Passivity-Based Distributed Control of Networked Euler-Lagrange Systems With Nonholonomic Constraints

Han Yu and Panos J. Antsaklis
Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556

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Abstract

In this report, we study the distributed control problem of networked Euler-Lagrange(EL) systems with nonholonomic constraints. The reason for singling out this particular topic is that distributed control of networked Euler-Lagrange systems with nonholonomic constraints captures a large class of contemporary engineering problems, such as rendezvous problem and formation problem. We propose a new set-up which allows us to use passivity as the design and analysis tool to solve our proposed problem here. By restricting ourselves to systems with physical constraints and by using passivity, we believe we can contribute to reverse the tide of "find a plant for my controller "which still permeates most of the research work on distributed control.

1 Introduction

In this report, we will study how to use passivity as the design and analysis tool for the distributed control of networked Euler-Lagrange systems. As an illustration of our results here, we study the passivity-based control for the rendezvous problem of car-like robots with nonholonomic constraints. First, we give some basic definitions and concepts of passivity and their relations to Lyapunov stability. Then we re-examine the Euler-Lagrange equations which is the classical mathematical model for mechanical systems. After that, we will propose a new setup which relates passivity based output synchronization with the consensus problem; scattering transformation (or wave variables) will be used to deal with the time-varying delays existing in the communication channel.

Finally, we show that by combing passivity results with some existing results of consensus, we can solve the rendezvous problem of the car-like robots with nonholonomic constraints. Simulation results are also provided.

2   Previous work

To introduce the concept of *passivity*, first we should discuss *dissipativity*. Although the concept of passivity appeared in literature first, the more general and systematic concept -*dissipativity*, will give us a more explicit way for understanding what is passivity and when a system can be defined as passive.

Dissipativity has first been introduced by Willems[8] and is motivated by the concept of passivity from the electrical network theory. Generally speaking, dissipativity theory claims that the changing rate of the energy stored in a dissipative system is always bounded from below by the rate of the energy supplied into the system. One would easily relate dissipativity theory to Lyapunov stability theorem: that is if the energy supply rate is zero, the energy storage function which is at least positive semi-definite, could be chosen as a valid Lyapunov function candidate.

The extended results of dissipativity theory and its applications have been discussed a lot recently. Dissipativity theory is consistent with some well known system and control theories, such as small gain theory, input to state stability and minimum phase property, see [9]. Moreover, the stability analysis for the hybrid systems based on multiple energy storage functions derived from dissipativity has been shown to have a more general form compared with the analysis based on multiple Lyapunov functions, see[10]-[11].

Furthermore, *passivity*, which uses the inner product of the system's input and output as the energy supply rate, is a special case of dissipativity. Passivity has appeared to be a very efficient analysis and design tool for the control of distributed system because the property of passivity is preserved when systems are interconnected in parallel or in negative feedback configuration, see[1]-[2],[12] and [16].

The challenges for the control of distributed systems include how to deal with the increasingly complex nature of the large scale interconnected dynamical systems, how to reduce the communication load and how to handle the unavoidable time-varying delays in the communication channels. Moreover, one interesting topic is how to reach agreement among those interconnected distributed subsystems. This is also well known as *distributed consensus* problem, and this topic has been studied extensively during the last decade[5].

The topic we are interested in here is closely related to those topics discussed above. However, while most of the work on distributed control system has been applying Lyapunov function as the basic tool to analyze the stability of interconnected systems, we are interested whether passivity can be used as an efficient alternative. Since then we can apply passivity as the framework for many applications in distributed control, such as formation control, rendezvous problem and consensus problem. Recent work related with this topic has been shown in [1]-[2] and [16]. While [1]and [16] focus more on using passivity as an alternate of the Lyapunov function technique for the stability analysis, [2] emphasizes more the special properties of passivity-the feedback interconnection of two passive systems is still passive. We would like to show a more comprehensive work in this report, which involves how to use passivity as the basic design tool for the distributed control of interconnected physical systems, in particular how to deal with the physical system's dynamics, and what else needs to be considered when we want to maintain passivity of the interconnected system when there are time-varying communication delays in the communication channels; also what kind of agreement among the interconnected agents we can achieve when taking the communication delays into consideration.

3   Passivity

Definition 1[2]: The dynamic system

$$
\begin{cases} \dot{x} = f(x, u) \\ y = h(x, u) \end{cases}
\tag{1}
$$

where $x \in \mathbb{R}^n$ and $u, y \in \mathbb{R}^p$, is said to be passive if there exists a C^1 storage function $V(x) \geq 0$ such that

$$
\dot{V} = \nabla V(x)^T f(x, u) \leq -S(x) + u^T y
\tag{2}
$$

for some positive semidefinite function $S(x)$. We say it is strictly passive if $S(x) > 0$. Moreover, a static nonlinearity $y = h(u)$ is passive if, for all $u \in \mathbb{R}^m$,

$$
u^T y = u^T h(u) \geq 0
\tag{3}
$$

and strictly passive if the inequality holds with $\forall u \neq 0$.

4 Euler-Lagrange Systems subject to Nonholonomic Constraints

4.1 Introduction

What is an Euler-Lagrange System? An Euler-Lagrange system is a system whose motion is described by the Euler-Lagrange equations, and the Euler-Lagrange equations are very important because they are very powerful modeling tools which describe the behavior of a large class of physical systems.

As we have discussed earlier, a passive system cannot store more energy than that is supplied to it from the outside, with the difference being the dissipated energy. It has already been shown that with appropriate storage functions and supply rates, the Euler-Lagrange systems define passive maps, see[13] and [15] for details.

Also we have mentioned before that passivity is invariant under feedback interconnection, so this motivates us to find a class of passivity-based distributed controllers which will stabilize the closed loop for the networked Euler-Lagrange systems. This is very useful since this enable us to design controllers for a large class of interconnected physical systems which could be modeled by the Euler-Lagrange equations. The more interesting thing is whether the passivity results will still be preserved when an Euler-Lagrange system is subject to certain nonholonomic constraints. This is important since most of the physical systems are subject to some nonholonomic constraints, so we need to take these nonholonomic constraints into considerations when we derive the model for the physical system.

Recent work that is related to passivity-based distributed control of networked Euler-Lagrange systems is shown in [16]. In [16], it has been shown that we can use the sum of the storage functions of interconnected Euler-Lagrange system as the Lyapunov function candidate; then if the underlying communication graph is balanced and strongly connected, the networked Euler-Lagrange systems can achieve output synchronization. There are three important remarks we want to highlight on the work shown in [16]: first, [16] shows that passivity can be used as a very efficient tool to analyze the stability of the networked distributed systems if all the interconnected systems are passive; second, the underlying communication graph plays an important role for the analysis of passivity for the interconnected systems, and it required that the communication graph is balanced and strongly connected so that the interconnection is lossless, which is consistent with Willem's dissipativity theorem shown in [8] ; third, many physical systems can be modeled as Euler-Lagrange systems, we can apply passivity results for the distributed control of a large class of physical systems. However, the analysis in [16] did not include nonholonomic constraints, and in the following we derive passivity results for the distributed control of networked

Euler-Lagrange systems which are subject to nonholonomic constraints.

4.2 Dynamic Equations of Motion

Consider a mechanical system with n generalized coordinates q subject to k nonholonomic constraints whose equations of motion are described by the Euler-Lagrange equations :

$$\frac{d}{dt}\frac{\partial L(q,\dot{q})}{\partial \dot{q}} - \frac{\partial L(q,\dot{q})}{\partial q} = A^T(q)\lambda + Q$$

$$A(q)\dot{q} = 0 \tag{4}$$

where $q, Q \in R^n$ are the generalized coordinates and the external forces respectively. $L(q,\dot{q}) = T(q,\dot{q}) - E(q)$ is the Lagragian function, $T(q,\dot{q})$ is the kinetic energy, which is assumed to be of the form $T(q,\dot{q}) = \frac{1}{2}\dot{q}^T D(q)\dot{q}$, where $D(q) = D^T(q) > 0$ is the $n \times n$ inertia matrix; $E(q)$ is the potential energy (here we assumed $E(q)$ has the form $E(q) = \frac{1}{2}q^T q$); the external forces consist of dissipative and control actions, where $Q = Mu - \frac{\partial F(\dot{q})}{\partial \dot{q}}$, with the control signal $u \in R^m$, $m \leq n$ and $M \in R^{n \times m}$ has the full column rank(we could consider M as the transform matrix for the control input). $F(\dot{q})$ is the Rayleigh dissipation function which defines as a memoryless passive operator $\dot{q} \to \frac{\partial F(\dot{q})}{\partial \dot{q}}$, such that

$$\dot{q}^T \frac{\partial F(\dot{q})}{\partial \dot{q}} \geq \alpha \|\dot{q}\|^2 \tag{5}$$

holds for all $\dot{q} \in R^n$ and $\alpha \geq 0$. Here we assume that $F(\dot{q}) = \frac{1}{2}\dot{q}^T \dot{q}$, λ is the Lagrange multiplier. $A(q)$ is a $k \times n$ matrix which defines the nonholonomic constraints.

4.3 State-Space Representation

Assume that the motion equation of the mechanical system has k independent nonholonomic constraints, which are given by:

$$A(q)\dot{q} = 0 \tag{6}$$

where $A(q)$ is a $k \times n$ matrix.

Let $[g_1(q), g_2(q), \ldots, g_{n-k}(q)]$ be a set of smooth and linear independent vector fields in the null space of $A(q)$, i.e.,

$$A(q)g_i(q) = 0, \quad i = 1, 2, \ldots, n-k \tag{7}$$

Let $G(q)_{n \times (n-k)}$ be the full rank matrix made up of these vectors. Since $A(q)\dot{q} = 0$, the constrained velocity is always in the null space of $A(q)$, it is possible to define the $n - k$ velocities $v(t) = [v_1, v_2, \ldots, v_{n-k}]$ such that [3]

$$\dot{q} = G(q)v(t) \tag{8}$$

we now consider the system given by (4):

$$\frac{d}{dt}\frac{\partial L(q,\dot{q})}{\partial \dot{q}} - \frac{\partial L(q,\dot{q})}{\partial q} = \dot{D}\dot{q} + D(q)\ddot{q} - \frac{\partial L(q,\dot{q})}{\partial q} = A^T(q)\lambda + Q \tag{9}$$

So, we have

$$\dot{D}\dot{q} + D(q)\ddot{q} - \frac{\partial L(q,\dot{q})}{\partial q} = A^T(q)\lambda + Q \tag{10}$$

Let $n(q,\dot{q}) = \dot{D}(q)\dot{q} - \frac{\partial L(q,\dot{q})}{\partial q}$, then we can get

$$D(q)\ddot{q} + n(q,\dot{q}) = A^T(q)\lambda + Q \tag{11}$$

multiply both sides of (11) by $G^T(q)$, we get

$$G^T(q)D(q)\ddot{q} + G^T(q)n(q,\dot{q}) = G^T(q)A^T(q)\lambda + G^T(q)Q = G^T(q)Q \tag{12}$$

Note that $G(q)$ belongs to the null space of $A(q)$, so we have $G^T(q)A^T(q) = 0$. Since $\dot{q} = G(q)v$, we have

$$\ddot{q} = G(q)\dot{v} + \dot{G}(q)v \tag{13}$$

put (13) into (12) we can get

$$G^T(q)D(q)G(q)\dot{v} + G^T(q)D(q)\dot{G}(q)v + G^T(q)n(q,\dot{q}) = G^T(q)Q \tag{14}$$

Let $\hat{M}(q) = G^T(q)D(q)G(q)$ and $\hat{m}(q,v) = G^T(q)D(q)\dot{G}(q)v + G^T(q)n(q,G(q)v)$, then

$$\hat{M}(q)\dot{v} + \hat{m}(q,v) = G^T(q)Q \tag{15}$$

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The reduced state-space model becomes

$$\dot{q} = G(q)v$$

$$\hat{M}(q)\dot{v} + \hat{m}(q,v) = G^T(q)Q \tag{16}$$

where $Q = Mu - \frac{\partial F(\dot{q})}{\partial \dot{q}}$. Let $u = [G^T(q)M]^{-1}[\hat{M}(q)\beta + \hat{m}(q,v) + G^T(q)\frac{\partial F(\dot{q})}{\partial \dot{q}}]$, apply the u into Q and substitute Q in (16), we can get $\dot{v} = \beta$. The closed loop state space model becomes:

$$\begin{cases} \dot{q} = G(q)v \\ \dot{v} = \beta \end{cases} \tag{17}$$

Let $x = [q^T, v^T]^T$, then we can rewrite (17) as

$$\begin{cases} \dot{x} = f(x) + g(x)\beta \\ y = v = h(x) \end{cases} \tag{18}$$

where $f(x) = [v^T G^T(q) \ 0]^T$, $g(x) = [0 \ I_{m \times m}]^T$ and $h(x) = v$.

4.4 Analysis of Passivity

It is easy to verify that the state-space representation of the Euler-Lagrange system subject to nonholonomic constraints which is described in (18)is passive with respect to the input and output pair (β, v). We give a brief proof in the following.

Theorem 1[1]:Consider a control affine nonlinear system of the form

$$\sum \begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \tag{19}$$

where $x \in R^n, u \in R^m$, and $y \in R^m$, the function $f(.) \in R^n$, $g(.) \in R^{n \times m}$ and $h(.) \in R^m$ are assumed to be sufficiently smooth. The admissible inputs are taken to be piecewise continuous and locally square integrable. Assume for simplicity, $f(0) = 0$ and $h(0) = 0$. The following statements are equivalent:

(i) The system $\sum$ is passive.

(ii)There exists a C^1 scalar function $V : R^n \to R$ such that $V(x) \geq 0$, $V(0) = 0$, and $S(x) \geq 0$ such that

$$L_f V(x) = -S(x)$$
$$L_g V(x) = h^T(x)$$

(20)

where $L_f V(x) = \frac{\partial V}{\partial x}^T f(x)$ and $L_g V(x) = \frac{\partial V}{\partial x}^T g(x)$.

Lemma 1: The state-space representation of the dynamic system subject to k nonholonomic constraints ($k \leq n$) which is described in (18) is passive with respect to the input and output pair (β, v).

Proof: If we choose the storage function $V(x)$ for the nonlinear system (18) as

$$V(x) = \frac{1}{2} x^T \begin{pmatrix} A^T(q)A(q) & 0 \\ 0 & I_{(n-k)\times(n-k)} \end{pmatrix} x$$

(21)

then we will have

$$L_f V(x) = \frac{\partial V(x)}{\partial x}^T f(x) = [q^T A^T(q)A(q) \ v^T][v^T G^T(q) \ 0]^T = 0$$

(22)

and

$$L_g V(x) = \frac{\partial V(x)}{\partial x}^T g(x) = [q^T A^T(q)A(q) \ v^T][0 \ I]^T = v^T = h^T(x)$$

(23)

According to *Theorem 1*, the nonlinear system (18) is passive with respect to the input and output pair (β, v), and $V(x)$ is defined as in (21) and $S(x) = 0$.

Remarks: The output synchronization results for the networked Euler-Lagrange systems in [16] is based on a similar analysis of the passivity property of Euler-Lagrange systems shown above expect that they did not take nonholonomic constraints into consideration. From the above analysis, we can see that if we take the nonholonomic constraints into consideration, then the nonlinear system (18) is passive with respect to the particular input-output pair (β, v), and if we follow the results shown in [16], we can achieve the output synchronization of v since now v is the output. One may claim that this is not a desired result since most of the time, we may be more interested in the synchronization results of the state q, and here v may not even have a physical meaning. We will first introduce Input-Output Linearization technique in the following section and later we will use it again in our proposed setup to solve this problem.

4.5 Input-Output Linearization

It has been shown in [3] that a system with nonholonomic constraints is not input-to-state linearizable, but it is input-to-output linearizable if a proper set of output equations are chosen. The necessary and sufficient condition for input-to-output linearization is that the decoupling matrix has full rank [4].

Since the motion of the dynamic system is subject to k nonholonomic constraints, we may have at most $n-k$ independent position output equations[3]:

$$y = h(q) = [h_1(q) \dots h_{n-k}(q)] \tag{24}$$

and the decoupling matrix $\Phi(q)$ for the nonlinear system (21) is the $(n-k) \times (n-k)$ matrix given by:

$$\Phi(q) = J_h(q)G(q) \tag{25}$$

where $J_h(q)$ is the $(n-k) \times n$ Jacobian matrix. If we introduce a new state space variable defined as:

$$Z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} h(q) \\ L_f h(q) \end{pmatrix} = \begin{pmatrix} h(q) \\ \Phi(q)v \end{pmatrix} \tag{26}$$

where $L_f h(q) = [f,h] = \frac{\partial h}{\partial x}f - \frac{\partial f}{\partial x}h$. The system under the new state variable Z is characterized by

$$\dot{z}_1 = \frac{\partial h}{\partial q}\dot{q} = z_2$$
$$\dot{z}_2 = \dot{\Phi}(q)v + \Phi(q)\beta \tag{27}$$

If we use the following state feedback

$$\beta = \Phi^{-1}(q)(\mu - \dot{\Phi}(q)v) \tag{28}$$

then we could achieve the input to output linearization:

$$\dot{z}_1 = z_2 \quad \dot{z}_2 = \mu \quad y = z_1 \tag{29}$$

Remarks: One may notice that after the input-to-output linearization, the passive nonlinear systems (18) becomes a non-passive system (29). To get our passivity results, we need to introduce our proposed setup in the

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following section.

5   Distributed Consensus Problem and Passivity-based Output Synchronization

Distributed Consensus problem has been explored a lot during the last decade. And the corresponding appli-
cations of the consensus algorithm have also been discussed a lot, i.e., synchronization of coupled oscillators,
flocking, formation control, fast consensus in small-word networks, Markov processes and gossip-algorithm, load
balancing in networks, rendezvous in space, distributed sensor fusion in sensor networks, and belief propagation
[5]. In [1] and [2], passivity-based stability analysis has been shown as a very efficient tool for the multi-agent
coordination problem. In this section, we would like to give our results for the Passivity-based Output Synchro-
nization problem, which is consistent with the results shown in [1] and [2]. Later we will use our results to solve
the multi-agent rendezvous problem.

In the previous section, we have shown that by applying the input-to-output linearization technique, the affine
nonlinear system (21) can be transformed to its input-to-output linearized form, which is shown in (27). Actually,
(27) can be re-written as

$$\ddot{z} = \mu \quad y = z \tag{30}$$

Assume there is a group of N agents with each agent's dynamics described by:

$$\ddot{z}_i = \mu_i \quad y_i = z_i \tag{31}$$

Consider the control law given by:

$$\mu_i = -\alpha_i[\dot{z}_i(t) - \upsilon_i(t)] + \dot{\upsilon}_i(t) + u_i(t)$$

$$\alpha_i = \alpha_i^T \quad \alpha_i > 0 \tag{32}$$

where $\upsilon_i(t)$ is some external reference point, $u_i(t)$ is the control input.

Let $\xi_i(t) = \dot{z}_i(t) - \upsilon_i(t)$, then we can get the state-space representation of the new variable $\xi_i(t)$ which is given by:

$$H_i : \begin{cases} \dot{\xi}_i = -\alpha_i \xi_i + u_i \\ p_i = \xi_i \end{cases} \tag{33}$$

10

In the following, we will show that under certain conditions and with properly designed control input $u_i(t)$, the multi-agent system with state-equation described by(32) and control law (33) will achieve output synchronization based on the analysis of passivity for the feedback configuration of two subsystems.
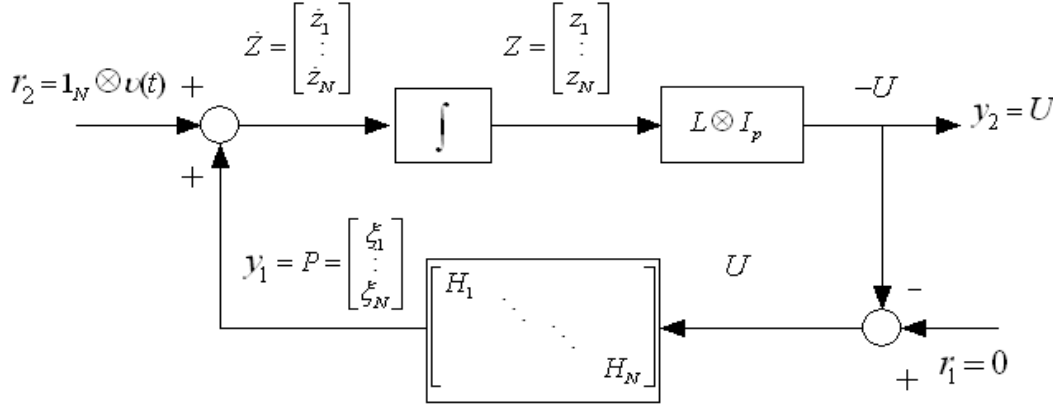

Figure 1: Feedback Configuration of the Interconnected Multi-agent System

Theorem 2: Consider the feedback configuration shown in Figure 1, where $\upsilon(t)$ is a bounded and piecewise continuous external reference point, and H_i, $i = 1,\ldots,N$ represents the state-space equation of each agent i, which is shown in (34). L is the graph Laplacian that represents the underlying communication graph of the interconnected agents, I_p is the $p \times p$ identity matrix, where p is the dimension of the states of each agent. Assume that the underlying communication graph is bidirectional (it follows that L is positive-semidefinite and symmetric, with zero row sums and zero column sums). Then:

   (i) The feed forward paths from $\dot{Z}$ to $-U$ and from P to $-U$ are both passive;

   (ii) The feedback path from U to P is passive;

   (iii) The closed-loop system is passive from r to y, where $r = [r_1^T, r_2^T]^T$, $y = [y_1^T, y_2^T]^T$;

   (iv) The interconnected system under the linear coupling $L \otimes I_p$ will achieve output synchronization asymptotically.

Proof:

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(i) Consider the storage function for the feed-forward part given by:

$$V_f(z) = \frac{1}{2} Z^T (L \otimes I_p) Z \tag{34}$$

$$\dot{V}_f(z) = ((L \otimes I_p)Z)^T \dot{Z} = (-U)^T \dot{Z} \tag{35}$$

thus the feedforward path from $\dot{Z}$ to $-U$ is passive. Moreover, since

$$\dot{Z} = 1_N \otimes \upsilon(t) + P(t) \tag{36}$$

we then have

$$\dot{V}_f(z) = ((L \otimes I_p)Z)^T \dot{Z} = Z^T (L^T \otimes I_p)(1_N \otimes \upsilon(t) + P(t)) = Z^T (L^T \otimes I_p)P(t) = (-U)^T P(t) \tag{37}$$

here $(L^T \otimes I_p)(1_N \otimes \upsilon(t)) = 0$ follows from L is Laplacian and we have made the assumption that L is symmetric with zero row sums and zero column sums. Thus the feed forward path from $P(t)$ to -U is also passive.

(ii) To prove that the feedback path from U to P is passive, we first need to show that each subsystem H_i is passive. Consider the storage function for each subsystem given by:

$$V_{bi}(\xi_i) = \frac{1}{2} \xi_i^T \xi_i \tag{38}$$

then we have

$$\dot{V}_{bi}(\xi_i) = \xi_i^T \dot{\xi}_i = \xi_i^T(-\alpha_i \xi_i + u_i) = -\alpha_i \xi_i^T \xi_i + \xi_i^T u_i \leq u_i^T p_i \tag{39}$$

This shows that each subsystem H_i is passive, then storage function for the feedback path is given by:

$$V_b(\xi) = \sum_{i=1}^{N} V_{bi}(\xi_i) \tag{40}$$

It follows that

$$\dot{V}_b(\xi) = \sum_{i=1}^{N} \dot{V}_{bi}(\xi_i) \leq \sum_{i=1}^{N} u_i^T p_i = U^T P \tag{41}$$

which means the feedback path from U to P is passive.

(iii) If we define the storage function for the closed-loop as

$$V(Z,\xi) = V_f(Z) + V_b(\xi) \tag{42}$$

then we have

$$\dot{V}(Z,\xi) = \dot{V}_f(Z) + \dot{V}_b(\xi) = (-U)^T(P + r_2) - \xi^T \alpha \xi + U^T P = y_2^T r_2 - \alpha \xi^T \xi \leq y_2^T r_2 = y^T r \tag{43}$$

where $r_1 = 0$, $r_2 = 1_N \otimes v(t)$, $y_1 = P$ and $y_2 = -U$. This proves that the closed-loop is passive with the input-output pair (r,y);

(iv) As seen from (37), $\dot{V}_f(Z) = (-U)^T P$, we have

$$\dot{V}(Z,\xi) = \dot{V}_f(Z) + \dot{V}_b(\xi) = (-U)^T P - \xi^T \alpha \xi + U^T P = -\xi^T \alpha \xi \leq 0 \tag{44}$$

where $\alpha = diag\{\alpha_i\}$, $\alpha = \alpha^T > 0$.

The storage function $V(Z,\xi)$ could be chosen as the Lyapunov function for the closed-loop system, and then (44) indicates that as $t \to \infty, \xi \to 0$. Let's re-examine the state-space representation for ξ:

$$\begin{aligned} \dot{\xi} &= -\alpha \dot{\xi} + U \\ P &= \xi \end{aligned} \tag{45}$$

where $\xi = [\xi_1^T \dots \xi_N^T]^T$. We can see that $\lim_{t\to\infty} \xi(t) = 0$ indicates that $\lim_{t\to\infty} U(t) = 0$. In view of $U(t) = (L \otimes I_p)Z(t)$, it follows that $\lim_{t\to\infty}(L \otimes I_p)Z(t) = 0$, this implies that $\lim_{t\to\infty} z_i(t) = \lim_{t\to\infty} z_j(t)$, $\forall i \neq j$ and $j \in \mathcal{N}_i$, where $\mathcal{N}_i$ denotes the set of the neighbors' of agent i. This proves that the interconnected agents will achieve output synchronization asymptotically. $\square$

6 Scattering Transformation and Passivation Scheme for the Networked Distributed System

Another important issue about networked control for distributed systems is how we deal with the time delays in the communication channel. In case of time-varying delays existing in the communication channel, the intercon-

nected system may not be able to preserve passivity[6]. It has been shown in [1],[6]-[7] and [15] that scattering transformation (or wave variables) can be used as the passivation scheme for the networked systems with time-varying delays. In this section we will give a more general passivation scheme for the distributed control of multi-agent system based on a linear consensus algorithm.

Assumptions: We will assume that the underlying communication graph for the networked systems discussed here is undirected and strongly connected with time-varying communication delays $T_{ij}(t)$ and $T_{ji}(t)$, where $T_{ij}(t)$ is not necessary equal to $T_{ji}(t)$.

Let the agents be coupled together with the control law

$$u_i = \sum_{j \in \mathcal{N}_i} K(z_{js} - z_i) \quad i = 1, \dots, N \tag{46}$$

where K is a positive constant. We can see that (46) is defined in the same way as the control input for the linear consensus problem, see [5]. The passivation configuration is illustrated schematically in Figure 2. The superscript +,- for the scattering variables is a convention for the direction of the power flow; the agents transmit the scattering variables instead of their outputs. The i th agent transmits the scattering variable S_{ij}^+ to the j th agent who receives it as the scattering variable S_{ji}^+. The j th agent then uses the control law $K(z_{is} - z_j)$ to extract the signal z_{is} out of the incoming scattering variable S_{ji}^+. The reference variables z_{js}, z_{is} are derived out of the scattering transformation. The scattering transformation is given by:

$$
\begin{aligned}
S_{ij}^+ &= \frac{1}{\sqrt{2}}[-K(z_{js} - z_i) + z_{js}] \\
f_{ij}S_{ji}^+ &= \frac{1}{\sqrt{2}}[K(z_{is} - z_j) + z_{is}] \\
S_{ji}^- &= \frac{1}{\sqrt{2}}[K(z_{is} - y_j) - z_{is}] \\
f_{ji}S_{ij}^- &= \frac{1}{\sqrt{2}}[-K(z_{js} - z_i) - z_{js}]
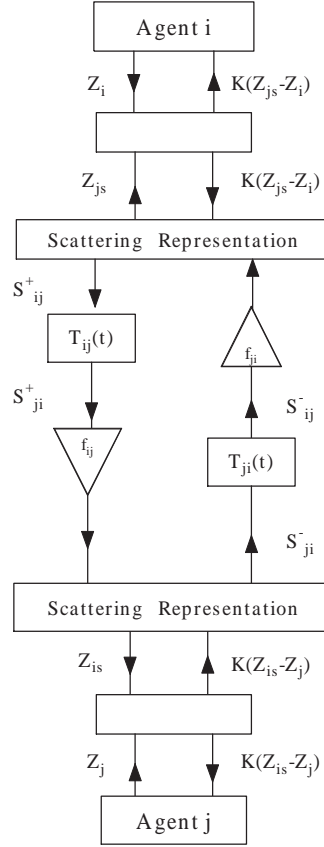\end{aligned}
\tag{47}
$$

Figure 2: scattering transformation

The energy stored in the communication channel between agent i and agent j is given by:

$$
\begin{aligned}
V^{ij}(t) &= \frac{1}{2}\int_{t_0}^{t}\left(\parallel S_{ij}^{+}(\tau)\parallel^2 - f_{ij}^2\parallel S_{ji}^{+}(\tau)\parallel^2 + \parallel S_{ji}^{-}(\tau)\parallel^2 - f_{ji}^2\parallel S_{ij}^{-}(\tau)\parallel^2\right)d\tau \\
&= \frac{1}{2}\Big(\int_{t-T_{ij}(t)}^{t}\parallel S_{ij}^{+}(\tau)\parallel^2 d\tau + \int_{t-T_{ji}(t)}^{t}\parallel S_{ji}^{-}(\tau)\parallel^2 d\tau + \int_{t_0}^{t-T_{ij}(t)}\parallel S_{ij}^{+}(\tau)\parallel^2 d\tau + \int_{t_0}^{t-T_{ji}(t)}\parallel S_{ji}^{-}(\tau)\parallel^2 d\tau \\
&\quad - \int_{t_0}^{t} f_{ij}^2\parallel S_{ij}^{+}(\tau - T_{ij}(\tau))\parallel^2 d\tau - \int_{t_0}^{t} f_{ji}^2\parallel S_{ji}^{-}(\tau - T_{ji}(\tau))\parallel^2 d\tau\Big) \\
&= \frac{1}{2}\Big(\int_{t-T_{ij}(t)}^{t}\parallel S_{ij}^{+}(\tau)\parallel^2 d\tau + \int_{t-T_{ji}(t)}^{t}\parallel S_{ji}^{-}(\tau)\parallel^2 d\tau \\
&\quad + \int_{t_0}^{t-T_{ij}(t)}\frac{1-T_{ij}^{'}(\sigma)-f_{ij}^2}{1-T_{ij}^{'}(\sigma)}\parallel S_{ij}^{+}(\sigma)\parallel^2 d\sigma + \int_{t_0}^{t-T_{ji}(t)}\frac{1-T_{ji}^{'}(\sigma)-f_{ji}^2}{1-T_{ji}^{'}(\sigma)}\parallel S_{ji}^{-}(\sigma)\parallel^2 d\sigma\Big)
\end{aligned}
\tag{48}
$$

where $\sigma = \tau - T_{ij}(\tau), \quad \forall i, j$. Also we should notice that

$$
\begin{aligned}
V^{ij}(t) &= \frac{1}{2} \int_{t_0}^{t} \left(\| S_{ij}^+(\tau) \|^2 - f_{ij}^2 \| S_{ji}^+(\tau) \|^2 + \| S_{ji}^-(\tau) \|^2 - f_{ji}^2 \| S_{ij}^-(\tau) \|^2 \right) d\tau \\
&= \int_{t_0}^{t} \left(-K(z_{js} - z_i)^T z_{js} - K(z_{is} - z_j)^T z_{is} \right) d\tau
\end{aligned}
\tag{49}
$$

From (48) we can see that as long as $\frac{1 - T'_{ij}(t) - f_{ij}^2}{1 - T'_{ij}(t)} \geq 0, \quad \forall i, j$, then we will have $V^{ij}(t) \geq 0$ which means the communication channel in the passiviation scheme is consuming energy at each time t. So if both of the interconnected systems are passive, then after we insert proper gains into the communication channel, we can guarantee that the whole networked system will still be passive in the presence of the time-varying delays. Here we have only shown the passivation scheme for the two interconnected systems. However, it is not difficult to show this is also true for multiple interconnections.

Now we can study the stability of the networked passive system when all the agents are coupled through the control law (46).

Theorem 3: Consider the feedback configuration of a group of networked passive agents as shown in figure 1, where each agent's state-space representation is given by (34). Assume that:

(1) The underlying communication graph $\mathbf{G} := [V(\mathscr{G}), \mathcal{E}(\mathscr{G})]$ is undirected and strongly connected;

(2) The communication delay $\{T_{ij}(t) : (i, j) \in \mathcal{E}(\mathscr{G})\}$ between each coupled agent (agent i and agent j) is time varying and $T_{ij}(t)$ is not necessary equal to $T_{ji}(t)$;

(3) Wave variables are transmitted through the communication channel according to the passivation scheme shown in Figure 2;

(4) The inserted gain of each communication channel is bounded by: $\frac{1 - T'_{ij}(t) - f_{ij}^2}{1 - T'_{ij}(t)} \geq 0, \quad \forall (i, j) \in \mathcal{E}(\mathscr{G})$;

then under the control law given by:

$$
u_i = \sum_{(i,j) \in \mathcal{E}(\mathscr{G})} K(z_{js} - z_i) \quad i = 1, \ldots, N
\tag{50}
$$

the networked passive systems are globally stable. Moreover, with properly designed gains f_{ij}s and f_{ji}s, the linear coupled agents will achieve output synchronization as long as the communication graph is strongly connected.

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Proof: In theorem 2 we have proved that in the absence of time-varying communication delays, the feedback configuration shown in Figure 1 is passive, and the storage function given by (43) is indeed a Lyapunov function candidate for the closed loop system. And we have proved that $\lim_{t\to\infty}\xi(t)=0$ implies $\lim_{t\to\infty}U(t)=0$ and we get the conclusion that the interconnected agents will achieve output synchronization asymptotically. However, in the presence of time-varying communication delays, we need to insert properly designed gains into the communication channel according to assumption (4) to maintain the passivity of the interconnected systems. As long as the above assumptions are satisfied, we can still assume that the interconnected systems is passive, then we use (43) again as the energy storage function and Lyapunov function candidate for the closed-loop system. It follows that $\lim_{t\to\infty}\xi(t)=0$, which again implies that $\lim_{t\to\infty}U(t)=0$. Since the control law for each agent is given by (50), $\lim_{t\to\infty}U(t)=0$ implies that

$$\lim_{t\to\infty}(z_{js}-z_i)=0 \quad or \quad \lim_{t\to\infty}(z_{is}-z_j)=0 \tag{51}$$

From the scattering transformation (47) we have

$$
\begin{aligned}
f_{ij}S_{ji}^{+} &= \frac{1}{\sqrt{2}}[K(z_{is}-z_j)+z_{is}] \\
&= f_{ij}S_{ij}^{+}(t-T_{ij}) \\
&= \frac{f_{ij}}{\sqrt{2}}[-K(z_{js}(t-T_{ij})-z_i(t-T_{ij}))+z_{js}(t-T_{ij})]
\end{aligned}
\tag{52}
$$

From (51) we know that

$$\lim_{t\to\infty}K(z_{is}-z_j)=0$$
$$\lim_{t\to\infty}K(z_{js}(t-T_{ij})-z_i(t-T_{ij}))=0 \tag{53}$$

Thus, we will have

$$\lim_{t\to\infty}z_{is} = \lim_{t\to\infty}f_{ij}z_{js}(t-T_{ij}) = \lim_{t\to\infty}f_{ij}z_i(t-T_{ij}) \tag{54}$$

which implies that

$$\lim_{t\to\infty}z_j = \lim_{t\to\infty}f_{ij}z_i(t-T_{ij}) \tag{55}$$

Remarks:

(1) Assumption (4) is necessary to preserve the passivity of the interconnected systems;

(2) According to assumption (4), if $T'_{ij}(t) < 1$, then $f_{ij}^2 \leq 1 - T'_{ij}(t)$; if $T'_{ij}(t) > 1$, then f_{ij} could be arbitrarily chosen; but one should notice that unbounded $T_{ij}(t)$ will significantly degenerate the performance of the interconnected systems;

(3) (55) implies that if $f_{ij} \neq 1$, then

$$\lim_{t \to \infty} z_i(t) = 0, \quad \forall i = 1, \dots, N \tag{56}$$

so outputs of the interconnected agents will converge to the origin; if $f_{ij} = 1, \quad \forall(i,j) \in \mathcal{E}(\mathcal{G})$, then see from (55), the outputs of the interconnected systems will achieve synchronization asymptotically; however, $f_{ij} = 1, \quad \forall(i,j) \in \mathcal{E}(\mathcal{G})$ requires that the $T'_{ij}(t) \leq 0, \quad \forall(i,j) \in \mathcal{E}(\mathcal{G})$, which imposes bound on the change rate of the delay in the communication channel.

7 Example:Passivity-based Rendezvous for a group of Car-Like Robots with Nonholonomic Constraints

In this section, we will give an example on the rendezvous problem for a group of car-like robots with non-holonomic constraints. We will apply all the results which we have discussed and presented in the previous sections.

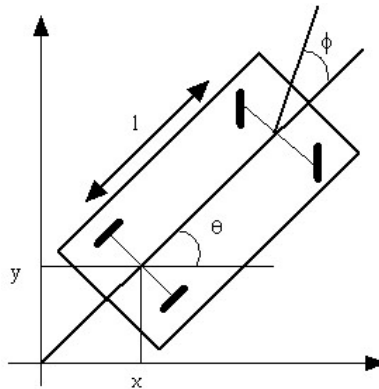## 7.1 State-Space Representation of the Car-Like Robot with Nonholonomic Constraints

Figure 3: Car-Like Robot with Nonholonomic Constraints

Description of the Car-Like Robot Model:

- 'Bicycle' model: front and rear wheels collapse into two wheels at the axle midpoints;

- Generalized coordinates $q = (x, y, \theta, \phi)$ θ: heading angle, ϕ: steering angle ;

- Nonholonomic constraints:

$$\dot{x}_f sin(\theta + \phi) - \dot{y}_f cos(\theta + \phi) = 0 \ (front \ wheel)$$

$$\dot{x} sin(\theta) - \dot{y} cos(\theta) = 0 \ (rear \ wheel)$$

(57)

- The front wheel position:

$$x_f = x + l cos(\theta) \quad y_f = y + l sin(\theta)$$

(58)

- The constraint matrix is:

$$A^T(q) = \begin{pmatrix} sin(\theta + \phi) & -cos(\theta + \phi) & -lcos(\phi) & 0 \\ sin(\theta) & -cos(\theta) & 0 & 0 \end{pmatrix}$$

(59)

If we chose

$$G(q) = \begin{pmatrix} cos(\theta)cos(\phi) & 0 \\ sin(\theta)sin(\phi) & 0 \\ \frac{1}{l}sin(\phi) & 0 \\ 0 & 1 \end{pmatrix}$$

(60)

then

$$\dot{q} = g_1(q)v_1 + g_2(q)v_2$$

(61)

where v_1 = front driving(the force applied to drive the front wheel), v_2 = steering input(the force applied to steer the direction).

In this case, the state-space representation for the closed-loop system can be written as:

$$\dot{X} = \begin{pmatrix} \dot{q} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} G(q)v \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} \beta$$

$$y = v$$

(62)

where $v = [v_1^T, v_2^T]^T$

7.2 Input-to-Output Linearization

In our case, in order to use the input-to-output linearization technique we need to define two new independent outputs since the robot has 2 degrees of freedom. We can define any point$(\hat{x}, \hat{y})$ with an arbitrarily distance d away from the position of the front wheel of the robot as the new independent outputs, the equation of $(\hat{x}, \hat{y})$ is given by:

$$h(q) = [h_1^T(q) \ h_2^T(q)]^T = \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} x + lcos(\theta) + dcos(\theta + \phi) \\ y + lsin(\theta) + dsin(\theta + \phi) \end{pmatrix} \tag{63}$$

then

$$J_h(q) = \begin{pmatrix} \frac{\partial \hat{x}}{\partial q} \\ \frac{\partial \hat{y}}{\partial q} \end{pmatrix} = \begin{pmatrix} 1 & 0 & -lsin(\theta) - dsin(\theta + \phi) & -dsin(\theta + \phi) \\ 0 & 1 & lcos(\theta) + dcos(\theta + \phi) & dcos(\theta + \phi) \end{pmatrix} \tag{64}$$

Then we have the decoupling matrix $\Phi(q)$

$$\Phi(q) = \begin{pmatrix} cos(\theta + \phi) - \frac{d}{l}sin(\theta + \phi)sin(\phi) & -dsin(\theta + \phi) \\ sin(\theta + \phi) + \frac{d}{l}cos(\theta + \phi)sin(\phi) & dcos(\theta + \phi) \end{pmatrix} \tag{65}$$

then according to (24)-(27), we will get the new state-space representation:

$$\dot{Z}_1 = Z_2 \quad \dot{Z}_2 = \mu \quad P = Z_1 \tag{66}$$

where $Z_1 = [\hat{x}^T \quad \hat{y}^T]^T$

7.3 Passivity-Based Rendezvous

Assume that we have N car-like robots to complete a rendezvous task. Each agent has the same model which is shown in(62)-(67). And by applying input-to-output linearization, we can express each agents as:

$$\dot{Z}_{i1} = Z_{i2} \quad \dot{Z}_{i2} = \mu_i \quad P_i = Z_{i1} \tag{67}$$

where $Z_{i1} = [\hat{x}_i^T \quad \hat{y}_i^T]^T$. For the Rendezvous problem, we need achieve $\lim_{t \to \infty} x_i(t) = x_d$ and $\lim_{t \to \infty} y_i(t) = y_d$ for $i = 1, \dots, N$, where (x_d, y_d) is the location for rendezvous. This also implies that we need to achieve the output

synchronization of P_i.

Assume that now we have three car-like robots to complete the task, and the underlying communication graph is bidirectional and strongly connected which can be shown in the figure below: Assume that the control law μ_i is
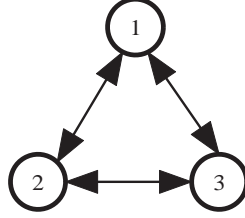

Figure 4: balanced graph and strongly connected communication graph

given by:

$$\mu_i = -K_i[\dot{Z}_i(t) - \upsilon_i(t)] + \dot{\upsilon}_i(t) + u_i(t) \tag{68}$$

where $K_i = K_i^T > 0$ and $\upsilon_i(t)$ could be considered as the predefined reference velocity for all the robots, and all the robots are coupled and controlled according to the configuration shown in Figure 1. Then according to Theorem 2, the robots will achieve output synchronization asymptotically. However, since there are time-varying communication delays existing in each communication channels, we need to use scattering transformation to maintain the passivity of the interconnected agents. Thus all the robots are transmitting wave variables through each communication channel according to the scheme shown in Figure 2. First let's examine what will happen if the change rate of the time-delays is negative, which means in this case, the inserted gains can be chosen for any values. If we choose all the inserted gains to be one, then according to Theorem 1, the robots will achieve output synchronization which means they will complete the rendezvous task. The simulation result is shown in Figure 5, which is the same as we have expected (here we assume that one of the robots know the rendezvous location while the other tow robots don't know where is the final destination), we can see from the simulation that the three robots will reach the rendezvous location, which is the same as we have expected.

Let's examine what will happen if the change rate of the delays is positive but bounded by one: the simulation result is shown in Figure 6, we can see that the robot which knows the rendezvous location will still goes to the predefined destination; however, the other two robots will go to the origin. This result is consistent with our discussion on Theorem 2.

One should notice that the above results can also be applied to the path-following problem. When we assign a virtual leader to one of the robots, then we can expect that the all the robots will follow the desired path of the virtual leader. The simulation results for the path-following problem in shown in Figure 7 and Figure 8, we also compare the case when the inserted gain can be chosen to be one and the case when the inserted gain cannot be chosen to be one.

8   Conclusion

In this report, we have explored how to use passivity as the design and analysis tool for the distributed control of networked Euler-Lagrange systems, and we have applied our results to solve the rendezvous problem of car-like robots with nonholonomic constraints. The contributions of this report include that we extend the work of Chopra and Spong [16] to the networked Euler-Lagrange systems subject to nonholonomic constraints; we also explore the setup shown in Murat's work [2] and we propose a new setup in this report to solve the output synchronization problem of networked Euler-Lagrange systems with nonholonomic constraints based on the analysis of passivity. The rendezvous example shown in this report actually indicates that the proposed framework could be used in many applications in distributed control system, such as the consensus problem and the multi-agent coordination problem. Future work may include passivity-based control on distributed hybrid systems, i.e., when the underlying communication graph is time-varying or the dynamics of the interconnected agents possess a behavior-based switching nature.

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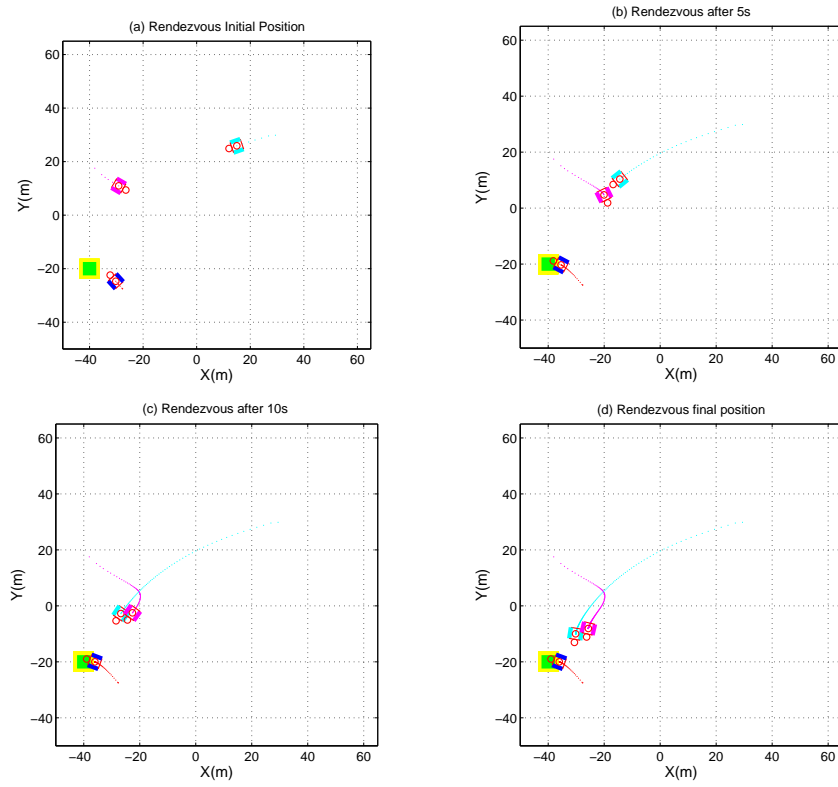
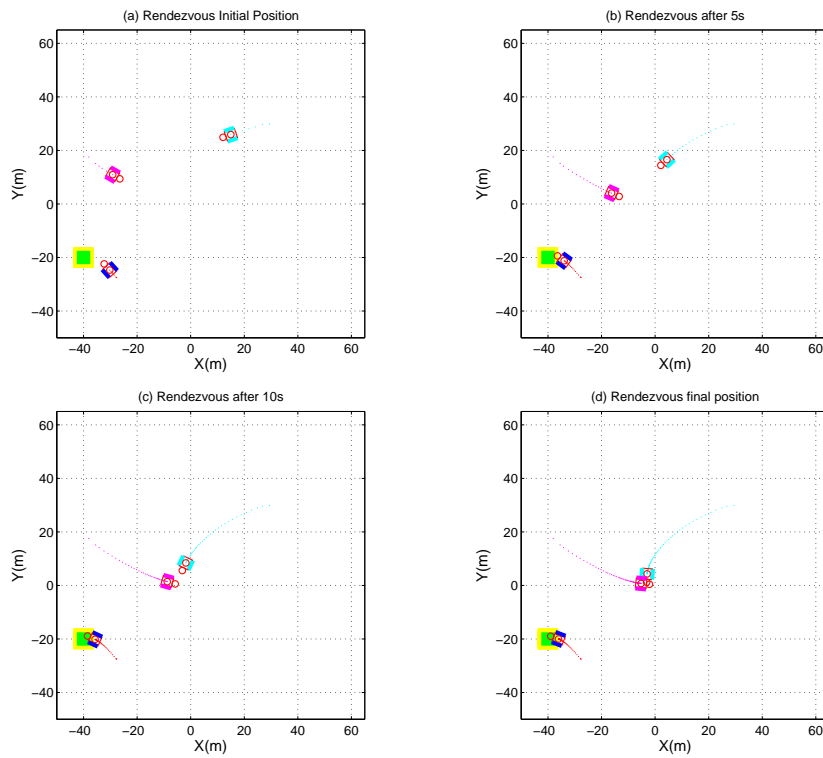Figure 5: Rendezvous of the Car-Like Robots when the inserted gains are 1

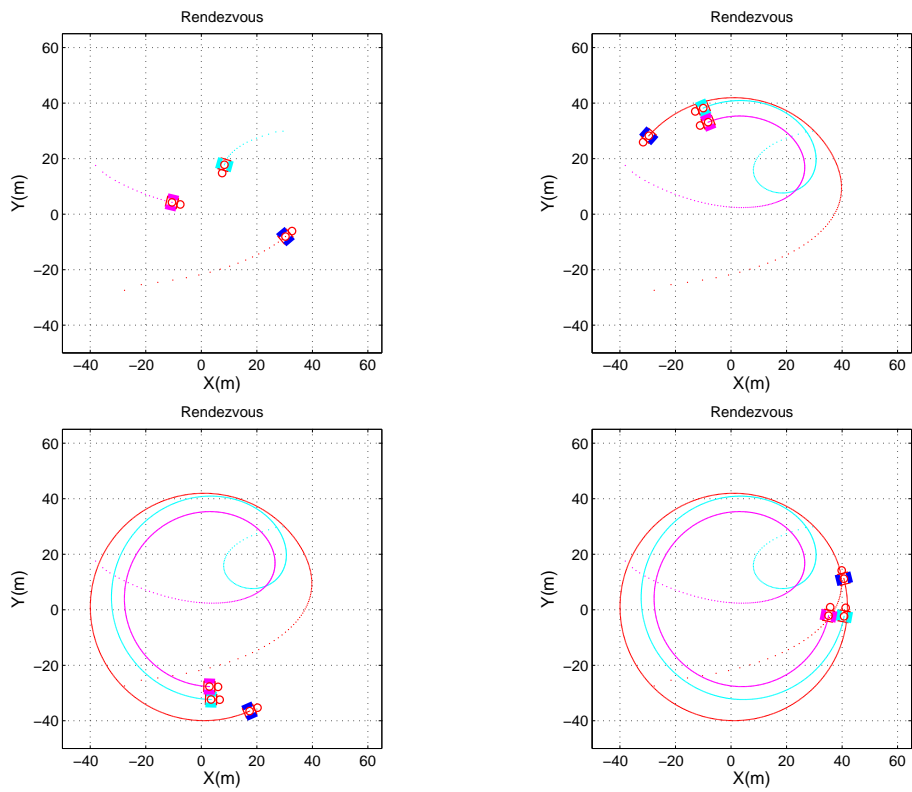Figure 6: Rendezvous of the Car-Like Robots when the inserted gains cannot be 1

Figure 7: Path following of the Car-Like Robots when the inserted gains could be 1
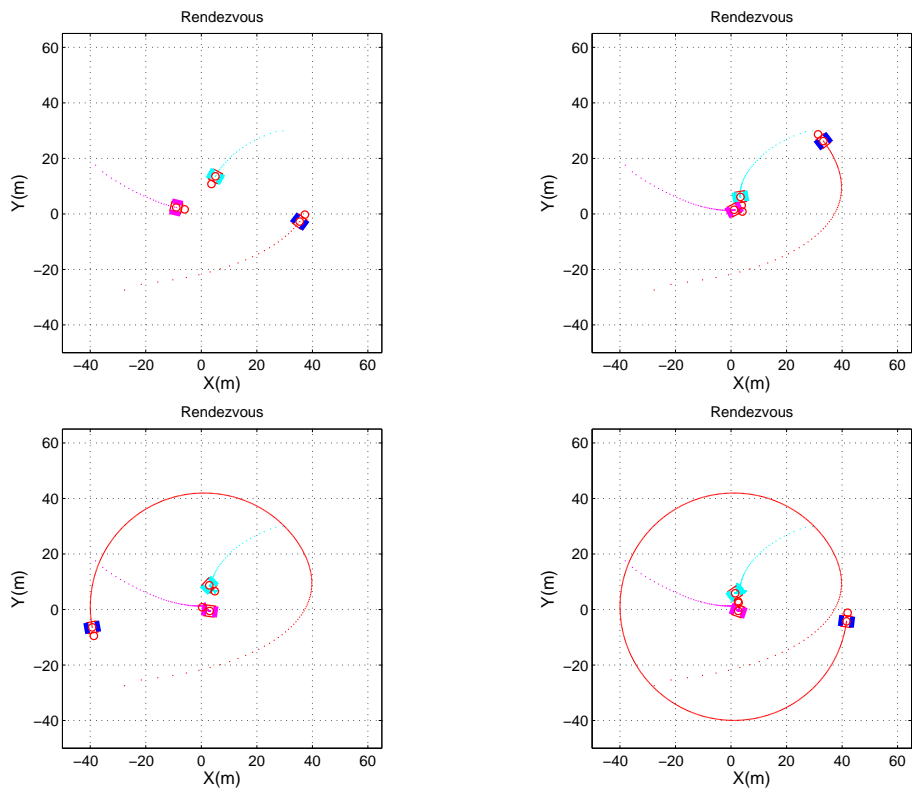

Figure 8: Path following of the Car-Like Robots when the inserted gains cannot be 1