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ON FINITE AND INFINITE ZEROS IN THE MODEL MATCHING PROBLEM

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Abstract

In the model matching problem, proper plant P and model T are given and a proper M is to be found such that T = PM. M can then be realized via feedback and feedforward compensation. For internal stability T and M must be stable. A proper and stable solution M exists only when the unstable finite and infinite zeros of P also appear in T. This is studied using the interactor and the Hermite forms of P and T, directly using factorizations of the transfer matrices and by utilizing and extending results of the related nominal synthesis problem. How to choose an appropriate T in control design is also discussed using polynomial matrix interpolation.

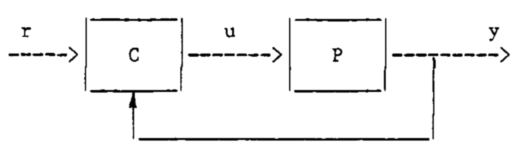
I. Introduction

In the past decade there has been significant interest in the equation

$$T = PM \tag{1}$$

where $T(p \times q)$, $P(p \times m)$, and $M(m \times q)$ are rational matrices, as such equations often appear in systems and control problems. The problem of determining solution M when T and P are given is usually referred to as the model matching problem for reasons discussed below.

Consider a linear time-invariant m-input, p-output plant $y = Pu$ described by its $(p \times m)$ proper transfer matrix $P(s)$. Let the general control law $u = C_y y + C_r r$ be used



where $C = [-C_y \; C_r]$ is the controller. The control action u can be expressed in terms of the reference signal r only, as a mathematically equivalent open loop control law. In particular $u = Mr$ with $M = (I+C_yP)^{-1}C_r$ a proper $(m \times q)$ transfer matrix. In the Model Matching Problem (MMP) a desired response $y_m = T_m r$ is given (T_m the model) which should be matched exactly by $y = PMr$ for all r of interest. And the problem then becomes to determine proper M which satisfies the equation $T_m = PM$ where T_m and P are given proper transfer matrices. M can then be realized via feedback and feedforward compensation [1-4]. Note that for $T_m = I$ we have the Inverse Problem. For internal stability in the control structure it is necnecessary that T_m and M be stable. Therefore in (1), P proper and T proper and stable are given and we are interested in the existence and properties of proper and stable solutions M.

It should be pointed out that in typical control applications T (or T_m) is not exactly fixed since a number of different T do perhaps suffice to satisfy the control specifications. Furthermore, the model of the plant P only approximately describes the behavior of the physical system, in which case attempting to exactly match a given T is unrealistic. The study of the mathematical MMP is however useful in in control, because it provides insight to the properties of the realizable closed loop transfer matrices T and to the required control M.

There is a number of control problems which benefit from the understanding of the MMP. Among these: The Decoupling Problem where T in (1) is not completely specified but it must be a (block) diagonal transfer matrix; this is one case of a class of problems where T is not given but restrictions are imposed on its structure or other properties; e.g., T must have specific poles. In the Model Following Problem it is required that the error $e = y - y_m$ asymptotically approaches zero. This is equivalent to determining proper stable M so that $PM - T_m = \underline{T}$ where the proper, stable T_m is given and $\underline{T}$ is any stable transfer matrix. If $\underline{T}$ is specified to exactly determine the behavior of the error, then this is the MMP with $T = T_m + \underline{T}$ in (1).

The MMP has received a lot of attention in the literature both because of its importance in control and its attractive mathematical formulation. The equation T = PM has been studied over rationals, polynomials, over rings, using a variety of mathematical tools and methods. And the literature is rich with results which offer insight and suggest alternative methods of solution.

Here we concentrate on the finite and infinite zeros T must have for solution M to exist. And we express the existence conditions in terms of those zeros and their associated structure (multivariable case). Two approaches are used: canonical forms in Section II and polynomial matrix factorizations in Section IV. In Section III the Nominal Synthesis Problem which is closely related to MMP is discussed, results are reviewed and extended and then used in the other sections. It is shown that for a proper P and a proper and stable T, proper and stable solution M exists if and only if T has all the unstable finite and infinite zeros of P together with their associated structure. This is made precise in this paper so it can be used in control design. And in Section V these results are utilized to derive simple guidelines to choose appropriate T; for this, polynomial matrix interpolation is used.

This paper formalizes the fact that in any control design, where the control law can be described by the above general controller $u = -C_y y + C_r r$, all the RHP zeros of the plant P must appear as RHP zeros of the compensated system transfer matrix T if internal stability is to be preserved or attained; further-

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more, T should be "more proper" than P. This important fact involving the RHP zeros, is perhaps more obvious in the single-input, single-output case although it does not appear to have been formally stated in the classical control literature. In the multi-input, multi-output case it is not easily detectable due to the character of the multivariable zeros and the widespread use of state-space, with its feedback first design, which tends to further obscure this fact. However, it has been implied in, shown in different degrees of detail, or shown in abstract settings by a number of authors. Among others: In [5] the "fixed poles" of M in (1) are defined which are in effect those poles of T not in P and those zeros of P not in T; for M stable, since T is already stable, those zeros of P not in T must be stable and therefore all the RHP zeros of P should be zeros of T. In [6] this result is shown for a particular control structure and with only passing reference to associated zero structures of P which must also appear in T. In [3] the Λ - structure matrices defined in the $\lambda = 1/(s+a)$ domain, are used in the same way as the canonical forms in Section II. In [7, Th. 3] it is shown in a coordinate-free way to what extent the zeros of P "appear in" the zeros of T; this is done in terms of pole and zero modules.

The emphasis of this paper is not so much on the novelty of the results but on the derivation via alternative methods so to gain insight and perhaps more importantly on useful formulation of the results so that they can be directly used in control design -- And this is shown in the last section.

II. Canonical Forms

Consider the equation

$$T = PM \qquad (1)$$

where P(p×m), T(p×q), rational matrices are given. Solution M(m×q), exists over the field of rationals if and only if

$$\text{rank } [T, P] = \text{rank } P \qquad (\text{Im } T \subset \text{Im } P) \qquad (2)$$

Suppose P and T are proper, i.e. $\lim_{s\to\infty} P(s) < \infty$. The existence of proper solutions M can be studied using the interactors or the Hermite forms of P and T. The interactor ξ_P of a proper P and its extension, the Hermite normal form H_P were introduced in [8, 9] respectively as appropriate canonical forms of P under dynamic compensation.

The interactor ξ_P of a full row rank matrix P (rank P = p) was defined as the unique nonsingular polynomial matrix of certain canonical structure for which $\lim_{s\to\infty} \xi_P P(s) = K$ where rank K = p.

It was shown in [8, Th. 4.5] that if rank T = rank P = p then a proper solution M exists if and only if $\xi_P \xi_T^{-1}$ is a proper matrix.

The interactor was generalized in [10] and defined for P where rank P = r $\leq$ p. In addition, the relation between ξ_P and H_P (for P proper and denominators of H_P at $\pi = s$) was shown to be $\xi_P H_P = \text{diag } [I_r, 0]$. [10, 11].

H_P(p×m), the Hermite normal form of P [9], is a basis of Im P. In general P is an R_S -matrix meaning that the entries in P are in R_S the ring of real transfer functions with denominators in S; S is a multiplicative subset of R[s] consisting of 1 together with all monic polynomials (of positive degree) generated by a set of monic prime factors. $P = H_P \underline{P}$ where $\underline{P}$ and $\underline{P}^{-1}$ are R_S -matrices. It has been shown that:

Theorem 2.1 [9]

Given T and P, R_S -matrices, there exists solution M, an R_S -matrix, which satisfies (1) if and only if

$$H_T = H_P \underline{M}, \qquad \underline{M} \text{ an } R_S \text{ -matrix} \qquad (3)$$

Let R_S denote the proper transfer functions. The above result then deals with the existence of proper solutions M of (1) given proper P and T. (3) in this case can also be written in terms of the interactor ξ_P and ξ_T as discussed above.

The zeros at infinity of proper P(s) are the zeros at w = 0 of P(1/w). They are exactly the zeros at infinity of H_P which implies that (3) can be used to study the zeros at infinity of T when a proper solution M to (1) does exist. Note that in this case H_P does not have any finite zeros.

Let rank T = rank P = p.

Corollary 2.2 Any zero at infinity of P is a zero at infinity of T.

Proof: In view of $H_P = [\xi_P^{-1} \ \ 0]$ (3) can be written as $\xi_P \xi_T^{-1}$ = a proper matrix which is the condition in [8, Th. 4.5]. $H_P(1/w)$ and therefore $\xi_P^{-1}(1/w)$ is a polynomial matrix, in Hermite canonical form, with all of its zeros at w = 0 [10]. Since $\xi_P \xi_T^{-1}$ must be a proper matrix in s, $\xi_P(1/w)\xi_T^{-1}(1/w)$ should not have any pole (at w = 0) which implies that all the zeros at infinity of P must be zeros at infinity of T.

Notice that the structure of the zeros at infinity of P (in the sense of (3)) will also appear in T for proper solutions M to exist.

Given P proper and T proper and stable we are interested in proper and stable solutions M of (1). The existence of solutions M in this case cannot be studied directly via Theorem 2.1 since P is not necessarily stable and the theorem demands that all matrices involved should be R_S -matrices, here taken to be proper and stable matrices. It is however possible to show that a solution M of (1) exists if and only if there exists a solution to an equation which only involves proper and stable matrices.

Theorem 2.3 Given P proper and T proper and stable, there exists proper and stable solution M in (1) if and only if there exists proper and stable solution X' in

$$T = D'X', \qquad (4)$$

where $P = N'D'^{-1}$ a proper, stable right coprime factorization. Furthermore, all solutions M are given by

$$M = D'X'. \qquad (5)$$

Proof Direct in view of Theorem 3.2 in next section.

Theorem 2.1 can now be used to study solutions of (4). In particular solution X' exists if and only if $H_T = H_N \underline{M}$ where $\underline{M}$ a proper and stable matrix. H_T and H_N contain the unstable (RHP) finite zeros and the zeros at infinity of T and P and only those. Consequently this relation can be used to study the conditions on the zeros of T for solutions to exist. The analysis is simplified if rank T = rank P = p which is common in control practice. Also note that H_T (derived with $\pi = s+a$ a > 0 [9]) has as denominators of its entries powers of π and it is of the form

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$H_T = [H_T^*, 0]$ with H_T^* nonsingular.

Corollary 2.4 Any unstable finite zero of P is a zero of T.

Proof: Similar to the proof of Corollary 2.2. Take $w = 1/s+a$ $a > 0$. Actually, using this mapping, not only results on the RHP zeros of T are derived but also on its zeros at infinity.

Notice that the structure of the RHP zeros of P will also appear in T in the sense of (3).

Although ξ_P and H_P provide significant insight they are difficult to compute. This reduces their applicability and in the next sections we consider alternative ways to study (1).

III. Nominal Synthesis Problem

In the Nominal Synthesis Problem it is assumed that P in T = PM is given and it is of interest to determine the solution pairs (T, M). This problem was studied in [7] and in the following the characterization of all solution pairs is presented without proof.

Given P, we are interested in the good solution pairs (T, M) of T = PM. A transfer matrix T is said to be good if its minimal polynomial is good. A polynomial p(s) will be good if $p(s) \in Sg$ where $Sg \subset R[s]$ is closed under multiplication in R[s], it includes the polynomial 1 and it excludes the zero polynomial; i.e. all the roots of p(s) are in symmetric regions, with respect to the real axis, in the complex plane.

Let $P = ND^{-1}$ be a right coprime polynomial factorization of P.

Theorem 3.1 The pair (T, M) is a good solution of (1) if and only if there exists a good transfer matrix X such that

$$\begin{bmatrix} T \\ M \end{bmatrix} = \begin{bmatrix} N \\ D \end{bmatrix} X . \qquad (6)$$

Proof: Direct from [7, Theorem 2].

Note that in [7], the above theorem is shown in a coordinate free framework. $Sg \subset k[s]$ is a principal ideal domain of polynomials in s with coefficients in in an arbitrary field k and T, M etc. are morphisms of k[s] -vector spaces.

The above theorem deals with general good transfer matrices and it will be used in this form to show results in the next section. A case of a good transfer matrix is of course a stable one and many times in the following we will be using stable and unstable instead of good and not good or bad.

Using Theorem 3.1 the following result is obtained which characterizes all proper and stable solutions of T = PM. Note that it has already been used in Section II to prove Theorem 2.3

Consider T = PM where P proper is given

Theorem 3.2 The pair (T, M) is a proper and stable solution to (1) if and only if there exists proper and stable X' such that

$$\begin{bmatrix} T \\ M \end{bmatrix} = \begin{bmatrix} N' \\ D' \end{bmatrix} X' , \qquad (7)$$

where $P = N'D'^{-1}$ is a right coprime proper and stable factorization of P.

Proof: It was shown in [12] that

$$\begin{bmatrix} N' \\ D' \end{bmatrix} = \begin{bmatrix} N \\ D \end{bmatrix} \Pi , \qquad (8)$$

where Π, Π^{-1} stable and $D\Pi$ biproper. If (T, M) is a proper, stable solution pair, it is given by (6) or by (7) where $X' = \Pi^{-1}X$ which is stable; X' is also proper since M = D'X' and D' is biproper. Conversely (7) is a solution for any proper and stable X'.

Theorems 3.1 and 3.2 can be used to study the zeros of T and the results are derived in the next section.

IV. Direct Study of The Zeros of T and Existence Theorem

Given P, let T be a good transfer matrix such that a good solution M to T = PM does exist. It is of interest to study the zeros of T. It is shown that the requirement T, M to be good implies that all the finite zeros of P which are not good must appear as zeros of T (under mild conditions). It is also shown that the requirement for T, M to be proper, assuming that P is proper, implies that all the zeros at at infinity of P will also be zeros at infinity of T.

We shall assume that rank P = p which is a common case in practice. Let $P = ND^{-1}$ a right coprime (rc) polynomial factorization and write

$$N = N_b\underline{N} \qquad (9)$$

where the roots of $|N_b|(\neq 0)$ are exactly those zeros of P which are not good (also called bad). This can always be achieved by using, for example, the Smith form of N. Note that since P has full row rank, N_b is a left divisor of a greatest left divisor (gld) of the rows of N, the determinant of which has roots the finite (transmission) zeros of P.

Let $T = N_T D_T^{-1}$, a rc polynomial factorization.

Lemma 4.1 $N_T = N_b\underline{N}_T$. (10)

Proof: In view of Theorem 3.1, T = NX where X is a good transfer matrix. Then $N_T D_T^{-1} = NX$ or $N_T = N_b(\underline{N}XD_T)$ which shows the result since $\underline{N}XD_T = \underline{N}_T$ a polynomial matrix.

This lemma can be used to study the relation between the bad zeros of P and T and their associated structures. However, it does not necessarily imply that the bad zeros of P in N_b should appear in N_T and therefore in T as the following example shows:

Consider $T = N_T D_T^{-1}$ good (stable) with $N_T = [0 \ \ 1]^T$, $P = ND^{-1}$ with $N = N_b = \text{diag}[s-1, 1]$. $M = D[0 \ \ 1]^T D_T^{-1}$ is a good (stable) solution of (1). However the bad (RHP) zero of P at 1 does not appear in T.

In general, if rank T < rank P = p the lemma does not necessarily imply that z_i, a bad zero of P, will be a zero of T since rank $N_T(z_i)$ < rank N_T (the normal normal rank) will not be necessarily true.

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Let $$\text{rank } T = \text{rank } P = p \, . \tag{11}$$

Theorem 4.2 Assume that T and M, good transfer matrices, satisfy (1). If G_T is a gld of the rows of N_T then

$$G_T = N_b \underline{G_T} \tag{12}$$

Proof: Since rank T = p, N_b will be a left divisor of G_T in view of (10).

Note that the roots of $|G_T| = |N_b||\underline{G_T}|$ are the zeros of T and they include the bad zeros of P in N_b; therefore,

Corollary 4.3 Any finite zeros of P which is not good is also a zero of T.

T may of course have more bad zeros in addition to the zeros of P.

The theorem implies more than the corollary indicates. Not only the bad zeros of P appear in T but also the structure associated with them (in N_b) also appears in T.

Based on Theorem 4.2 results on the zeros at infinity will now be directly derived.

Suppose that P is proper and given a T proper, a proper solution M to T = PM has been found. Let (11) be satisfied.

Corollary 4.4 Any zero at infinity of P is a zero at infinity of T.

Proof: In T(s) = P(s)M(s) let s = 1/w to obtain

$\tilde{T}(w) = \tilde{P}(w)\tilde{M}(w)$. The zeros at infinity of P(s) are exactly the zeros at w = 0 in $\tilde{P}(w)$. Consider w = 0 to be the zeros of $\tilde{P}$ which are not good; then $\tilde{N}_b$ defined in (9) only contains zeros at w = 0. Theorem 4.2 then directly implies the result.

Note that the structure at infinity of P is also repeated in T. Corollary 4.4 says that for existence of a proper solution M, T should be "more proper" than P.

The above results involving the finite and infinite zeros of P could have been derived simultaneously as follows: Use w = 1/s+a a > 0 fixed to obtain $\tilde{P}(w)$ and consider the bad zeros of $\tilde{P}$ to be the mappings of the bad finite zeros of P together with the zeros at w = 0 which correspond to the zeros at in-infinity of P; then the Theorem and Corollary 4.3, in terms of $\tilde{P}$ and $\tilde{T}$, imply both desired results.

Theorem 4.5 Given P proper, T proper and stable with rank P = rank T = p, there exists proper and stable solution M if and only if T has as its zeros all the RHP finite zeros and all the zeros at infinity of P in the sense of (12).

Proof: Necessity has been shown. The sufficiency proof is constructive. First, let p = m and work in the w -domain (w = 1/s+a a > 0). $M = P^{-1}T = D(N_b\underline{N})^{-1}N_T D_T^{-1}$. Since (10) and Theorem 4.2 are satisfied, $M = D\underline{N}^{-1}\underline{N_T}D_T^{-1}$ which does not have any poles at w = 0 nor at any bad locations. Therefore M is proper and stable. If p < m, write $P = [N_p \quad 0]D^{-1}$ and choose $X_p = \underline{N_p}^{-1}\underline{N_T}D_T^{-1}$ in

$$T = [N_p \quad 0]\begin{bmatrix} X_p \\ X_r \end{bmatrix} \text{; then } M = D\begin{bmatrix} X_p \\ X_r \end{bmatrix} \text{ where } X_r$$

is arbitrarily chosen for stability and properness.

V. Selecting T in Control Design

In control, T in T = PM is chosen so that the system response y = Tr to test inputs satisfies the control design specifications. The relation

$$N_T = N_b \underline{N_T} \tag{10}$$

which characterizes the unstable finite zeros and the zeros at infinity T must have for a proper and stable solution M to exist, does not provide a convenient and direct way to choose appropriate T. Note that the transfer function entries in T are individually chosen to satisfy specifications. And although they can be easily chosen to include the zeros at infinity of P, i.e. T is chosen to be "more proper" than P, the unstable zeros of P do not necessarily appear as zeros of individual entries of T. Therefore there is a need for simple and direct conditions which will help the designer to choose T containing the unavoidable unstable zeros together with the appropriate structure.

Let z_i i = 1,...,ℓ be the unstable zeros of P (roots of $|N_b|$). Then rank $N_b(z_i) < p$ which implies that there exists a real 1×p nonzero vector a_i such that $a_i N_b(z_i) = 0$. Post multiplying by $\underline{N}(z_i)$, the solutions do not change and the a_i can be determined from $a_i N(z_i) = 0$.

Assume that z_i i = 1,...,ℓ are distinct or if z_j is a multiple zero the rank reduction in $N(z_i)$ equals the multiplicity of z_j.

Theorem 5.1 The unstable zeros z_i i = 1,...,ℓ of P together with their structure will appear in T if and only if

$$a_i N_T(z_i) = 0 \qquad i = 1,\dots,\ell \tag{13}$$

where a_i are determined from

$$a_i N(z_i) = 0 \qquad i = 1,\dots,\ell \, . \tag{14}$$

Proof: Relations (13) and (14) are the necessary and sufficient conditions for N_b to be a left divisor of N_T. This is shown in [13]. Note that if z_i includes multiple zeros which do not satisfy the above conditions, (13) and (14) should be modified as it is shown in [13]; this was not included here for simplicity.

The theorem can also be written in terms of transfer functions:

(13) is always equivalent to $a_i T(z_i) = 0$ since T is stable;

(14) can be written as $a_i P(z_i) = 0$ when P does not have any poles at z_i.

Under this assumption:

Corollary 5.2 The unstable zeros of P together with their structure will appear in T if and only if

$$a_i T(z_i) = 0 \tag{13a}$$

where a_i are determined from

$$a_i P(z_i) = 0 \, . \tag{14a}$$

As an example, consider a diagonal T; that is the control specifications demand diagonal decoupling of the system. Let

$$P = \frac{1}{s+1}\begin{bmatrix} s-1 & 0 \\ 1 & 1 \end{bmatrix} \text{ with a zero at } s = 1. \text{ Then}$$

$aP(1) = 0 \Rightarrow a = [1 \quad 0]$ and T must satisfy $aT(1) = [1 \quad 0]T(1) = 0$. Since T must be diagonal (square and stable), $t_{11}(1) = 0$; that is the RHP zero of the plant should appear in the (1,1) entry of T only. Certainly T can be chosen to have 1 as a zero in both diagonal entries; note that if $T(z_i) \equiv 0$ (13a) is always satisfied. However, the RHP zeros are undesirable in control and the minimum possible number should be included in T.

Let

$$P = \frac{1}{s+1}\begin{bmatrix} s-1 & 1 \\ 0 & 1 \end{bmatrix} \text{ with also a zero at } s = 1.$$

Then $aP(1) = 0 \Rightarrow a = [1, \ -1]$ and $aT(1) = 0 \Rightarrow t_{11}(1) = 0$ and $t_{22}(1) = 0$. That is, there are cases where the structure of P and the requirement that T be diagonal imply that the RHP zero must appear in both diagonal entries of T.

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