrices: } \tilde{A} = A - \hat{A}, \tilde{B} = B - \hat{B}, \tilde{C} = C - \hat{C}, \tilde{D} = D - \hat{D}$$

The state response of the system is summarized in the following proposition.

*Proposition 3.* The system described above has a state response:

$$z(t) = \begin{cases} e^{\Lambda_c(t-t_k)}\Sigma^k z_0, & t \in [t_k, t_k + \tau) \\ e^{\Lambda_o(t-(t_k+\tau))}e^{\Lambda_c(\tau)}\Sigma^k z_0, & t \in [t_k + \tau, t_{k+1}) \end{cases} \quad (2)$$

where  $\Sigma = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} e^{\Lambda_o(h-\tau)}e^{\Lambda_c(\tau)} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}$ , and

$$\Lambda_o = \begin{bmatrix} A & BK & -BK \\ LC & \hat{A} - L\hat{C} + \hat{B}K + L\tilde{D}K & -\hat{B}K - L\tilde{D}K \\ LC & L\tilde{D}K - L\hat{C} & A - L\tilde{D}K \end{bmatrix},$$

$$\Lambda_c = \begin{bmatrix} A & BK & -BK \\ LC & \hat{A} - L\hat{C} + \hat{B}K + L\tilde{D}K & -\hat{B}K - L\tilde{D}K \\ 0 & 0 & 0 \end{bmatrix}.$$

The following gives a necessary and sufficient condition for stability.

*Theorem 4.* The system described above is globally exponentially stable around the solution  $z = \begin{bmatrix} x \\ \bar{x} \\ e \end{bmatrix} = \mathbf{0}$  if and

only if the eigenvalues of  $\Sigma$  are strictly inside the unit circle, where  $\Sigma = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} e^{\Lambda_o(h-\tau)}e^{\Lambda_c(\tau)} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}$ , and

$\Lambda_o, \Lambda_c$  as before.

For the proof, see Estrada [2008].

#### 3.1 Examples

We now run simulations to illustrate the above results. Figure 4 displays the model and plant state for a high value of  $\tau$ , while an analogous plots are displayed in Figure 5 for low values. Finally, in Figure 6 we show the maximum eigenvalue of the system (the system becomes unstable when this value exceeds 1), verifying the added stability range provided by increased intermittent feedback.

For the purpose of these simulations, we used the following values:  $A = [0 \ 1; 0 \ 0.25]$ ,  $B = [0; 1]$ ,  $C = [1 \ 0]$ ,  $D = 0$ ,  $\hat{A} = [0.0958 \ 1.0604; -0.0066 \ -0.0134]$ ,  $\hat{B} = [-0.0518; 1.0269]$ ,  $\hat{C} = [0.9734 \ -0.0137]$ ,  $\hat{D} = -0.396$ ,  $K = [-1 \ -2]$ ,  $L = [20; 100]$ .

The above results are useful for situations when the full state of the plant is unavailable. An extension of our results to nonlinear plants is presented in the next section.

4. NONLINEAR PLANTS

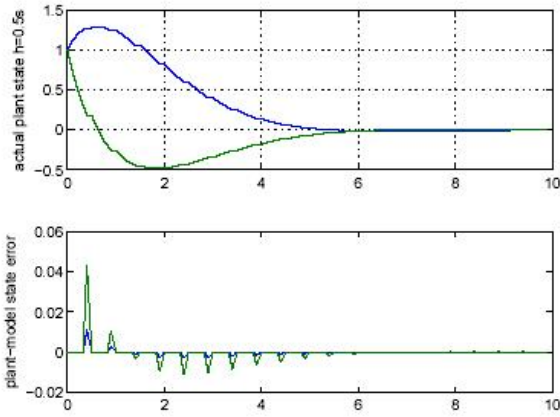


Fig. 4. Plant and model state. State observer case,  $h = 0.5$ ,  $\tau = 0.4$

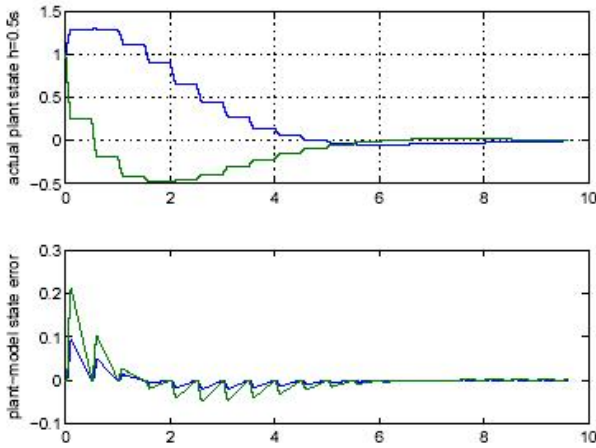


Fig. 5. Plant and model state. State observer case,  $h = 0.5$ ,  $\tau = 0.1$

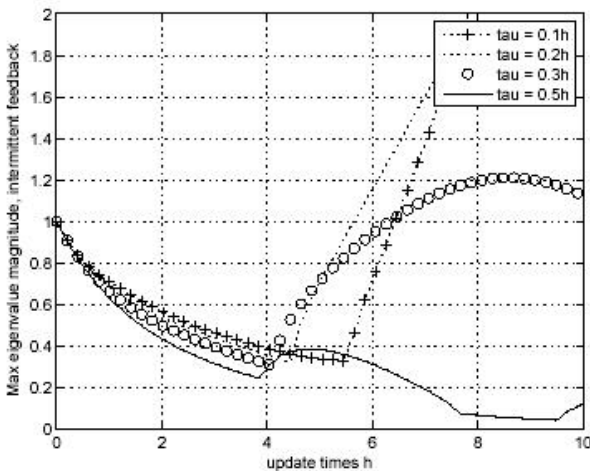


Fig. 6. Maximum eigenvalue search. State observer case, intermittent feedback

In the previous sections we have restricted our study to the cases where the plant is linear. Let us now lift this restriction and seek to find the corresponding stability properties for nonlinear plants with intermittent feedback.

The setup and procedure that follows closely mirrors that proposed by Montestruque [2003] for traditional MB-NCS. The sufficient conditions obtained relate the stability of the nonlinear MB-NCS with the value of a function that depends on the Lipschitz constants of the plant and model as well as the stability properties of the compensated non-networked model. The results are obtained by studying the worst-case behavior of the norm of the plant state and the error, thus leading to conservative results.

Let the plant be given by:

$$\dot{x} = f(x) + g(u) \tag{3}$$

We use a model on the actuator side of the plant to estimate the actual state of the plant. The controller will be assumed to be a nonlinear state feedback controller. The control signal  $u$  is generated by taking into account the plant model state. The plant state sensor will send through the network the real value of the plant state to the model (that is, the loop will be closed) every  $h$  seconds, and the loop will remain closed for  $\tau$  seconds during each cycle. During these times, the state of the model is set to be the same as that of the plant. We will assume the plant model dynamics are given by:

$$\hat{x} = \hat{f}(x) + \hat{g}(u) \tag{4}$$

And the controller has the following form:

$$u = \hat{h}(\hat{x}) \tag{5}$$

We define as the error between the plant state and the plant model state,  $e = x - \hat{x}$ . Combining the above, we obtain:

$$\begin{aligned} \dot{x} &= f(x) + g(\hat{h}(\hat{x})) = f(x) + m(\hat{x}) \\ \dot{\hat{x}} &= \hat{f}(x) + \hat{g}(\hat{h}(\hat{x})) = f(x) + \hat{m}(\hat{x}) \end{aligned} \tag{6}$$

Assume also that the plant model dynamics differ from the actual plant dynamics in an additive fashion:

$$\begin{aligned} \hat{f}(\zeta) &= f(\zeta) + \delta_f(\zeta) \\ \hat{m}(\zeta) &= m(\zeta) + \delta_m(\zeta) \end{aligned} \tag{7}$$

Thus:

$$\begin{aligned} \dot{x} &= f(x) + m(\hat{x}) \\ \dot{\hat{x}} &= f(x) + \hat{m}(\hat{x}) + \delta_f(\hat{x}) + \delta_m(\hat{x}) \end{aligned} \tag{8}$$

Assume that  $f$  and  $\delta$  satisfy the following local Lipschitz conditions for with  $x, y \in B_L$ , a ball centered on the origin:

$$\begin{aligned} \|f(x) - f(y)\| &\leq K_f \|x - y\| \\ \|\delta(x) - \delta(y)\| &\leq K_\delta \|x - y\| \end{aligned} \tag{9}$$

It is to be noted that if the plant model is accurate the Lipschitz constant  $K_\delta$  will be small.

Assume that the non-networked compensated plant model is exponentially stable when  $\hat{x}(t_0) \in B_S$ ,  $\hat{x}(t) \in B_\tau$ , for  $t \in [t_0, t_0 + \tau)$  with  $B_S$  and  $B_\tau$  balls centered on the origin.

$$\|\hat{x}(t)\| \leq \alpha \|\hat{x}(t_0)\| e^{-\beta(t-t_0)} \text{ with } \alpha, \beta > 0. \quad (10)$$

*Theorem 5.* The non-linear MB-NCS with dynamics described above, and that satisfies the Lipschitz conditions described and with exponentially stable compensated plant model satisfying is asymptotically stable if:

$$1 - \alpha \left( e^{-\beta(h-\tau)} + \left( e^{K_f(h-\tau)} - e^{-\beta(h-\tau)} \right) \left( \frac{K_\delta}{K_f + \delta} \right) \right) > 0 \quad (11)$$

For the proof, see Estrada [2008]. It should be noted that these results are conservative, and the condition is sufficient only. Finding tighter bounds for nonlinear plants in model-based networked control systems remains an open problem.

## 5. STABILITY OF MB-NCS WITH INTERMITTENT FEEDBACK AND TIME-VARYING UPDATES

Until now we have only considered the case where the parameters  $\tau$  and  $h$  are constant. Let us now take a closer look at what happens when these parameters vary with time. The definitions for Lyapunov stability and mean square stability used throughout this section are the same as those in Montestruque [2004].

### 5.1 Lyapunov stability with bounded intervals

We shall first analyze the case where the parameters are time-varying, but their probability distributions are unknown. Let the plant, model, and controller have the same dynamics as described in Section 2. The following result describes the state response of the system. The derivation of this result is analogous to that for constant  $\tau$  and  $h$ .

*Proposition 6.* The system described above with initial conditions  $z = \begin{bmatrix} x(t_0) \\ 0 \end{bmatrix} = z_0$  has the following response:

$$z(t) = \begin{cases} e^{\Lambda_o(t-t_k)} \left( \prod_{j=1}^k M(j) \right) z_0, & t \in [t_k, t_k + \tau) \\ e^{\Lambda_o(t-(t_k+\tau))} e^{\Lambda_c(\tau)} \left( \prod_{j=1}^k M(j) \right) z_0, & t \in [t_k + \tau, t_{k+1}) \end{cases}$$

where  $M(j) = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda_o(h(j)-\tau(j))} e^{\Lambda_c(\tau(j))} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ ,  $\Lambda_o = \begin{bmatrix} A + BK & -BK \\ \tilde{A} + \tilde{B}K & \tilde{A} - \tilde{B}K \end{bmatrix}$ ,  $\Lambda_c = \begin{bmatrix} A + BK & -BK \\ 0 & 0 \end{bmatrix}$ ,  $t_{k+1} - t_k = h(k)$ , and  $\tau(j) < h(j)$ .

We now present a condition for Lyapunov stability of this system.

*Theorem 7.* The system described above is Lyapunov asymptotically stable for  $h \in [h_{\min}, h_{\max}]$  and  $\tau \in [\tau_{\min}, \tau_{\max}]$  (with  $\tau_{\max} < h_{\min}$ ) if there exists a symmetric positive definite matrix  $X$  such that  $Q = X - MXM^T$  is positive definite for all  $h \in [h_{\min}, h_{\max}]$  and  $\tau \in [\tau_{\min}, \tau_{\max}]$ , where  $M = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda_o(h-\tau)} e^{\Lambda_c(\tau)} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ .

Note that the output norm can be bounded by

$$\begin{aligned} & \left\| e^{\Lambda_o(t-(t_k+\tau))} e^{\Lambda_c(\tau)} \left( \prod_{j=1}^k M(j) \right) z_0 \right\| \\ & \leq \left\| e^{\Lambda_o(t-(t_k+\tau))} \right\| \left\| e^{\Lambda_c(\tau)} \right\| \left\| \prod_{j=1}^k M(j) \right\| \|z_0\| \\ & \leq e^{\bar{\sigma}(\Lambda_o)h_{\max} - \tau_{\min}} \left\| e^{\Lambda_c(\tau)} \right\| \left\| \prod_{j=1}^k M(j) \right\| \|z_0\| \end{aligned}$$

That is, since  $e^{\Lambda_o(t-(t_k+\tau))}$  has finite growth and will grow for at most from  $\tau_{\min}$  to  $h_{\max}$ , then convergence of the product of matrices  $M(j)$  to zero ensures the stability of the system. Such convergence to zero is guaranteed by the existence of a symmetric positive definite matrix  $X$  in the Lyapunov equation.

### 5.2 Mean square stability of discrete MB-NCS with IF with i.i.d update times

Now, let us consider the case where  $\tau$  is constant, but  $h(k)$  are independent identically distributed with probability distribution  $F(h)$ . This corresponds to the situation where we might not know how frequently we can access the network, but when we do obtain access to it, we continue to have access to it for a fixed amount of time, so as to, for example, complete a given task or transmit a certain set of packets. We present a stability condition for this case:

*Theorem 8.* The system described above with update times  $h(j)$  independent identically distributed random variable with probability distribution  $F(h)$  is globally mean square asymptotically stable around the solution  $z = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$  if  $K = E \left[ (e^{\bar{\sigma}(\Lambda_o)(h-\tau)})^2 \right] < \infty$  and the maximum singular value of the expected value  $M^T M$ ,  $\|E[M^T M]\| = \bar{\sigma}(E[M^T M])$  is strictly less than one, where  $M = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda_o(h-\tau)} e^{\Lambda_c(\tau)} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ .

The proof is similar to that found in Montestruque [2004] for the case of instantaneous feedback.

### 5.3 Mean square stability of discrete MB-NCS with IF with Markov chain-driven update times

We now consider the situation where the parameter  $h$  is driven by a Markov chain and provide a stability condition.

*Theorem 9.* The system described above with update times  $h(k) = h_{\omega_k} \neq \infty$  driven by a finite state Markov chain  $\{\omega_k\}$  with state space  $\{1, 2, \dots, N\}$  and transition probability matrix  $\Gamma$  with elements  $p_{i,j}$  is globally mean square asymptotically stable around the solution  $z = [x^T e^T]^T = \mathbf{0}$  if there exist positive definite matrices  $P(1), P(2), \dots, P(N)$  such that

$$\left( \sum_{j=1}^N p_{i,j} \left( H(i)^T P(j) H(i) \right) - P(i) \right) < 0 \quad \forall i, j \in 1, \dots, N$$

with  $H(i) = e^{\Lambda_o(h_i-\tau)} e^{\Lambda_c(\tau)} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ .

Once again, the proof follows that in Montestruque [2004] for the case of instantaneous feedback.

## 6. CONCLUDING REMARKS AND FUTURE WORK

In this paper, we have provided a set of results for model-based networked control systems with intermittent feedback. We focused first on deriving stability results and provided necessary and sufficient conditions for the basic setup as well as the case with state observers. We also provided sufficient conditions for nonlinear systems. Finally, we investigated the situation where the update times  $\tau$  and  $h$  are time-varying, first addressing the case where they have upper and lower bounds, then moving on to the case where their distributions are i.i.d or driven by a Markov chain, providing stability conditions in each case.

The area of performance of networked control systems, both under the model-based architecture and otherwise, remains a relatively unexplored ground for research. In future work, we expect to provide results on performance of model-based networked control systems with intermittent feedback, and will consider other issues, such as robustness, filtering, and improving control as time elapses, as well.

## REFERENCES

- P. Antsaklis and A. Michel. *Linear Systems* 1st edition, McGraw-Hill, New York, 1997.
- P. Antsaklis and J. Baillieul. Special Issue on Networked Control Systems. *Proceedings of the IEEE*, 95(1), 2007.
- B. Azimi-Sadjadi. Stability of Networked Control Systems in the Presence of Packet Losses. *Proceedings of the 42nd Conference of Decision and Control*, December 2003.
- J. Baillieul and P. Antsaklis. Control and communication challenges in networked real-time systems. *Proceedings of the IEEE*, 95(1), 2007.
- M.S. Branicky, S. Phillips, and W. Zhang. Scheduling and feedback co-design for networked control systems. *Proceedings of the 41st Conference of Decision and Control*, December 2002.
- R. Brockett and D. Liberzon. Quantized Feedback Stabilization of Linear Systems *IEEE Transactions on Automatic Control*, Vol 45, no 7, pp 1279-89, July 2000.
- D. Delchamps. Stabilizing a Linear System with Quantized State Feedback. *IEEE Transactions on Automatic Control*, Vol 35, no. 8, pp. 916-924.
- N. Elia, and S. K. Mitter. Stabilization of linear systems with limited information. *IEEE Transactions on Automatic Control*, 2001, 46(9): 1384-1400.
- T. Estrada, H. Lin, and P.J. Antsaklis. Model-based control with Intermittent Feedback *Proceedings of the Mediterranean Control Conference*, 2006, Ancona, Italy.
- T. Estrada and P.J. Antsaklis. Control with Intermittent Communication (Feedback) over a Network: Recent Results *Proceedings of the Workshop on Networked Distributed Systems for Intelligent Sensing and Control*, 2007, Kalamata, Greece.
- T. Estrada and P.J. Antsaklis. ISIS Technical Report on Model-Based Control with Intermittent Feedback <http://www.nd.edu/isis/tech.html>.
- J. Hespanha, P. Naghshtabrizi, Yonggang Xu. A survey of recent results in networked control systems *Proceedings of the IEEE*, 95(1), 2007.
- D. Hristu-Varsakelis. Feedback Control Systems as Users of a Shared Network: Communication Sequences that Guarantee Stability, *Proceedings of the 40th Conference on Decision and Control*, December 2001, pp. 3631-36.
- H. Ishii and B. A. Francis. *Limited Data Rate in Control Systems with Networks*. Lecture Notes in Control and Information Sciences, vol. 275, Springer, Berlin, 2002.
- L.A. Montestruque. *A Dissertation: Model-Based Networked Control Systems*. University of Notre Dame, Notre Dame, IN, USA, September 2004.
- L.A. Montestruque and P.J. Antsaklis. Model-Based Networked Control Systems: Necessary and Sufficient Conditions for Stability. *Proceedings of the 10th Mediterranean Conference on Control and Automation*, July 2002.
- L.A. Montestruque and P.J. Antsaklis. State and Output Feedback Control in Model-Based Networked Control Systems *Proceedings of the 41st IEEE Conference on Decision and Control*, December 2002.
- L.A. Montestruque and P.J. Antsaklis. Stability of Model-Based Networked Control Systems with Time-Varying Transmission Times *IEEE Transactions on Automatic Control, Special Issue on Networked Control Systems*, Vol. 49, No. 9, pp.1562-1572, September 2004.
- G. Nair and R. Evans. Communication-Limited Stabilization of Linear Systems. *Proceedings of the Conference on Decision and Control*, 2000, pp. 1005-1010.
- E. Ronco and D. J. Hill. Open-loop intermittent feedback optimal predictive control: a human movement control model. *Proceedings of the Neural Information Processing Systems conference*, 1999.
- L. Schenato. To zero or to hold control inputs in lossy networked control systems. *Proceedings of the European Control Conference*, 2007, Kos, Greece.
- Richard A. Schmidt. *Motor Control and Learning - A Behavioral Emphasis* 4th edition, Human Kinetics, 2005.
- J. Took, D. Tilbury, and N. Soparkar. Trading Computation for Bandwidth: Reducing Computation in Distributed Control Systems using State Estimators. *IEEE Transactions on Control Systems Technology*, July 2002, Vol 10, No 4, pp 503-518.
- S. Tatikonda and S. Mitter. Control under communication constraints *IEEE Transactions on Automatic Control*, 2004, 49(7): 1056-1068.
- G. Walsh, H. Ye, and L. Bushnell. Stability Analysis of Networked Control Systems. *Proceedings of American Control Conference*, June 1999.
- W. S. Wong and R. W. Brockett. Systems with finite communication bandwidth constraints I: Stabilization with limited information feedback. *IEEE Transactions on Automatic Control*, 1999, 44(5): 1049-1053.
- W. S. Wong and R. W. Brockett. Systems with finite communication bandwidth constraints II: Stabilization with limited information feedback. *IEEE Transactions on Automatic Control*, 1999, 44(5): 1049-1053.
- W. Zhang, M. S. Branicky, and S. M. Phillips. Stability of networked control systems. *IEEE Control Systems Magazine*, 2001, 21(1): 84-99.