Design of Digital Control Networks for Continuous Passive Plants Subject To Delays and Data Dropouts

Nicholas Kottenstette *and Panos J. Antsaklis †
*Corresponding Author
Institute for Software Integrated Systems
Vanderbilt University
PO BOX 1829, Station B
Nashville, TN 37203
Email: nkottens@isis.vanderbilt.edu
†Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556
Email: antsaklis.1@nd.edu

Abstract

We present a framework to synthesize l_2^m-*stable* control networks which are subject to delays and data dropouts. This framework can be applied to control systems which use "soft-real-time" cooperative schedulers and wired or wireless network feedback. The approach applies to *passive* plants and controllers that can be either linear, nonlinear, and (or) time-varying. This framework is based on fundamental results presented here related to *passive* control, and scattering theory. It loosens the requirements for the *passive* systems to possess specialized storage functions and state space descriptions as is typically done in showing Lyapunov stability for *passive* force-feedback telemanipulation systems, of which we provide a short review. The benefits of loosening these requirements will become quite obvious to the reader as we make connections between the general input-output definitions of *passivity* and the more specific *passive dissipative* system definitions.

Theorem 4 states how a (non)linear (*strictly input* or *strictly output*) *passive* plant can be transformed to a discrete (*strictly input*) *passive* plant using a particular digital sampling and hold scheme. Furthermore, Theorem 5(6) provide new sufficient conditions for l_2^m (and L_2^m)-*stability* in which a *strictly-output passive* controller and plant are interconnected with only *wave-variables*. Lemma 2 shows it is sufficient to use discrete *wave-variables* when data is subject to fixed time delays and dropouts in order to maintain *passivity*. Lemma 3 shows how to safely handle time varying discrete *wave-variable* data in order to maintain *passivity*. Proposition 1 shows how to synthesize a discrete *passive LTI* system from a continuous *passive LTI* system, which leads to Corollary 1 that shows how to synthesize a discrete *strictly-output passive LTI* system from a continuous *passive LTI* system. Corollary 2 provides a suitable method to appropriately scale the synthesized discrete *strictly-output passive LTI* system. Corollaries 3 and 4 show how to integrate the discrete *strictly-output passive LTI* system with *wave variables*. Proposition 2 shows how a *LTI strictly-output passive* observer can be implemented for a *strictly-output passive LTI* continuous plant. Corollaries 5 and 6 result from Proposition 2 as they relate to an observer using *wave variables*. We then present a new cooperative scheduler algorithm to implement a l_2^m-*stable* control network. We also provide an illustrative simulated example followed by a discussion of future research.

Index Terms

passivity theory, scattering theory, *wave variables*, telemanipulation, (wireless) networked control systems, digital control systems, control synthesis, observers, non-linear control theory, linear control theory, *strictly-positive real* systems, *passive* systems, *strictly-output passive* systems, *positive real* systems, control with cooperative schedulers, *LTI* systems, l_2^m-stability theory, *LMI*'s, adaptive control theory, *Hamiltonian* systems, *Euler-Lagrange systems*

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Note: All proofs in Appendix B and the corresponding Theorems are in the following paper:
N. Kottenstette, P. Antsaklis, "Stable Digital Control Networks for Continuous Passive Plants Subject to Delays and Data Dropouts". (in the Proceedings of the 46^{th} IEEE Conference on Decision and Control, December 2007)
The result in regards to preservation of passivity using the *IPESH* will appear in:
N. Kottenstette, J. Hall, X. Koutsoukos, P. Antsaklis, and J. Sztipanovits, "Digital control of multiple discrete passive plants over networks," International Journal of Systems, Control and Communications (IJSCC), Special Issue on Progress in Networked Control Systems, 2009.
, however, the more general case in regards to the *IPESH* preserving *strictly-input passive* and *strictly-output passive* are presented here in Theorem 4.

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I. INTRODUCTION

This work has been motivated by the urgent need to develop reliable wireless control networks. These networks typically consist of distributed-wireless sensors, actuators and controllers which communicate with low cost devices such as the MICA2 and MICAz motes [1]. The operating systems for these devices, typically consist of a very simple scheduler, known as a cooperative scheduler [2]. The cooperative scheduler provides a common time-base to schedule tasks to be executed, however, it does not provide a context-switch mechanism to interrupt tasks. Thus, tasks have to cooperate in order not to delay other pending tasks which is difficult to satisfy without careful testing and auditing of the software being run. The analysis can be quite complex if some of the tasks are event driven and take a moderately long time to run to completion. Even deterministic schedulers can run into significant difficulties dealing with issues such as priority inversion of tasks, for example. As a result, a controller needs to be designed to tolerate time-varying delays caused by disruptive tasks which share the cooperative scheduler. Although, other operating systems can be designed to provide a tighter real-time scheduling performance, the time varying delays which will ultimately be encountered with wireless sensing and actuation will be comparable, if not more significant. The reason for these time varying delays arises from the fundamental fact that digital communication systems are subject to noise which limits their average capacity and determines the average time varying delays in which information is transmitted. Furthermore, as information is transmitted over a network and stored in queues while waiting to be routed, the variance in the delays continues to increase. If the network becomes congested and the queues fill up data will have to be dropped and random drop outs will occur.

The primary aim of this paper is to provide a theoretical framework to build l_2^m*-stable* controllers which can be subject to time-varying scheduling delays and data dropouts. Such results are also of importance as they will eventually allow the plant-controller network depicted in Fig. 5 to run entirely isolated from the plant as is done with telemanipulation systems. Telemanipulation systems have had to address wireless control problems [3] and the corresponding literature provides results to address how to design stable control systems subject to transmission delays in such systems. Much of the theory presented in this paper is inspired and related to work on telemanipulation systems. Thus, Section II provides a brief review of telemanipulation, and how it relates to *passive* control and scattering theory in order to provide the reader with some physical insight related to the framework presented in Section III.

Telemanipulation systems are distributed control systems where a human operator uses a local manipulator to control a remotely located robot in order to modify a remote environment. The position tracking between the human operator and the robot is typically maintained by a *passive* proportional-derivative controller. In fact, a telemanipulation system typically consists of a series network of interconnected two-port *passive* systems in which the human operator and environment terminate each end of the network [4]. These *passive* networks can remain stable in-spite of system uncertainty; however, delays as small as a few milliseconds may cause force feedback telemanipulation systems to become unstable. Fixed delayed power variables, force (effort) and velocity (flow), make the communication channel non *passive* which typically results in instabilities except for special cases. For example, global stability can be guaranteed if there exists zero-state detectable plants and controllers with positive definite storage functions in $\mathcal{C}^1$ in which the product of their L_2^m-gains is less than or equal to one [5, Theorem 3.1].

It was shown In [3], [6] that by using the scattering transformation, power variables can be transformed into *wave variables* [7] and the communication channel will remain *passive* in spite of arbitrary, fixed delays. In [5] additional passivity conditions are provided for stability results when using continuous-time plants and controllers in conjunction with *wave variables*. For continuous-time systems, if additional information is transmitted along with the continuous *wave variables*, the communication channel will also remain *passive* in the presence of time varying delays [8]. However, only recently has it been shown how discrete *wave variables* can remain *passive* in spite of time varying delays and dropouts [9], [10]. We verify this to be true for fixed time delays and data dropouts (Lemma 2). However, we provide a simple counter example showing that this is not the case for all time-varying delays. We then provide a lemma which states how to properly handle time varying discrete wave variable data and maintain *passivity* (Lemma 3). [11], [12] build upon the novel digital sample and hold scheme introduced in [9] which allows the resulting discrete-time inner-product to be equal to the continuous-time inner-product.

We will build on the results in [11] to show in general how to transform a (non)linear (*strictly input*) or (*strictly output*) *passive* system into a discrete (*strictly input*) or (*strictly output*) *passive* system (Theorem 4). We then formally present l_2^m*-stability* results related to *strictly-output passive* networks. In particular Theorem 3 shows how to make a discrete *passive* plant *strictly-output passive* and l_2^m*-stable*. Theorem 3 also makes it possible to synthesize discrete *strictly-output passive* systems from discrete *passive LTI* systems such as those consisting of *passive* wave digital filters [13]. We will then use the scattering transform to interconnect the controller to the plant with *wave variables*. We use Lemma 3 to show that the cooperative scheduler can allow time varying data transmission delays and maintain passivity between the plant and controller. As a result our digital control system implemented with a cooperative scheduler will remain l_2^m-*stable*.

Section II provides a brief discussion of telemanipulation systems including *passivity* and scattering theory from a continuous time and classic control framework. Section III provides definitions and theorems necessary to present our main results in Section IV. Some of the initial theorems and lemmas were given in [14], therefore the corresponding proofs have been moved to the appendix and may be omitted. New *passive* discrete *LTI* system synthesis results are provided in Section IV-A which includes Proposition 2 that shows how to implement a *strictly-output passive* observer. In Section V presents results from a simulation of a *passive* motor being digitally-controlled over a network subject to various fixed time-delays in which a novel observer is used to recover passivity in the discrete time domain. Section VI summarizes our key findings and discusses future research directions.

II. TELEMANIPULATIONSYSTEMS.

Telemanipulation systems typically consist of a network of interconnected *passive* systems. In particular [3] showed how to design telemanipulation which can tolerate arbitrary fixed time delays. We review these results and introduce *wave variables* in order to lead the reader into our main results. Passive systems are an important class of systems for which Lyapunov like functions exist [15]–[18]. The Lyapunov like function arises from the definition of passivity (1). In *passive* systems (1), the rate of change in stored energy E_{store} is equal to the amount of power put in to the system P_{in} minus the amount of power dissipated P_{diss} (≥ 0).

$$\dot{E}_{\mathsf{store}} = P_{\mathsf{in}} - P_{\mathsf{diss}} \tag{1}$$

As long as all internal states x of the system are associated with stored energy in the system, it can be shown that a *passive* system is stable when no input power is present, simply by setting $P_{\mathsf{in}} = 0$. $P_{\mathsf{diss}} \geq 0$ implies that $\dot{E}_{\mathsf{store}} \leq 0$ which shows that the system is Lyapunov stable. By using either the invariant set theorem or Barbalat's Lemma [16] asymptotic stability can be shown [4]. These *passive* systems, which can be interconnected in parallel and feedback configurations which result in additional *passive* systems, are fundamental components of telemanipulation systems [4]. Instabilities can occur when a telemanipulation system incurs communication delays between the master controller and slave manipulator; note that delays as small as a few milliseconds can cause instability. Instabilities may occur when the communication channel becomes a non-passive element in the telemanipulation system [7]. *Wave variables* are used here to communicate commands and provide feedback in telemanipulation systems, because they allow the communication channel to remain *passive* for arbitrary fixed delays. The variables, the values of which were most commonly communicated in the past over a telemanipulation channel were *power variables*, such as force

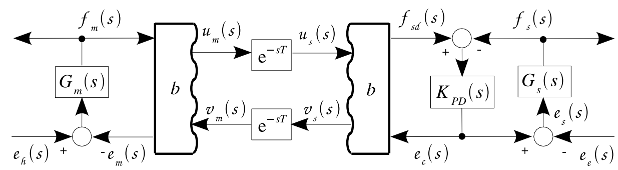

Fig. 1. Telemanipulation system depicted in the s-Domain, subject to communication delays.

and velocity (F,$\dot{x}$). *Power variables*, generally denoted by an *effort* and *flow* pair (e_*,f_*) whose product is power, are typically used to show the exchange of energy between two systems using *bond graphs* [19], [20]. Some other examples of *effort* and *flow* pairs of *power variables* are voltage and current (V,$\dot{q}$), and magnetomotive force and flux rate ($\mathcal{F}$,$\dot{\varphi}$). In this paper *wave variables* are used and described by the pair of variables (u_*,v_*) and determined by the transmission wave impedance b (> 0) [7]. The fixed channel communication time delay is T seconds. The transmission between the master and slave controller (as depicted in Fig. 1 in the s-Domain) are governed by the following delay equations for the wave variables:

$$u_s(t) = u_m(t-T) \tag{2}$$

$$v_m(t) = v_s(t-T) \tag{3}$$

in which the input *wave variables* are computed using

$$u_m(t) = \frac{1}{\sqrt{2b}}(bf_m(t) + e_m(t)) \tag{4}$$

$$v_s(t) = \frac{1}{\sqrt{2b}}(bf_{sd}(t) - e_c(t)) \tag{5}$$

These simple wave variable transformations, which can be applied to vectors, allow us to show that the wave communication channel is both *passive* and lossless assuming zero initial conditions.

$$E_{\mathsf{store}}(t) = \int_0^t P_{\mathsf{in}} d\tau = \int_{t-T}^t \left(\frac{1}{2}u_m^{\mathsf{T}}u_m + \frac{1}{2}v_s^{\mathsf{T}}v_s\right) d\tau \geq 0 \tag{6}$$

In Fig. 1, the transfer function associated with the master manipulator is denoted by $G_m(s)$ and is typically a *passive* mass. Furthermore, the slave manipulator is denoted by the transfer function, $G_s(s)$ and is typically a *passive* mass. The *passive* “proportional-derivative” plant controller $K_{PD}(s)$ has the following form:

$$K_{PD}(s) = \frac{Bs + K}{s} \tag{7}$$

The plant controller is “proportional-derivative” in the sense that the integral of the flow variable f_* yields a displacement variable q_* which is then multiplied by a proportional gain K and derivative term B. With $r_s(s) = e_d(s) = 0$ the system is *passive* and Lyapunov stable in regards to the plant's velocity and the velocity equilibrium point equals 0 (note that the final position of the plant is dependent on the systems initial condition). This velocity equilibrium holds in spite of arbitrary fixed delays since *passivity* is preserved. See [21] and

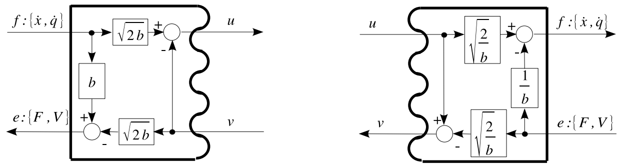

Fig. 2. Block diagrams depicting the wave variable transformation (simplified version of Fig. 3 in [23]).

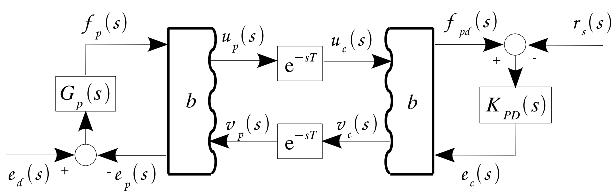

Fig. 3. A delay-insensitive system in which a *passive* controller commands a *passive* plant.

[22, Theorem 2] for tests which can be applied to determine an appropriate value for b to satisfy either stability or L_2^m stability in which $e_d(s) = 0$. We will show that it is sufficient for $K_{PD}(s)$ and $G_P(s)$ to be modified to be *strictly-output passive* in order to satisfy L_2^m stability for $\forall b > 0$ and both $e_d(t)$ and $r_s(t)$ can be signals in L_2^m. The sufficient proof for both L_2^m and l_2^m stability is given in Section IV. Although the *wave variables* (u_*,v_*) do not need to be associated with a particular direction as do the power variables, when interconnected with a pair of effort and flow variables an effective direction is implied. Fig. 2 shows how to implement the wave transform for both cases. Fig. 1 can be modified to yield the following system depicted in Fig. 3 in which a *passive* controller $K_{PD}(s)$ is able to command a *passive* plant $G_p(s)$. The plant will follow the flow set-point $\mathbf{r}_s(s)$. If we precede $\mathbf{r}_s(s)$ with a causal derivative filter $G_d(s) = \frac{s}{\tau s+1}$ such that $\mathbf{r}_s(s) = G_d(s)\mathbf{q}_s(s)$ then the plant will track a desired fixed displacement set-point $\mathbf{q}_s(s)$ at steady state.

Simulations offers some insights into this system's behavior: for example when using velocity feedback of a *passive* mass ($G_p(s) = \frac{1}{Ms}$), then the plant displacement $\int f_p(t)dt$ will equal the displacement set-point $q_s(t) = \int r_s(t)dt$ at steady state. If the plant is *passive* and stable such as a mass-spring-damper system ($G_p(s) = \frac{s\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$), then steady state error will occur ($\int f_p(t)dt \neq \int r_s(t)dt$). So far the discussion has taken place with respect to the continuous time domain and has been pointed out that even under fixed delayed data to and from the controller $K_{PD}(s)$, with the use of *wave variables*, a *passive* control system can be designed which is Lyapunov stable.

III. PASSIVE CONTROL THEORY

Passive systems (1) are a special class of *dissipative* dynamical systems which have storage function $w(u, y) = u^\mathsf{T} y$ [24]–[26]. Passive control theory is general and broad in that it applies to a large class of linear, non-linear, continuous and discrete control systems. In [15] results for continuous and discrete *passive* systems are presented. Passive control theory has been used in digital *adaptive control* to show stability of various *parameter adaptation algorithms* [27]. Additional texts which discuss *passive* control theory for non-linear continuous systems are [16]–[18]. In [28] a comprehensive treatment is dedicated to the *passive* control of a class of non-linear systems, known as *Euler-Lagrange systems*. *Euler-Lagrange systems* can be represented by a *Hamiltonian* which has a Dirac structure that allows dissipative and energy storage elements to be interconnected to ports without causal specification [29, p. 124]. In [29] an extensive treatment of intrinsically *passive* control using Generalized Port-Controlled Hamiltonian Systems is presented, in particular as it relates to telemanipulation and scattering theory. Our presentation of *passive* control theory focuses on laying the groundwork for discrete *passive* control theorems, mirrors the continuous counterpart results presented in [17], and extends the continuous and discrete results in [15].

A. l_2^m STABILITY THEORY FOR PASSIVE NETWORKS

This section covers some basic results related to discrete time *passivity* theory some of which are novel. In particular how *strictly-output passivity* relates to l_2^m-stability, how to transform a *passive* system to a *strictly-output passive* system with negative feedback, and how an *IPESH* (Definition 5) converts a continuous *passive* system to a discrete time *passive* system.

Mathematical Preliminaries: The l_2^m space, is the real space of all bounded, infinitely summable functions $f(i) \in \mathbb{R}^n$. We assume $f(i) = 0$ for all $i < 0$. The *inner product* is denoted $\langle \cdot, \cdot \rangle$ in which for example $\langle u, y \rangle = u^\mathsf{T} y$ is a valid inner product [30, p.68]. More generally the *inner product* will apply to functions in the l_2^m space, which is the set of all functions $f(i)$ which satisfy the inequality given by (8).

$$\langle f(i), f(i) \rangle \triangleq \|f(i)\|_2^2 = \sum_{i=0}^{\infty} f^\mathsf{T}(i) f(i) < \infty \tag{8}$$

A truncation operator will be defined as follows:

$$f_N(i) = \begin{cases} f(i), & \text{if } 0 \le i < N \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

Likewise the extended l_2^m space, $l_{2_e}^m$, is the set of all functions $f(i)$ which satisfy the following inequality (10).

$$\langle f(i), f(i) \rangle_N = \sum_{i=0}^{N-1} f^\mathsf{T}(i) f(i) < \infty, \ N \ge 1 \tag{10}$$

Note that $l_2^m \subset l_{2_e}^m$. Typically $l_{2_e}^m$ is defined with the summation to N and the truncation includes N [27, p. 75] and [15, p. 172], however, these definitions are equivalent. Finally we can define our l_2^m norms (11) and truncation of the l_2^m norm (12) as follows:

$$\|f(i)\|_2 \triangleq \langle f(i), f(i) \rangle^{\frac{1}{2}} \tag{11}$$

$$\|f(i)_N\|_2^2 \triangleq \langle f(i), f(i) \rangle_N = \sum_{i=0}^{N-1} f^\mathsf{T}(i) f(i) \tag{12}$$

The following definition for l_2^m-stability is similar to the one given in [31] which refers to [17] in regards to stating that *finite l_2^m-gain* is sufficient for l_2^m-stability, for the continuous time case only. We provide a short proof for the discrete time case and we note for completeness where the developments parallel each other [17].

Definition 1: Let the set of all functions $u(i) \in \mathbb{R}^n$, $y(i) \in \mathbb{R}^p$, which are either in the l_2^m space or $l_{2_e}^m$ space, be denoted by $l_2^m(U)$/$l_{2_e}^m(U)$ and $l_2^m(Y)$/$l_{2_e}^m(Y)$ respectively. Define now G as an input-output mapping $G : l_{2_e}^m(U) \to l_{2_e}^m(Y)$, such that it is *$l_2^m$-stable if*

$$u \in l_2^m(U) \Rightarrow G(u) \in l_2^m(Y) \tag{13}$$

The map G has *finite l_2^m-gain* if there exist finite constants γ and b such that for all $N \geq 1$

$$\|(G(u))_N\|_2 \leq \gamma \|u_N\|_2 + b, \forall u \in l_{2_e}^m(U) \tag{14}$$

holds. Equivalently G has *finite l_2^m-gain* if there exist finite constants $\hat{\gamma} > \gamma$ and $\hat{b}$ such that for all $N \geq 1$ [17, (2.21)]

$$\|(G(u))_N\|_2^2 \leq \hat{\gamma}^2 \|u_N\|_2^2 + \hat{b}, \forall u \in l_{2_e}^m(U) \tag{15}$$

holds.

Remark 1: If G has *finite l_2^m-gain* then it is sufficient for l_2^m-stability. Let $u \in l_2^m(U)$ and $N \to \infty$ which leads (14) to

$$\|(G(u))\|_2 \leq \gamma \|u\|_2 + b, \forall u \in l_2^m(U) \tag{16}$$

which implies (13) (see [17, p. 4] for the continuous time case).

Lemma 1: [17, Lemma 2.2.13] The *l_2^m-gain* $\gamma(G)$ is given as

$$\gamma(G) = \inf\{\hat{\gamma} \mid \exists\ \hat{b} \text{ s.t. (15) holds}\} \tag{17}$$

Next we will present definitions for various types of passivity for discrete time systems.

Definition 2: [15], [17] Let $G : l_{2_e}^m(U) \to l_{2_e}^m(U)$ then for all $u \in l_{2_e}^m(U)$ and all $N \geq 1$:

I. G is *passive* if there exists some constant $\beta > 0$ such that (18) holds.

$$\langle G(u), u\rangle_N \geq -\beta \tag{18}$$

II. G is *strictly-output passive* if there exists some constants $\beta,\ \epsilon > 0$ such that (19) holds.

$$\langle G(u), u\rangle_N \geq \epsilon \|(G(u))_N\|_2^2 - \beta \tag{19}$$

III. G is *strictly-input passive* if there exists some constants $\beta,\ \delta > 0$ such that (20) holds.

$$\langle G(u), u\rangle_N \geq \delta \|u_N\|_2^2 - \beta \tag{20}$$

Remark 2: Denote a discrete time state space system Σ in which the state $x \in \mathbb{X} \subseteq \mathbb{R}^n$ and output $y \subseteq \mathbb{Y} \in \mathbb{R}^m$ evolves according to

$$\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= h(x(k), u(k)) \end{aligned} \tag{21}$$

in which $u \in \mathbb{U} \subseteq \mathbb{R}^m$ and $u \in l_2^m(U)$ is an input to the system. If G can be described by Σ, and satisfies any of the definitions listed in Definition 2, in which $0 \leq \beta < \infty$ then Σ is a corresponding *passive dissipative* system in which the available storage $S_a(x(0)) \leq \beta$ exists (note that β implicitly depends on $y(0)$). $S_a(x(0))$ represents the maximum amount of energy which can be extracted from Σ for any $x(0) \in \mathbb{X}$. Depending on the type of *passivity* which is satisfied $\epsilon \geq 0$ and $\delta \geq 0$ will satisfy (22).

$$0 \leq S_a(x(0)) \triangleq \sup_{\substack{x(0) \\ u(\cdot),\ N \geq 0}} -(\langle y, u\rangle_N - \delta \|u_N\|_2^2 - \epsilon \|y_N\|_2^2) \leq \beta < \infty \tag{22}$$

Note that when $N = 0$, the truncated inner product and norms are defined to be equal to zero in order to satisfy $S_a(x(0)) \geq 0$. Also note that it is the existence of the available storage $0 \leq S_a(x(0)) < \infty,\ \forall x(0) \in \mathbb{X}$ and $S_a(0) = 0$ which is a necessary and sufficient condition for Σ to be *dissipative* [25, Theorem 1] [32, Theorem 13.17].

Remark 3: For a *dissipative passive* system a storage function $S(x) : x \in \mathbb{X} \mapsto \mathbb{R}_+$ exists in which $S(x) \geq 0,\ \forall x \in \mathbb{X}$ and $S(0) = 0$ In particular, the following inequality is satisfied:

$$S(x(0)) + \langle y, u\rangle_N - \delta \|u_N\|_2^2 - \epsilon \|y_N\|_2^2 \geq S(x(N)) \tag{23}$$

in which $\delta > 0$ for a *strictly-input passive* system, $\epsilon > 0$ for a *strictly-output passive* system. Equivalently

$$y^\mathsf{T}(k)u(k) - \delta u^\mathsf{T}(k)u(k) - \epsilon y^\mathsf{T}(k)y(k) \geq S(x(k+1)) - S(x(k)). \tag{24}$$

We note that any proofs associated with β for *passive* systems (Definition 2) apply equally to *dissipative passive* systems in which we can substitute $-\beta = S(x(N)) - S(x(0))$. (see [33] [17, Section 3.1], [25] for a further discussion related to continuous time *dissipative* systems and [34, Appendix C] and [32, Section 13.9] for discrete time *dissipative* systems). The *dissipative* dynamical systems storage function $S(x)$ is a "Lyapunov-like" function which can be shown to be a Lyapunov function in which $S(x) > 0,\ x \neq 0$ if Σ is *completely reachable* and *zero-state observable*.

Definition 3: [32, Definition 13.12] A dynamical system Σ is completely reachable if for all $x(k_0) \in \mathbb{X} \subseteq \mathbb{R}^n$, there exists a $k_1 < k_0$ and a square summable input $u(k)$ defined on $[k_1, k_0]$ such that the state, $x(k),\ k \geq k_1$, can be driven from $x(k_1) = 0$ to $x(k_0)$.

Definition 4: [32, Definition 13.15] A dynamical system Σ is zero-state observable if $u(k) = 0$ and $y(k) = 0$ implies $x(k) = 0$.

Theorem 1: If a *passive dissipative* system Σ described by (21) which is zero-state observable, completely reachable and there exists a function $\kappa : l_2^m(Y) \to l_2^m(U)$ such that

$$\begin{aligned} &\kappa(0) = 0 \text{ and} \\ &y^\mathsf{T}(k)\kappa(y(k)) - \epsilon y^\mathsf{T}(k)y(k) - \delta \kappa(y(k))^\mathsf{T}\kappa(y(k)) < 0, \\ &\qquad y(k) \neq 0 \end{aligned} \tag{25}$$

in which $\epsilon > 0$ for the case when the system is *strictly-output passive* ($\epsilon = 0$ otherwise) and $\delta > 0$ for the case when the system is *strictly-input passive* ($\delta = 0$ otherwise) then Σ has a zero solution $x(k) = 0$ which is Lyapunov stable. If Σ is also *strictly-output passive* then Σ is asymptotically stable. Finally, if $S(x)$ is also proper ($S(x) \to \infty,$ as $\|x\|_2 \to \infty$) then Σ is globally asymptotically stable.

Proof: [32, Theorem 13.18] shows that $S(x) > 0,\ \forall x \in \mathbb{X}$ if a *passive dissipative* system is zero-state observable, completely reachable and $\kappa(y)$ satisfies (25). Setting $u[k] = 0$ in (24) results in

$$S(x(k+1)) - S(x(k)) = 0 \leq 0 \tag{26}$$

which satisfies the conditions for $S(x(k))$ to be a Lyapunov function. When Σ is zero state observable and *strictly-output passive* then setting $u[k] = 0$ and noting that $y[k] \neq 0$ when $x[k] \neq 0$ implies that (24) results in

$$S(x(k+1)) - S(x(k)) \leq -\epsilon y^{\mathsf{T}}(k)y(k) < 0,\ \forall x \neq 0 \tag{27}$$

which satisfies the conditions for Σ to be asymptotically stable. Finally, if $S(x)$ is also proper then Σ is globally asymptotically stable (see [32, Theorem 13.2] for the corresponding Lyapunov stability conditions just discussed). ■

Remark 4: For a *passive* system ($\epsilon = \delta = 0$) $\kappa(y(k)) = -y(k)$ satisfies (25). For a *strictly-output passive* system $\delta = 0,\ \epsilon > 0,\ \kappa(y(k)) = 0$ satisfies (25). Finally, for a *strictly-input passive* system $\delta > 0,\ \epsilon = 0,\ \kappa(y(k)) = \gamma y(k),\ \gamma > \frac{1}{\delta} > 0$ satisfies (25). Therefore (25) is a redundant condition. Analogous statements can be made for the continuous time case, if in addition $S(x)$ is a C^1 (continuously differentiable) function (see [32, Theorem 5.3] for similar observability and reachability conditions to imply that $S(x) > 0$).

Theorem 2: If $G : l^m_{2_e}(U) \to l^m_{2_e}(U)$ is *strictly-output passive* then G is *passive* and has *finite* l^m_2*-gain*.

Proof: We denote $y = G(u)$, and rewrite (19)

$$\begin{aligned}\epsilon\|y_N\|_2^2 &\leq \langle y, u\rangle_N + \beta \\ &\leq \langle y, u\rangle_N + \beta + \frac{1}{2}\|\frac{1}{\sqrt{\epsilon}}u_N - \sqrt{\epsilon}y_N\|_2^2 \\ &\leq \beta + \frac{1}{2\epsilon}\|u_N\|_2^2 + \frac{\epsilon}{2}\|y_N\|_2^2\end{aligned} \tag{28}$$

thus moving all terms of y to the left, (28), has the final form of (15) with l^m_2*-gain* $\hat{\gamma} = \frac{1}{\epsilon}$ and $\hat{b} = \frac{2\beta}{\epsilon}$. ■

Remark 5: See [17, Theorem 2.2.14] for the continuous time case, and [34, Appendix C] for the discrete time *dissipative strictly-output passive* case. Note that this theorem does apply for *dissipative strictly-output passive* systems (see Remark 2) and the proof can be applied analogously to continuous time systems as well.

The requirement for *strictly-output passive* is a relatively easy requirement to obtain for a *passive* plant with map G and input u and output y. This is accomplished by closing the loop relative to a reference vector r with a real positive definite feedback gain matrix $K > 0$ such that $u = r - Ky$.

Theorem 3: Given a *passive* system with input u, output $G(u) = y$, a real positive definite matrix $K > 0$, and reference vector r. If the input $u = r - Ky$, then the mapping $G_{cl} : r \to y$ is *strictly-output passive* which implies l^m_2*-stability*.

Remark 6: Theorem 2 implies that the output energy of a *strictly-output passive* system will be bounded by a constant times the supplied input energy. A *LTI strictly-output passive* system can be considered with $\beta = 0$, therefore the output y will stay at $y = 0$ if $u = 0$ for all i. Practically speaking the output $y(i)$ should return to 0 when $u(i)$ returns to 0 with a *strictly-output passive* system as $N \to \infty$. As we shall see later, we will be able to maintain *strictly-output passivity* when interconnecting two *strictly-output passive* systems with *wave variables* which can be subject to arbitrary data delays and dropouts.

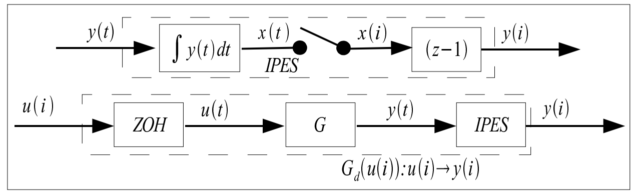

Fig. 4. A representation of the *IPESH* for *SISO LTI* systems [35].

Remark 7: Theorem 3 can be stated in a more general form in which K is replaced by a *strictly-input passive* system, however, we want this transformation of a *passive* system to a *strictly-output passive* system to be as simple as possible (see [17, Theorem 2.2.11(b)] for the continuous time case). Furthermore a *LTI passive* systems which has zeros on the unit circle can still be made *strictly-output passive* by closing the loop, however it can not be made *strictly-input passive* without having to and an additional feed-forward term.

B. INNER-PRODUCT EQUIVALENT SAMPLE AND HOLD

In this section we show how a (non)linear (*strictly input*) or (*strictly output*) *passive* plant can be transformed to a discrete (*strictly input*) or (*strictly output*) *passive* plant using a particular digital sampling and hold scheme (Theorem 4). This novel zero-order digital to analog hold, and sampling scheme introduced in [11] results in a combined system such that the energy exchange between the analog and digital port is equal. This equality allows one to interconnect an analog to a digital Port-Controlled Hamiltonian (*PCH*) system which yields an overall passive system. In [12], a correction was made to the original scheme proposed in [11]. In order to prove Theorem 4, we will restate the sample and hold algorithm with a slightly modified nomenclature.

Definition 5: [11], [12] Let a continuous one-port plant be denoted by the input-output mapping $G_{ct} : L^m_{2_e}(U) \to L^m_{2_e}(U)$. Denote continuous time as t, the discrete time index as i, the sample and hold time as T_s, the continuous input as $u(t) \in L^m_{2_e}(U)$, the continuous output as $y(t) \in L^m_{2_e}(U)$, the transformed discrete input as $u(i) \in l^m_{2_e}(U)$, and the transformed discrete output as $y(i) \in l^m_{2_e}(U)$. The *inner-product equivalent sample and hold* (*IPESH*) is implemented as follows:

I. $x(t) = \int_0^t y(\tau)d\tau$
II. $y(i) = x((i+1)T_s) - x(iT_s)$
III. $u(t) = u(i), \forall t \in [iT_s, (i+1)T_s)$

As a result

$$\langle y(i), u(i)\rangle_N = \langle y(t), u(t)\rangle_{NT_s}, \forall N \geq 1 \tag{29}$$

holds.

Remark 8: Fig. 4 shows an implementation of the *IPESH* for a single-input and single-output (*SISO*) *LTI* system $G(s)$

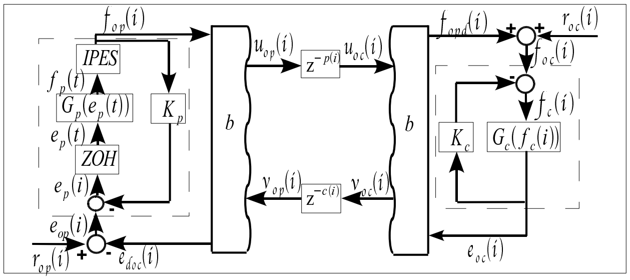

Fig. 5. l_2^m*-stable* digital control network for cooperative scheduler

when the output is scaled by $\frac{1}{T}$ it is referred to as the *IPESH*-Transform [35]. The corresponding *passive* zero-order-hold equivalent is

$$G_d(z) = \frac{(z-1)^2}{z}\mathcal{Z}\left\{\frac{G(s)}{s^2}\right\}. \tag{30}$$

Theorem 4: Using the *IPESH* given in Definition 5, the following relationships can be stated between the continuous one-port plant, G_{ct}, and the discrete transformed one-port plant, $G_d : l_{2_e}^m(U) \to l_{2_e}^m(U)$:

I. If G_{ct} is *passive* then G_d is *passive*.
II. If G_{ct} is *strictly-input passive* then G_d is *strictly-input passive*.
III. If G_{ct} is *strictly-output passive* then G_d is *strictly-output passive*.

Proof: See Appendix B-B. ■

Remark 9: This is a general result, in which Theorem 4-I includes the special case where the input is a force and the output is a velocity [12, Definition 2] and it includes the special case for interconnecting *PCH* systems [11], [36, Theorem 1]. Theorem 4-III corrects [14, Theorem 3-III].

Remark 10: Note that the storage function β for the discrete-time mapping G_d is simply a discrete-time sampled version related to the continuous-time mapping G_{ct}. Therefore, if $\beta \geq 0$ for the continuous time system G_{ct}, then $\beta \geq 0$ for the discrete time system G_d. Therefore, when G_{ct} is a *dissipative passive* system, then as long as there still exists a discrete time state realization Σ as governed by (21) then all the above results apply equally for a discrete time *dissipative passive* system Σ. In particular, for *LTI* systems this always will be the case.

IV. MAIN RESULTS

Fig. 5 depicts our proposed control scheme that guarantees l_2^m stability under variable delays in the feedback and control channels. Depicted is a continuous *passive* plant $G_p(e_p(t)) = f_p(t)$ which is actuated by a zero-order hold and sampled by an *IPES*. Thus G_p is transformed into a discrete passive plant $G_{dp}(e_p(i)) = f_{op}(i)$. Next, a positive definite matrix K_p is used to create a discrete *strictly-output passive* plant $G_{op}(e_{op}(i)) = f_{op}(i)$ outlined by the dashed line. Next G_{op} is interconnected in the following feedback configuration such that

$$\langle f_{op}, e_{doc}\rangle_N = \frac{1}{2}(\|(u_{op})_N\|_2^2 - \|(v_{op})_N\|_2^2) \tag{31}$$

holds due to the wave transform. Moving left to right towards the *strictly-output passive* digital controller $G_{oc}(f_{oc}) = e_{oc}$ we first note that

$$\langle f_{opd}, e_{oc}\rangle_N = \frac{1}{2}(\|(u_{oc})_N\|_2^2 - \|(v_{oc})_N\|_2^2) \tag{32}$$

holds due to the wave transform. The *wave variables* $u_{oc}(i), v_{op}(i)$ are related to the corresponding wave variables $u_{op}(i), v_{oc}(i)$ and by the discrete time varying delays $p(i), c(i)$ such that

$$u_{oc}(i) = u_{op}(i - p(i)) \tag{33}$$
$$v_{op}(i) = v_{oc}(i - c(i)) \tag{34}$$

(33) and (34) hold. Finally the positive definite matrix K_c is used to make the *passive* digital controller $G_c(f_c(i)) = e_{oc}(i)$ *strictly-output passive*. Typically, r_{oc} can be considered the set-point in which $f_{opd}(i) \approx -r_{oc}(i)$ at steady state, while $r_{op}(i)$ can be thought as a discrete disturbance. Which leads us to the following theorem.

Theorem 5: The system depicted in Fig. 5 is l_2^m*-stable* if

$$\langle f_{op}, e_{doc}\rangle_N \geq \langle e_{oc}, f_{opd}\rangle_N \tag{35}$$

holds for all $N \geq 1$.

Theorem 6: The system depicted in Fig. 5 without the *IPESH* in which i and t denote continuous time is L_2^m*-stable* if

$$\langle f_{op}, e_{doc}\rangle_\tau \geq \langle e_{oc}, f_{opd}\rangle_\tau \tag{36}$$

holds for all $\tau \geq 0$.

Proof: The proof is completely analogous to the proof given for Theorem 5 in Appendix B-D, the differences being that the *IPESH* is no longer involved and the discrete time delays are replaced with continuous time delays. ■

In order for (35) to hold, the communication channel/ data-buffer needs to remain *passive*. The following lemmas state under what time delays and data dropouts these conditions hold.

Lemma 2: [36, Proposition 1] If the discrete time varying delays are fixed $p(i) = p, c(i) = c$ and/or data packets are dropped then (35) holds.

[36, Proposition 2] appears to be too broad in stating that the communication channel is *passive* in spite of variable time delays when only the transmission of one data packet per sample period occurs. For instance, a simple counter example is to assume $p(i) = i$, then (99) will not hold if $N\|(u_{op})_1\|_2^2 > (\|(u_{op})_N\|_2^2 + \|(v_{oc})_N\|_2^2)$. Clearly other variations can be given such that $p(i)$ eventually becomes fixed and never changes after sending old *duplicate samples*, and still (35) will not hold. Therefore, we state the following lemma:

Lemma 3: The discrete time varying delays $p(i), c(i)$ can vary arbitrarily as long as (99) holds. Thus, the main assumption (35) will hold if either:

1) Duplicate transmissions are dropped at the receivers. This can be accomplished by transmitting the tuple (i,$u_{op}(i)$), if $i \in \{$ the set of received indexes $\}$ then set $u_{oc}(i) = 0$.
2) we drop received data so that (99) holds. This requires us to track the current energy storage in the communication channel.

Remark 11: Examples of similar energy-storage audits as stated in Lemma 3-2 are given in [37, Section IV] which does not use wave-variables, and in [8] which treats the continuous time case.

A. PASSIVE DISCRETE LTI SYSTEM SYNTHESIS

In [38], using dissipative theory and a longer proof than we will provide, it was shown how to synthesize a discrete passive plant from a linear time invariant (*LTI*) plant. The advantage of the observer described in [38] is that it does not require a measurement of the integrated output of the *passive* plant. However, if one is concerned with controlling an integrated output such as position, one will probably have this measurement available as well as the corresponding *passive* output such as velocity. We will also show how an observer, based on the integrated output measurement can still be used. Such an observer maintains passivity and eliminates the need to directly measure the actual *passive* output such as the velocity. The proof for the observer will follow a similar proof by [39].

A *passive* continuous time *LTI* system [40], $H(s)$, which has a corresponding minimal state space representation given by (37) and denoted by the matrices
$\{\mathbf{A} \in \mathbb{R}^{\mathbf{n}\times\mathbf{n}}, \mathbf{B} \in \mathbb{R}^{\mathbf{n}\times\mathbf{p}}, \mathbf{C} \in \mathbb{R}^{\mathbf{p}\times\mathbf{n}}, \mathbf{D} \in \mathbb{R}^{\mathbf{p}\times\mathbf{p}}\}$

$$\begin{aligned} \dot{x}(t) &= \mathbf{A}x(t) + \mathbf{B}u(t) \\ y(t) &= \mathbf{C}x(t) + \mathbf{D}u(t) \end{aligned} \tag{37}$$

is cascaded in series with a diagonal matrix of integrators, $H_I(s)$, described by $\{\mathbf{A_I} = \mathbf{0}, \mathbf{B_I} = \mathbf{I}, \mathbf{C_I} = \mathbf{I}, \mathbf{D_I} = \mathbf{0}\}$. The combined system, $H_o(s) = H(s)H_I(s)$, is described by $\{\mathbf{A_o}, \mathbf{B_o}, \mathbf{C_o}\}$. Where

$$\mathbf{A_o} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{C} & \mathbf{0} \end{bmatrix} \in \mathbb{R}^{(n+p)\times(n+p)} \tag{38}$$

$$\mathbf{B_o} = \begin{bmatrix} \mathbf{B} \\ \mathbf{D} \end{bmatrix} \in \mathbb{R}^{(n+p)\times p} \tag{39}$$

$$\mathbf{C_o} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \end{bmatrix} \in \mathbb{R}^{p\times(n+p)} \tag{40}$$

Applying a zero-order-hold and an ideal sampler, the system is described by [41]

$$\begin{aligned} x(k+1) &= \mathbf{\Phi_o}x(k) + \mathbf{\Gamma_o}u(k) \\ p(k) &= \mathbf{C_o}x(k) \end{aligned} \tag{41}$$

in which

$$\begin{aligned} \mathbf{\Phi_o} &= e^{\mathbf{A_o}T} \\ \mathbf{\Gamma_o} &= \int_0^T e^{\mathbf{A_o}\eta} d\eta \mathbf{B_o} \end{aligned} \tag{42}$$

.

Proposition 1: Applying a zero-order-hold input to *passive* continuous time *LTI* system, $H(s)$, and sampling the output with the inner-product equivalent sampler at a sample rate T results in a discrete *passive LTI* system, $G_p(z)$ with discrete state equations

$$\begin{aligned} x(k+1) &= \mathbf{\Phi_o}x(k) + \mathbf{\Gamma_o}u(k) \\ y(k) &= \mathbf{C_p}x(k) + \mathbf{D_p}u(k) \end{aligned} \tag{43}$$

where $\mathbf{C_p} = \mathbf{C_o}(\mathbf{\Phi_o} - \mathbf{I})$, and $\mathbf{D_p} = \mathbf{C_o}\mathbf{\Gamma_o}$.

Proof: From Definition 5 it is a simple exercise to compute the *passive* output $y(k) = p(k+1) - p(k)$ as follows

$$\begin{aligned} x(k+1) &= \mathbf{\Phi_o}x(k) + \mathbf{\Gamma_o}u(k) \\ y(k) &= \mathbf{C_o}(\mathbf{\Phi_o} - \mathbf{I})x(k) + \mathbf{C_o}\mathbf{\Gamma_o}u(k) \end{aligned} \tag{44}$$

hence $\mathbf{C_p} = \mathbf{C_o}(\mathbf{\Phi_o} - \mathbf{I})$, and $\mathbf{D_p} = \mathbf{C_o}\mathbf{\Gamma_o}$. ■

Using Proposition 1 and Theorem 3 the following corollary can be shown:

Corollary 1: Given a positive definite matrix $\mathbf{K_x} > 0$ and discrete *passive* system described by (43), the system

$$\begin{aligned} x(k+1) &= \mathbf{\Phi_{sp}}x(k) + \mathbf{\Gamma_{sp}}u(k) \\ y(k) &= \mathbf{C_{sp}}x(k) + \mathbf{D_{sp}}u(k) \end{aligned} \tag{45}$$

is *strictly-output passive*. Here

$$\begin{aligned} \mathbf{\Phi_{sp}} &= \mathbf{\Phi_o} - \mathbf{\Gamma_o}\mathbf{K_x}(\mathbf{I} + \mathbf{D_p}\mathbf{K_x})^{-1}\mathbf{C_p} \\ \mathbf{\Gamma_{sp}} &= \mathbf{\Gamma_o}(\mathbf{I} - \mathbf{K_x}(\mathbf{I} + \mathbf{D_p}\mathbf{K_x})^{-1}\mathbf{D_p}) \\ \mathbf{C_{sp}} &= (\mathbf{I} + \mathbf{D_p}\mathbf{K_x})^{-1}\mathbf{C_p} \\ \mathbf{D_{sp}} &= (\mathbf{I} + \mathbf{D_p}\mathbf{K_x})^{-1}\mathbf{D_p} \end{aligned} \tag{46}$$

With our discrete *strictly-output passive* system we can scale the gain so that its steady state gain matches the *strictly-output passive* continuous systems steady state gain.

Corollary 2: Given a diagonal matrix $\mathbf{K_s} > 0$ and discrete *strictly-output passive* system described by (45), the following system is *strictly-output passive*

$$\begin{aligned} x(k+1) &= \mathbf{\Phi_{sp}}x(k) + \mathbf{\Gamma_{sp}}u(k) \\ y(k) &= \mathbf{K_s}\mathbf{C_{sp}}x(k) + \mathbf{K_s}\mathbf{D_{sp}}u(k) \end{aligned} \tag{47}$$

in which each diagonal element

$$k_s(i) = \begin{cases} y_c(i)/y_d(i) \forall i \in \{1, \ldots, p\} & \text{if } y_c(i) \text{ and } y_d(i) \neq 0; \\ \frac{1}{T} & \text{otherwise} \end{cases} \tag{48}$$

The vectors y_c/y_d correspond to the respective steady state continuous/discrete output of a *strictly-output passive* plant given a unit step input. These vectors can be computed as follows:

$$\begin{aligned} & y_c = (-\mathbf{C_c}\mathbf{A_c}^{-1}\mathbf{B_c} + \mathbf{D_c})\mathbf{1} \\ & y_d = H_{sp}(z=1)\mathbf{1},\ H_{sp}(z) = \mathbf{C_{sp}}(z\mathbf{I} - \mathbf{\Phi_{sp}})^{-1}\mathbf{\Gamma_{sp}} + \mathbf{D_{sp}} \end{aligned} \tag{49}$$

where

$$\begin{aligned} \mathbf{G_x} &= \mathbf{I} + \mathbf{D}\mathbf{K_x} \\ \mathbf{C_c} &= \mathbf{G_x}^{-1}\mathbf{C} \\ \mathbf{D_c} &= \mathbf{G_x}^{-1}\mathbf{D} \\ \mathbf{A_c} &= \mathbf{A} - \mathbf{B}\mathbf{K_x}\mathbf{C_c} \\ \mathbf{B_c} &= \mathbf{B}(\mathbf{I} - \mathbf{K_x}\mathbf{D_c}) \end{aligned} \tag{50}$$

Next, the following corollary provides a method to compute $u_{op}(k),\ f_{op}(k)$ given $r_{op},\ v_{op},\ b$. We can also synthesize the digital controller from a continuous model using the *IPES* with *ZOH* as well, so an additional corollary will show how to compute $v_{oc}(k), e_{oc}(k)$ given $u_{oc}(k), r_{oc}(k)$.

Corollary 3: The following state equation describes the relationship between the inputs r_{op}, v_{op} and scattering gain b to the outputs $u_{op}(k), f_{op}(k)$.

$$\begin{aligned} x(k+1) &= \mathbf{\Phi_{ef}} x(k) + \mathbf{\Gamma_{ef}}(\sqrt{2b} v_{op}(k) + r_{op}(k)) \\ f_{op}(k) &= \mathbf{C_{ef}} x(k) + \mathbf{D_{ef}}(\sqrt{2b} v_{op}(k) + r_{op}(k)) \\ u_{op}(k) &= \sqrt{2b} f_{op}(k) - v_{op}(k) \end{aligned} \tag{51}$$

Here

$$\begin{aligned} \mathbf{G} &= \mathbf{I} + b\mathbf{K_s}\mathbf{D_{sp}} \\ \mathbf{C_{ef}} &= \mathbf{G}^{-1}\mathbf{K_s}\mathbf{C_{sp}} \\ \mathbf{D_{ef}} &= \mathbf{G}^{-1}\mathbf{K_s}\mathbf{D_{sp}} \\ \mathbf{\Phi_{ef}} &= \mathbf{\Phi_{sp}} - b\mathbf{\Gamma_{sp}}\mathbf{C_{ef}} \\ \mathbf{\Gamma_{ef}} &= \mathbf{\Gamma_{sp}}(\mathbf{I} - b\mathbf{D_{ef}}) \end{aligned} \tag{52}$$

Corollary 4: The following state equation describes the relationship between the inputs r_{oc}, u_{oc} and scattering gain b to the outputs $v_{oc}(k), e_{oc}(k)$.

$$\begin{aligned} x(k+1) &= \mathbf{\Phi_{fe}} x(k) + \mathbf{\Gamma_{fe}}(\sqrt{\frac{2}{b}} u_{oc}(k) + r_{oc}(k)) \\ e_{oc}(k) &= \mathbf{C_{fe}} x(k) + \mathbf{D_{fe}}(\sqrt{\frac{2}{b}} u_{oc}(k) + r_{oc}(k)) \\ v_{oc}(k) &= u_{oc}(k) - \sqrt{\frac{2}{b}} e_{oc}(k) \end{aligned} \tag{53}$$

Where

$$\begin{aligned} \mathbf{G_1} &= \mathbf{I} + \frac{1}{b}\mathbf{K_s}\mathbf{D_{sp}} \\ \mathbf{C_{fe}} &= \mathbf{G_1}^{-1}\mathbf{K_s}\mathbf{C_{sp}} \\ \mathbf{D_{fe}} &= \mathbf{G_1}^{-1}\mathbf{K_s}\mathbf{D_{sp}} \\ \mathbf{\Phi_{fe}} &= \mathbf{\Phi_{sp}} - \frac{1}{b}\mathbf{\Gamma_{sp}}\mathbf{C_{fe}} \\ \mathbf{\Gamma_{fe}} &= \mathbf{\Gamma_{sp}}(\mathbf{I} - \frac{1}{b}\mathbf{D_{fe}}) \end{aligned} \tag{54}$$

In order to prove that a state observer can be used in a *strictly-input passive* manner, we require the following lemma.

Lemma 4: [42] The discrete *LTI* system (43) is *strictly-input passive* and has *finite* l_2^m*-gain* (*strictly-positive real* (*SPR*)) if and only if there exists a symmetric positive definite matrix $\mathbf{P}$ that satisfies the following *LMI*:

$$\begin{bmatrix} \mathbf{\Phi_o}^\mathsf{T}\mathbf{P}\mathbf{\Phi_o} - \mathbf{P} & (\mathbf{\Gamma_o}^\mathsf{T}\mathbf{P}\mathbf{\Phi_o} - \mathbf{K_s}\mathbf{C_p})^\mathsf{T} \\ \mathbf{\Gamma_o}^\mathsf{T}\mathbf{P}\mathbf{\Phi_o} - \mathbf{K_s}\mathbf{C_p} & -(\mathbf{K_s}\mathbf{D_p} + \mathbf{D_p}^\mathsf{T}\mathbf{K_s}^\mathsf{T} - \mathbf{\Gamma_o}^\mathsf{T}\mathbf{P}\mathbf{\Gamma_o}) \end{bmatrix} < 0 \tag{55}$$

Remark 12: Therefore by Theorem 4-II any continuous *strictly-input passive* with *finite* l_2^m*-gain LTI* (strongly positive real [32, Definition 5.18]) system which is sampled and actuated by an *IPESH* will satisfy (55). Note, that [42] has omitted the key assumption that the system must also have *finite* l_2^m*-gain*. The combined conditions of *strictly-input passive* and *finite* l_2^m*-gain* conditions require the discrete *LTI* systems corresponding z-transform to be of relative degree 0 and have no zeros on the unit circle.

Remark 13: We also added K_s in order to show that any positive diagonal matrix can be used to scale the output $y(k)$ as is done with our observer described by (56).

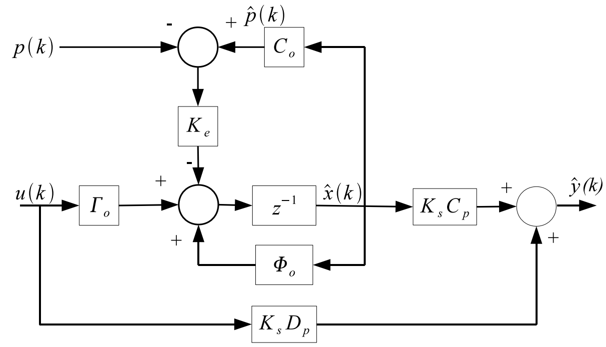

Fig. 6. Passive Observer Structure.

B. Passive Observers

We now propose the following state observer as depicted in Fig. 6 which is based on the sampled integrated output of the *strictly-input passive* plant with *finite* l_2^m*-gain* and the corresponding output estimate $\hat{y}(k)$:

$$\begin{aligned} \hat{x}(k+1) &= \mathbf{\Phi_o}\hat{x}(k) + \mathbf{\Gamma_o}u(k) - \mathbf{K_e}(\hat{p}(k) - p(k)) \\ \hat{p}(k) &= \mathbf{C_o}\hat{x}(k) \\ \hat{y}(k) &= \mathbf{K_s}\mathbf{C_p}\hat{x}(k) + \mathbf{K_s}\mathbf{D_p}u(k) \end{aligned} \tag{56}$$

This observer is similar to the observer proposed in [39] except that it is based on the sampled integrated output. Defining the error in the state estimate as $e(k) \triangleq \hat{x}(k) - x(k)$ and the augmented observer state vector as $x_{ob}(k) \triangleq [x(k), e(k)]$ the system dynamics are

$$\begin{aligned} x_{ob}(k+1) &= \mathbf{\Phi_{ob}} x_{ob}(k) + \mathbf{\Gamma_{ob}} u(k) \\ \hat{y}(k) &= \mathbf{K_s}\mathbf{C_{ob}} x_{ob}(k) + \mathbf{K_s}\mathbf{D_p} u(k) \end{aligned} \tag{57}$$

where

$$\begin{aligned} \mathbf{\Phi_{ob}} &= \begin{bmatrix} \mathbf{\Phi_o} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi_o} - \mathbf{K_e}\mathbf{C_o} \end{bmatrix} \\ \mathbf{\Gamma_{ob}} &= \begin{bmatrix} \mathbf{\Gamma_o} \\ \mathbf{0} \end{bmatrix} \\ \mathbf{C_{ob}} &= \begin{bmatrix} \mathbf{C_p} & \mathbf{C_p} \end{bmatrix} \end{aligned} \tag{58}$$

Proposition 2: If the sampled *LTI* system is strongly positive real and K_e is chosen such that the eigenvalues of $\mathbf{\Phi_o} - \mathbf{K_e}\mathbf{C_o}$ are inside the unit circle the observer described by (56) is both *SPR* and *strictly-output passive*.

Proof: First by choosing the eigenvalues to be inside the unit circle there exist two matrices $\mathbf{Q_2} > 0$ and $\mathbf{P_o} > 0$ such that the following Lyapunov inequality is satisfied

$$-\mathbf{Q_2} = (\mathbf{\Phi_o} - \mathbf{K_e}\mathbf{C_o})^\mathsf{T}\mathbf{P_o}(\mathbf{\Phi_o} - \mathbf{K_e}\mathbf{C_o}) < 0 \tag{59}$$

In order to satisfy the requirements of Lemma 4 we consider the following symmetric positive definite matrix

$$\mathbf{P_{ob}} = \begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mu\mathbf{P_o} \end{bmatrix} > 0 \tag{60}$$

and show that there exists a $\mu > 0$ that satisfies (64). Note the following inequalities hold from our discrete *SPR* system which results from a continuous strongly positive real system.

$$\begin{aligned}-\mathbf{Q_1} &= \mathbf{\Phi_o}^\top\mathbf{P}\mathbf{\Phi_o} - \mathbf{P} < 0\\ -\mathbf{Q_3} &= -(\mathbf{K_s}\mathbf{D_p} + \mathbf{D_p}^\top\mathbf{K_s}^\top - \mathbf{\Gamma_{ob}}^\top\mathbf{P_{ob}}\mathbf{\Gamma_{ob}})\\ &= -(\mathbf{K_s}\mathbf{D_p} + \mathbf{D_p}^\top\mathbf{K_s}^\top - \mathbf{\Gamma_o}^\top\mathbf{P}\mathbf{\Gamma_o}) < 0 \end{aligned} \tag{61}$$

To simplify the expression we define

$$\mathbf{C_1} \triangleq \mathbf{\Gamma_o}^\top\mathbf{P}\mathbf{\Phi_o} - \mathbf{K_s}\mathbf{C_p} \tag{62}$$

Therefore the proposed *SPR* system described by (57) has to satisfy

$$\begin{bmatrix}\mathbf{Q_1} & \mathbf{0} & -\mathbf{C_1}^\top\\ \mathbf{0} & \mu\mathbf{Q_2} & -\mathbf{C_p}^\top\mathbf{K_s}^\top\\ -\mathbf{C_1} & -\mathbf{K_s}\mathbf{C_p} & \mathbf{Q_3}\end{bmatrix} > 0 \tag{63}$$

Using a similarity transformation, (63) is equivalent to

$$\begin{bmatrix}\mathbf{Q_1} & -\mathbf{C_1}^\top & \mathbf{0}\\ -\mathbf{C_1} & \mathbf{Q_3} & -\mathbf{K_s}\mathbf{C_p}\\ \mathbf{0} & -\mathbf{C_p}^\top\mathbf{K_s}^\top & \mu\mathbf{Q_2}\end{bmatrix} > 0 \tag{64}$$

The following upper block matrix, $\mathbf{O}$, satisfies (55) due to Proposition 1, Theorem 4-(II,III), and Lemma 4.

$$\mathbf{O} = \begin{bmatrix}\mathbf{Q_1} & -\mathbf{C_1}^\top\\ -\mathbf{C_1} & \mathbf{Q_3}\end{bmatrix} > 0 \tag{65}$$

Since $\mathbf{O} > 0$, and $\mathbf{Q_2} > 0$, then from using Proposition 8.2.3-v in [43] which is based on the Schur Complement Theory we need to show that

$$\mathbf{O} > 0, \text{ and} \tag{66}$$

$$\begin{aligned}\mu\mathbf{Q_2} - \begin{bmatrix}\mathbf{0} & -\mathbf{C_p}^\top\mathbf{K_s}^\top\end{bmatrix}\mathbf{O}^{-1}\begin{bmatrix}\mathbf{0}\\ -\mathbf{K_s}\mathbf{C_p}\end{bmatrix} &> 0\\ \mu\mathbf{Q_2} - \mathbf{C_p}^\top\mathbf{K_s}^\top\mathbf{O}^{-1}\mathbf{K_s}\mathbf{C_p} &> 0\end{aligned} \tag{67}$$

Thus denoting $\lambda_m(\cdot)/\lambda_M(\cdot)$ as the minimum/maximum eigenvalues for a matrix, noting that the similarity transform of $\mathbf{Q_2} = \mathbf{P_2}\mathbf{\Lambda_2}\mathbf{P_2}^\top$, and defining $\mathbf{M} \triangleq \mathbf{C_p}^\top\mathbf{K_s}^\top\mathbf{O}^{-1}\mathbf{K_s}\mathbf{C_p}$, μ needs to satisfy

$$\mu > \frac{\lambda_M(\mathbf{P_2}^\top(\mathbf{M} + \mathbf{M}^\top)\mathbf{P_2})}{2\lambda_m(\mathbf{Q_2})} \tag{68}$$

Therefore μ exists and satisfies (64). ■

Remark 14: The proof given in [39] which shows *sufficiency* for *passive* systems implicitly assumes that the discrete sampled plant is strongly positive real, which applies only to continuous time systems which are *strictly-input passive*, asymptotically stable, and of relative degree 0. Furthermore, the results from [39] can not be applied to our desired observer design which uses the integrated output of a strongly positive real plant. We have found in practice, however, such a proposed observer system will typically preserve *passivity* for *passive* and *strictly-output passive LTI* plants which can be verified by finding a P which satisfies [34, Lemma C.4.2]. In fact the proposed observer will maintain either *strictly-output passive*, *strictly-input passive*, or *passive* mapping when the corresponding *known* plant is respectively either *strictly-output passive*, *strictly-input passive*, or *passive*.

Theorem 7: Assume an observer described by (56) which has precise knowledge of a *LTI* plant $H(s)$ which has a minimal state space representation described by (37) and denoted as $\{\mathbf{A} \in \mathbb{R}^{\mathbf{n}\times\mathbf{n}}, \mathbf{B} \in \mathbb{R}^{\mathbf{n}\times\mathbf{p}}, \mathbf{C} \in \mathbb{R}^{\mathbf{p}\times\mathbf{n}}, \mathbf{D} \in \mathbb{R}^{\mathbf{p}\times\mathbf{p}}\}$. The discrete counterpart

$$G_p(z) = \mathbf{C_p}(z\mathbf{I} - \mathbf{\Phi_o})^{-1}\mathbf{\Gamma_o} + \mathbf{D_p}, \tag{69}$$

with a corresponding state-space realization given by (43) is:

i) *passive* when $H(s)$ is *passive*,
ii) *strictly-input passive* when $H(s)$ is *strictly-input passive*, and
iii) *strictly-output passive* when $H(s)$ is *strictly-output passive*.

Denote $\hat{Y}(z)$ and $U(z)$ as the corresponding z-transforms of $\hat{y}(k)$ and $u(k)$. The corresponding observer system response

$$H_{ob}(z) = \frac{\hat{Y}(z)}{U(z)} = \mathbf{K_s}\left[\mathbf{C_{ob}}(z\mathbf{I} - \mathbf{\Phi_{ob}})^{-1}\mathbf{\Gamma_{ob}} + \mathbf{D_p}\right] \tag{70}$$

can be shown to have the final form

$$H_{ob}(z) = \mathbf{K_s}G_p(z),\ \mathbf{K_s} > 0. \tag{71}$$

Therefore the observer $H_{ob}(z)$ is:

i) *passive* when $H(s)$ is *passive*,
ii) *strictly-input passive* when $H(s)$ is *strictly-input passive*, and
iii) *strictly-output passive* when $H(s)$ is *strictly-output passive*.

Proof:

$$\begin{aligned}&\mathbf{K_s}^{-1}H_{ob}(z) - \mathbf{D_p} =\\ &\begin{bmatrix}\mathbf{C_p}\mathbf{C_p}\end{bmatrix}\begin{bmatrix}z - \mathbf{\Phi_o} & \mathbf{0}\\ \mathbf{0} & z - (\mathbf{\Phi_o} - \mathbf{K_e}\mathbf{C_o})\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{\Gamma_o}\\ \mathbf{0}\end{bmatrix}\\ &= \begin{bmatrix}\mathbf{C_p}\mathbf{C_p}\end{bmatrix}\begin{bmatrix}(z - \mathbf{\Phi_o})^{-1} & \mathbf{0}\\ \mathbf{0} & (z - (\mathbf{\Phi_o} - \mathbf{K_e}\mathbf{C_o}))^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{\Gamma_o}\\ \mathbf{0}\end{bmatrix}\\ &= \mathbf{C_p}\begin{bmatrix}(z - \mathbf{\Phi_o})^{-1} & (z - (\mathbf{\Phi_o} - \mathbf{K_e}\mathbf{C_o}))^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{\Gamma_o}\\ \mathbf{0}\end{bmatrix}\\ &\mathbf{K_s}^{-1}H_{ob}(z) = G_p(z)\end{aligned}$$

■

When the observer for a continuous *strictly-output passive LTI* system is also *strictly-output passive* we can set the feedback gain $K_p = 0$ in Fig. 5. Note that K_p can convert a continuous *passive* signal into a discrete *strictly-output passive* signal with an observer. Similar to Corollary 3, we present Corollary 5 as it applies to using a *strictly-output passive* observer of a *strictly-output passive* or strongly positive real plant.

Corollary 5: The observer described by (56) for a *LTI* system which is either *strictly-output passive* or strongly positive real is *strictly-output passive* and the following state equations describe the relationship between the inputs r_{op}, v_{op} and scattering gain b to the outputs $\hat{u}_{op}(k), \hat{f}_{op}(k)$.

$$\begin{aligned}\hat{x}(k+1) &= \mathbf{\Phi_{efo}}\hat{x}(k) + \mathbf{\Gamma_{efo}}(\sqrt{2b}v_{op}(k) + r_{op}(k)) + \mathbf{K_e}p(k)\\ \hat{f}_{op}(k) &= \mathbf{C_{efo}}\hat{x}(k) + \mathbf{D_{efo}}(\sqrt{2b}v_{op}(k) + r_{op}(k))\\ \hat{u}_{op}(k) &= \sqrt{2b}\hat{f}_{op}(k) - v_{op}(k)\end{aligned} \tag{72}$$

where

$$\begin{aligned}
\mathbf{G} &= \mathbf{I} + b\mathbf{K_s D_p} \\
\mathbf{C_{efo}} &= \mathbf{G}^{-1}\mathbf{K_s C_p} \\
\mathbf{D_{efo}} &= \mathbf{G}^{-1}\mathbf{K_s D_p} \\
\mathbf{\Phi_{efo}} &= \mathbf{\Phi_o} - \mathbf{K_e C_o} - b\mathbf{\Gamma_o C_{efo}} \\
\mathbf{\Gamma_{efo}} &= \mathbf{\Gamma_o}(\mathbf{I} - b\mathbf{D_{efo}})
\end{aligned} \tag{73}$$

Remark 15: When $b = 0$, the expression in Corollary 5 satisfies that of a standard observer which does not use *wave variables*.

Corollary 6: The observer described by (56) for a *LTI* system which is either *strictly-output passive* or strongly positive real is *strictly-output passive* and the following state equations describe the relationship between the inputs r_{oc}, u_{oc} and scattering gain b to the outputs $\hat{v}_{oc}(k), \hat{e}_{oc}(k)$.

$$\begin{aligned}
\hat{x}(k+1) &= \mathbf{\Phi_{feo}}\hat{x}(k) + \mathbf{\Gamma_{feo}}(\sqrt{\frac{2}{b}}u_{oc}(k) + r_{oc}(k)) + \mathbf{K_e}p(k) \\
\hat{e}_{oc}(k) &= \mathbf{C_{feo}}\hat{x}(k) + \mathbf{D_{feo}}(\sqrt{\frac{2}{b}}u_{oc}(k) + r_{oc}(k)) \\
\hat{v}_{oc}(k) &= u_{oc}(k) - \sqrt{\frac{2}{b}}\hat{e}_{oc}(k)
\end{aligned} \tag{74}$$

where

$$\begin{aligned}
\mathbf{G_1} &= \mathbf{I} + \frac{1}{b}\mathbf{K_s D_p} \\
\mathbf{C_{feo}} &= \mathbf{G_1}^{-1}\mathbf{K_s C_p} \\
\mathbf{D_{feo}} &= \mathbf{G_1}^{-1}\mathbf{K_s D_p} \\
\mathbf{\Phi_{feo}} &= \mathbf{\Phi_o} - \mathbf{K_e C_o} - \frac{1}{b}\mathbf{\Gamma_o C_{feo}} \\
\mathbf{\Gamma_{feo}} &= \mathbf{\Gamma_o}(\mathbf{I} - \frac{1}{b}\mathbf{D_{feo}})
\end{aligned} \tag{75}$$

V. SIMULATION

We shall control a motor with an ideal current source, which will allow us to neglect the effects of the motor inductance and resistance for simplicity. The fact that the current source is non-ideal, leads to a non-passive relationship between the desired motor current and motor velocity [23]. There are ways to address this problem using *passive* control techniques by controlling the motors velocity indirectly with a switched voltage source and a minimum phase current feedback technique [44], and more recently incorporating the motors back voltage measurement which provides an exact tracking error dynamics *passive* output feedback controller [45].

The motor is characterized by its torque constant, $K_m > 0$, back-emf constant K_e, rotor inertia, $J_m > 0$, and damping coefficient $B_m > 0$. The dynamics are described by

$$\dot{\omega} = -\frac{B_m}{J_m}\omega + \frac{K_m}{J_m}i \tag{76}$$

which are in a *strictly-output passive* form. We choose to use the *passive* "proportional-derivative" controller described by (7) and define $\tau = \frac{B}{K}$ in order to factor out K and yield

$$K_{PD}(s) = K\frac{\tau s + 1}{s} \tag{77}$$

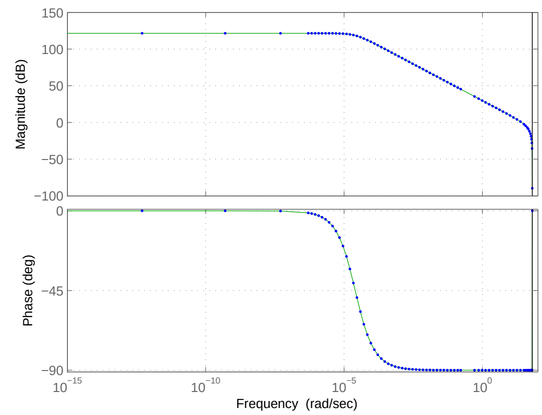

Fig. 7. Bode plot depicting crossover frequency for baseline plant with observer and controller.

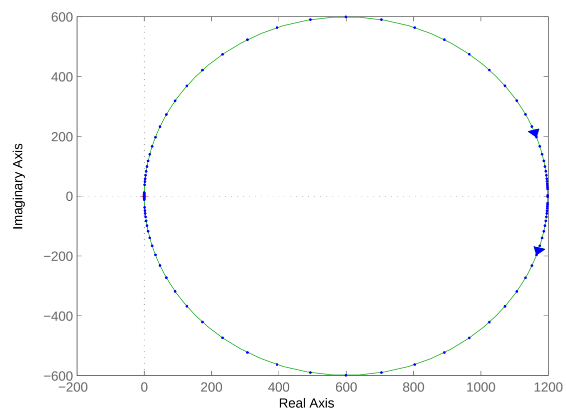

Fig. 8. Nyquist plot for the continuous plant (solid line) and the synthesized discrete counterpart (solid dots) with observer.

Using loop-shaping techniques we choose $\tau = \frac{J_m}{B_m}$ and choose $K = \frac{J_m\pi}{10K_mT}$. This will provide a reasonable crossover frequency at roughly a tenth the Nyquist frequency and maintain a 90 degree phase margin. We choose to use the same motor parameter values given in [45] in which $K_m = 49.13mV$rad sec, $J_m = 7.95 \times 10^{-3}kgm^2$, and $B_m = 41\mu Nm$sec. With $T = .05$ seconds, we use Corollary 4 to synthesize a *strictly-output passive* controller from our continuous model (77), and Corollary 5 to implement the *strictly-output passive* observer of our *strictly-output passive* plant. We also use Corollary 2 in order to compute the appropriate gains for both the controller $K_{s_c} = 1$ and the *strictly-output passive* plant $K_{s_p} = 20$. Note that by arbitrarily choosing $K_{s_c} = \frac{1}{T} = 20$ would have led to a incorrectly scaled system in which the crossover frequency would essentially equal the Nyquist frequency (only because a zero exists at -1 in the complex z-plane). Fig. 7, Fig. 8, and Fig. 9 indicates that our baseline system performs as expected.

We chose $K_e = [16.193271, 1.799768]^{\mathsf{T}}$ for our observer in which the poles are equal to a tenth of the poles of the discrete *passive* plant synthesized by Proposition 1, this by definition forces all the poles inside the unit circle. Since the plant is

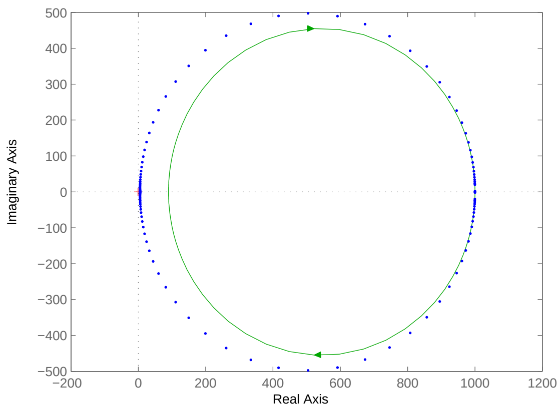

Fig. 9. Nyquist plot for the continuous controller (solid line) and the synthesized discrete counterpart (solid dots).

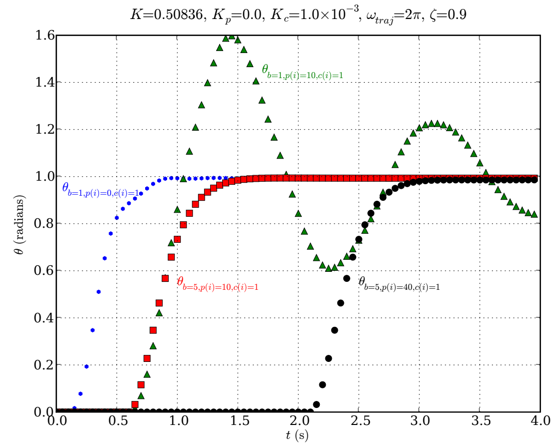

Fig. 11. Step response for motor with *strictly-output passive* digital controller and *strictly-output passive* observer as depicted in Fig. 5 with delays.

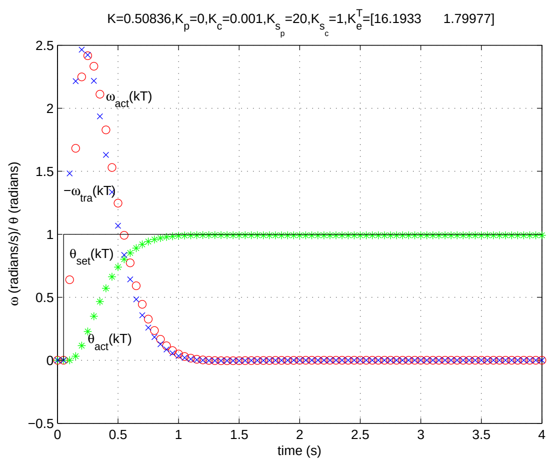

Fig. 10. Baseline step response for motor with *strictly-output passive* digital controller and *strictly-output passive* observer.

strictly-output passive we chose $K_p = 0$. For the controller we chose $K_c = 0.001$ in order to make it *strictly-output passive*. Fig. 10 shows the step response to a desired position set-point $\theta_d(k)$ which generates an approximate velocity reference for $r_{oc}(z) = -H_t(z)\theta_d(z)$. $H_t(z)$ is a zero-order hold equivalent of $H_t(s)$, in which $\omega_{traj} = 2\pi$ and $\zeta = .9$.

$$H_t(s) = \frac{\omega_{traj}^2 s}{s^2 + 2\zeta\omega_{traj} + \omega_{traj}^2} \tag{78}$$

Note, that it is important to use a second order filter in order to achieve near perfect tracking, a first order filter resulted in significant steady state position errors for relatively slow trajectories. Finally in Fig. 11 we see that the proposed control network maintains similar performance as the baseline system. Note that by increasing $b = 5$ significantly reduced the overshoot caused by a half second delay (triangles $b = 1$/squares $b = 5$). Also note that even a two second delay (large circles $b = 5$) results in only a delayed response nearly identical to the baseline system.

VI. CONCLUSIONS

We have presented a theory to design digital control networks which maintain $l_2^m - stability$ in spite of time varying delays caused by cooperative schedulers. We also provided the necessary conditions for Lyapunov stability and asymptotic stability as we connected the relationships between the more general input-output definitions for *passivity* and the more specific definitions for *passive dissipative* systems (see Remark 2). It is important to note that the delays can be either random or deterministic and require no tight bound on delay when discussing $l_2^m - stability$ (see Theorem 5, Theorem 6, Lemma 2, and Lemma 3). By using *wave variables*, and *passive* control theory we can effectively *separate* the controller design from the communication design. The control engineer can effectively shape the system response using a low sampling data rate confident that stability will be maintained while the communication engineer can focus on providing a suitable channel capacity to maintain a reasonable *average* delay. We presented a fairly complete and needed l_2^m stability analysis (which is lacking from much of the journal literature, however we did find a nice discussion in [34, Appendix C]) in particular Theorem 2 shows that *strictly-output passive* is sufficient for stability while Theorem 3 (for the discrete-time case) appeared to be lacking from the open literature. The remaining new results (not available in the open literature) which led to a l_2^m-*stable* controller design are as follows:

1) Theorem 4-I is an improvement which captures all *passive* systems (not just *PCH*) systems.
2) Theorem 4-II, and Theorem 4-III are completely original.
3) Theorem 5 is a new and general theorem to interconnect continuous nonlinear *passive* plants which should lead to more elaborate networks interconnected in the discrete time domain. Theorem 6 is also new. Neither Theorem 5 nor Theorem 6 require knowledge of the energy storage function in order to show l_2^m/L_2^m-*stability* of the network.
4) Proposition 1 showed how to synthesize a discrete *passive LTI* system from a continuous one.
5) Corollary 1 and Corollary 2 showed how to respectively

make the discrete *passive* plant *strictly-output passive* and scale the output so that it will match the steady state output for its continuous counterpart.

6) Corollary 3 and Corollary 4 showed how to implement the *strictly-output passive* network depicted in Fig. 5.
7) Proposition 2 showed how to implement a discrete *strictly-output passive LTI* observer for a strongly positive real continuous *LTI* system (which is fairly restrictive in its applicability).
8) Theorem 7, however shows how to implement a discrete *passive* observer which preserves the specific *passivity* properties of the *passive* plant it is tracking. Thus showing that an observer for either a strongly positive real plant or *strictly-output passive* plant will be *strictly-output passive*. [34, Lemma C.4.2] provides a necessary and sufficient test to determine when the observer described by (56) will maintain *passivity* when an imperfect model of the plant is present. It is of interest to determine what type of plant uncertainties can be tolerated when implementing such an observer.
9) Corollary 5 and Corollary 6 showed how to implement a *strictly-output passive* observer when attached to a scattering junction.

Note that Theorem 3 now allows us to directly design *low-sensitivity strictly-output passive* controllers using the *wave-digital filters* described in [13]. Recently we have extended this networking theory as it applies to multiple plants controlled by either a single or possibly multiple controllers. This is achieved using a "power junction" which combines multiple plant and controller inputs in a *passive* manner [46, Section 2.5]. Furthermore, memoryless input nonlinearities such as actuator saturation and those associated with Hammerstein systems can be effectively dealt with under this framework [46, Section 3.2]. This is important since much work focused on showing how to achieve stochastic stability of an unstable plant may be *impossible* to achieve when actuator saturation is present [46, Section 3.1]. Therefore, these fundamental results provide a solid basis for future controls research in which distributed wireless control systems can be designed.

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APPENDIX A
OBSERVER SIMULATION EQUATIONS

In order to simulate an observer for a continuous *LTI* plant in which the actual state space matrices for the actual *passive* plant are denoted $\{\mathbf{A_a} \in \mathbb{R}^{\mathbf{n}\times\mathbf{n}}, \mathbf{B_a} \in \mathbb{R}^{\mathbf{n}\times\mathbf{p}}, \mathbf{C_a} \in \mathbb{R}^{\mathbf{p}\times\mathbf{n}}, \mathbf{D_a} \in \mathbb{R}^{\mathbf{p}\times\mathbf{p}}\}$. The actual discrete equivalent matrices for a *passive* system are computed appropriately as described by (38), (39), (40), (41), and (42), and denoted as $\{\mathbf{\Phi_{oa}}, \mathbf{\Gamma_{oa}}, \mathbf{C_{oa}}\}$. If the observer is implemented on the plant side for a *LTI* strongly positive real or *strictly-output passive* plant as depicted in Fig. 5 and described by Corollary 5, then the system can be described by

$$\begin{bmatrix}\hat{x}(k+1)\\ x(k+1)\end{bmatrix} = \begin{bmatrix}\mathbf{\Phi_{efo}} & \mathbf{K_e C_{oa}}\\ -b\mathbf{\Gamma_{oa}C_{efo}} & \mathbf{\Phi_{oa}}\end{bmatrix}\begin{bmatrix}\hat{x}(k)\\ x(k)\end{bmatrix} + \begin{bmatrix}\mathbf{\Gamma_{efo}}\\ \mathbf{\Gamma_{efoa}}\end{bmatrix}(\sqrt{2b}v_{op}(k) + r_{op}(k))$$

$$\begin{bmatrix}\hat{f}_{op}(k)\\ p(k)\end{bmatrix} = \begin{bmatrix}\mathbf{C_{efo}} & \mathbf{0}\\ \mathbf{0} & \mathbf{C_{oa}}\end{bmatrix}\begin{bmatrix}\hat{x}(k)\\ x(k)\end{bmatrix} + \begin{bmatrix}\mathbf{D_{efo}}\\ \mathbf{0}\end{bmatrix}(\sqrt{2b}v_{op}(k) + r_{op}(k)) \quad (79)$$

in which

$$\mathbf{\Gamma_{efoa}} = \mathbf{\Gamma_{oa}}(\mathbf{I} - b\mathbf{D_{efo}}) \quad (80)$$

. Similarly, if we implement the observer for a continuous plant on the "controller side" (i.e. when the plant is more accurately depicted as having a flow input and corresponding effort output) as depicted in Fig. 5 and described by Corollary 6 then the system can be described by

$$\begin{bmatrix}\hat{x}(k+1)\\ x(k+1)\end{bmatrix} = \begin{bmatrix}\mathbf{\Phi_{feo}} & \mathbf{K_e C_{oa}}\\ -\frac{1}{b}\mathbf{\Gamma_{oa}C_{feo}} & \mathbf{\Phi_{oa}}\end{bmatrix}\begin{bmatrix}\hat{x}(k)\\ x(k)\end{bmatrix} + \begin{bmatrix}\mathbf{\Gamma_{feo}}\\ \mathbf{\Gamma_{feoa}}\end{bmatrix}(\sqrt{\frac{2}{b}}u_{oc}(k) + r_{oc}(k))$$

$$\begin{bmatrix}\hat{e}_{oc}(k)\\ p(k)\end{bmatrix} = \begin{bmatrix}\mathbf{C_{feo}} & \mathbf{0}\\ \mathbf{0} & \mathbf{C_{oa}}\end{bmatrix}\begin{bmatrix}\hat{x}(k)\\ x(k)\end{bmatrix} + \begin{bmatrix}\mathbf{D_{feo}}\\ \mathbf{0}\end{bmatrix}(\sqrt{\frac{2}{b}}u_{oc}(k) + r_{oc}(k)) \quad (81)$$

in which

$$\mathbf{\Gamma_{feoa}} = \mathbf{\Gamma_{oa}}(\mathbf{I} - \frac{1}{b}\mathbf{D_{feo}}) \quad (82)$$

.

APPENDIX B
ADDITIONAL PROOFS

A. Proof for Theorem 3

Proof: First we use the definition of passivity for G and substitute the feedback formula for u.

$$\langle y, u\rangle_N = \langle y, r - Ky\rangle_N \geq -\beta \quad (83)$$

Then we can obtain the following inequality

$$\langle y, r\rangle_N \geq \lambda_m(K)\|y_N\|_2^2 - \beta \quad (84)$$

in which $\lambda_m(K) > 0$ is the minimum eigenvalue for K. Hence, (84) has the form of (19) which shows *strictly-output passive* and implies l_2^m*-stability*. ∎

It is important to note that for very small maximum eigenvalues, the system is essentially the nominal *passive* system we started with. This is important, for we can design more general *passive* digital controllers and modify them with this simple transform to make them *strictly-output passive*.

B. Proof for Theorem 4

Proof:

I. Since the continuous *passive* system G_{ct} satisfies

$$\langle y(t), u(t)\rangle_\tau \geq -\beta, \forall\tau \geq 0 \quad (85)$$

then by substituting (29) into (85) results in

$$\langle y(i), u(i)\rangle_N \geq -\beta, \forall N \geq 1 \quad (86)$$

which satisfies (18).

II. Let $\tau = NT_s$, then since the continuous *strictly-input passive* system G_{ct} satisfies

$$\langle y(t), u(t)\rangle_\tau \geq \delta\|u(t)_\tau\|_2^2 - \beta, \forall\tau \geq 0 \tag{87}$$

and Definition 5-III implies

$$\|u(t)_\tau\|_2^2 = T_s\|u(i)_N\|_2^2 \tag{88}$$

substituting (88) and (29) into (87) results in

$$\langle y(i), u(i)\rangle_N \geq T_s\delta\|u(i)_N\|_2^2 - \beta, \forall N \geq 1 \tag{89}$$

therefore, the transformed discrete system G_d satisfies (20).

III. with $\tau = NT_S$, the continuous *strictly-output passive* system G_{ct} satisfies

$$\langle y(t), u(t)\rangle_\tau \geq \epsilon\|y(t)_\tau\|_2^2 - \beta, \forall\tau \geq 0 \tag{90}$$

from Definition 5-II and the *Schwarz's Inequality* we relate $\|y(i)_N\|_2^2$ to $\|y(t)_\tau\|_2^2$ as follows:

$$\begin{aligned}\|y(i)_N\|_2^2 &= \sum_{j=1}^{n}\left[\sum_{i=0}^{N-1} y_j^2(i)\right] \\ &= \sum_{j=1}^{n}\left[\sum_{i=0}^{N-1}\left(\int_{iT_s}^{(i+1)T_s} y_j(t)dt\right)^2\right] \\ &\leq T_s\sum_{j=1}^{n}\left[\sum_{i=0}^{N-1}\left(\int_{iT_s}^{(i+1)T_s} y_j^2(t)dt\right)\right] \\ &\leq T_s\|y(t)_\tau\|_2^2\end{aligned} \tag{91}$$

rewriting (91) as

$$\|y(t)_\tau\|_2^2 \geq \frac{1}{T_s}\|y(i)_N\|_2^2 \tag{92}$$

and substituting (92) into (90) results in

$$\langle y(i), u(i)\rangle_N \geq \frac{\epsilon}{T_s}\|y(i)_N\|_2^2 - \beta, \forall N \geq 1 \tag{93}$$

therefore, the transformed discrete system G_d satisfies (19).

■

C. Proof for Theorem 5

Proof: First, by theorem 4-I, G_p is transformed to a discrete *passive* plant. Next, by theorem 3 both the discrete plant and controller are transformed into a *strictly-output passive* systems. The *strictly-output passive* plant satisfies

$$\langle f_{op}, e_{op}\rangle_N \geq \epsilon_{op}\|(f_{op})_N\|_2^2 - \beta_{op} \tag{94}$$

while the *strictly-output passive* controller satisfies (95).

$$\langle e_{oc}, f_{oc}\rangle_N \geq \epsilon_{oc}\|(e_{oc})_N\|_2^2 - \beta_{oc} \tag{95}$$

Substituting, $e_{doc} = r_{op} - e_{op}$ and $f_{opd} = f_{oc} - r_{oc}$ into (35) yields

$$\langle f_{op}, r_{op} - e_{op}\rangle_N \geq \langle e_{oc}, f_{oc} - r_{oc}\rangle_N$$

which can be rewritten as

$$\langle f_{op}, r_{op}\rangle_N + \langle e_{oc}, r_{oc}\rangle_N \geq \langle f_{op}, e_{op}\rangle_N + \langle e_{oc}, f_{oc}\rangle_N \tag{96}$$

so that we can then substitute (94) and (95) to yield

$$\langle f_{op}, r_{op}\rangle_N + \langle e_{oc}, r_{oc}\rangle_N \geq \epsilon(\|(f_{op})_N\|_2^2 + \|(e_{oc})_N\|_2^2) - \beta \tag{97}$$

in which $\epsilon = \min(\epsilon_{op}, \epsilon_{oc})$ and $\beta = \beta_{op} + \beta_{oc}$. Thus (97) satisfies (19) in which the input is the row vector of $[r_{op}, r_{oc}]$, and the output is the row vector $[f_{op}, e_{oc}]$ and completes the proof. ■

D. Proof for Lemma 2

Before we begin the proof, we denote the partial sum from M to N of an extended norm as follows

$$\|x_{(M,N)}\|_2^2 \triangleq \langle x, x\rangle_{(M,N)} = \sum_{i=M}^{N-1}\langle x, x\rangle \tag{98}$$

Proof: In order to satisfy (35), (31) minus (32) must be greater than zero, or

$$\begin{aligned}(\|(u_{op})_N\|_2^2 - \|(v_{op})_N\|_2^2) - (\|(u_{oc})_N\|_2^2 - \|(v_{oc})_N\|_2^2) \geq 0 \\ (\|(u_{op})_N\|_2^2 - \|(u_{oc})_N\|_2^2) + (\|(v_{oc})_N\|_2^2 - \|(v_{op})_N\|_2^2) \geq 0 \\ (\|(u_{op})_N\|_2^2 - \|(u_{op}(i-p(i))_N\|_2^2) + \\ (\|(v_{oc})_N\|_2^2 - \|(v_{oc}(i-c(i))_N\|_2^2) \geq 0\end{aligned} \tag{99}$$

holds. Clearly (99) holds when the delays are fixed, as (99) can be written to show

$$(\|(u_{op})_{((N-p),N)}\|_2^2 + \|(v_{oc})_{((N-p),N)}\|_2^2) \geq 0 \tag{100}$$

the inequality always holds for all $0 \leq p, c < N$. Note if p and c equal zero, then inequality in (100) becomes an equality. If all the data packets were dropped then, $\|(u_{oc})_N\|_2^2) = 0$ and $\|(v_{op})_N\|_2^2) = 0$, such that (35) holds and all the energy is dissipated. If only part of the data packets are dropped, the effective inequality described by (99) serves as a lower bound ≥ 0; hence dropped data packets do not violate (35). ■