HIDDEN MODES IN TWO DEGREES OF FREEDOM DESIGN: \{R;G,H\} CONTROLLER

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Abstract

The stable and unstable hidden modes of the \{R;G,H\} controlled system are characterized by considering cancellations in products of transfer matrices. This characterization is complete because of the assumption that the plant description is controllable and observable, and that \(R\), \(G\) and \(H\) are implemented via irreducible realizations. Under these assumptions the hidden modes of the controlled system are due exclusively to the system interconnection. The characterizations given show how hidden modes are introduced in the controlled system. This information can be used by the control systems designer to avoid hidden modes if possible; otherwise, the effect of the hidden modes on the system performance can be determined.

Stable hidden modes need to be studied because they can degrade the performance and they can increase the necessary order of the compensators (see example below). Consequently, a complete understanding of stable hidden modes will lead to better control design algorithms.

Historical note: before internal stability in a feedback system became well understood, unstable hidden modes were of primary concern in early control designs using transfer functions. Under the assumption of internal stability in the controlled system only stable hidden modes will be present. Nevertheless, the characterization of hidden modes which follows is complete and includes also unstable hidden modes if internal stability is not present.

The hidden modes can be characterized using internal descriptions \([4-10]\). In this paper they will be characterized by considering "cancellations" in products of transfer matrices see for example \([10]\), \([11]\). Hidden modes can be characterized in terms of transfer matrices since it is assumed that each subsystem is completely described by its input-output description. By considering products of transfer matrices, that is, cascade connections, the interconnection that gives rise to the hidden modes is specified. This information can be used by the control systems designer to avoid hidden modes if possible; otherwise, the effect of the hidden modes on the system performance can be determined.

Theorem 1: The uncontrollable modes from \(r\) in the controlled \(\{R;G,H\}\) system correspond to:

- (i) poles of \(P\) which cancel in \(PG\), and
- (ii) poles of \(H\) which cancel in \(HP\), and
- (iii) poles of \(G\) which cancel in \(GH\), and
- (iv) poles of \(I+GHP\)^{-1} which cancel in \((I+GHP)^{-1}GR\).

Theorem 2: The unobservable modes from \(y\) in the controlled \(\{R;G,H\}\) system correspond to:

- (i) poles of \(G\) which cancel in \(PG\), and
- (ii) poles of \(H\) which cancel in \((PG)H\), and
- (iii) poles of \(R\) which cancel in \(P(I+GHP)^{-1}GR\), and
- (iv) poles of \((I+GHP)^{-1}\) which cancel in \(P(I+GHP)^{-1}\).

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Theorem 3: The unobservable modes from \( u \) in the controlled \( \{R;G,H\} \) system correspond to:

1. poles of \( H \) which cancel in \( GH_1 \) and \( GH_2 \), and
2. poles of \( P \) which cancel in \( GH_1 P \), and
3. poles of \( R \) which cancel in \( (G+HP)^{-1} GR \) and
4. poles of \( (G+HP)^{-1} GR \) which cancel in \( (G+HP)^{-1} G \).

Theorem 2 shows that only poles of the controller can correspond to unobservable modes from the output \( y \). This is intuitively pleasing because the controlled system can be thought of as a cascade connection of \( P \) and \( M \) with \( M = G(1+GHP)^{-1} R \). In this setup if any poles of \( P \) cancel in \( PM \) they will correspond to uncontrollable modes from \( r \) (systems in cascade), so only the poles of \( M \) and those that cancel in \( M \) can be unobservable from \( y \) as seen in Theorem 2.

Discussion: Theorems 1-3 give insight as to how the system interconnection introduces hidden modes. Consider the \( \{R;G,H\} \) controller as a separate system. This system consists of the parallel connection of \( R \) and \( M \) in cascade with \( G \) with input \( [r' \ v']' \) (\( ' \) denotes transposition) and output \( y_c \). The unobservable modes from \( y_c \) correspond to the poles of \( R \) which cancel in \( GR \) and the poles of \( H \) which cancel in \( GH \). The controlled system is put together by connecting \( y = Fu \) with the controller by setting \( u = y_c \) and \( v = y \). From Theorems 2 and 3 the unobservable modes of the controller remain unobservable from \( u \) and from \( y \) in the controlled system together with additional modes introduced by the interconnection. The uncontrollable modes from \( [r' \ v']' \) of the controller correspond to the poles of \( G \) which cancel in \( G[H,R] \). These modes remain also uncontrollable from \( r \) in the controlled system as seen from Theorem 1 (iii). Therefore, the unobservable modes from \( y_c \) and the uncontrollable modes from \( [r' \ v']' \) of the controller remain unobservable from \( y \) and uncontrollable from \( r \) in the controlled system.

Uncontrollable and unobservable closed loop eigenvalues can affect the closed loop system in different ways as shown in the examples below.

**Example 1** [5, p. 232]

Astrom and Wittenmark show how a stable hidden mode can degrade performance. They consider the plant \( P = K(z-b)/(z-1)(z-a) \) where \( b \) and \( a \) are the two degrees of freedom controller \( C = -C_y C_P \). It can be easily shown that their control law can be implemented via \( G = \epsilon_C(z-c_2)/((z-1)(z-a)) \), \( H = \epsilon_D z(1+\epsilon_C)/(z-c) \) and \( R = 1 \) in the \( \{G,H,R\} \) controller configuration. Note that \( a \) and \( c \) are chosen so that the compensated system is internally stable and \( t_0 \), \( K \) and \( a \) are real constants defined in [5]. Note that the particular implementation of the controller used here does not affect the following remarks: A simulation shows that the step response of the system contains an undesirable "ripple" or "ringing" in the control signal \( u \) while the output signal \( y \) is well behaved at the sampling instants. It is pointed out in [5] that the "ringing" is caused by the cancellation of the \( (z-b) \) factor. From Theorems 2 and 3 we can see that the reason for "ringing" is that the pole of \( G \) at \( z = 1 \) cancels in \( PG \) corresponds to an unobservable mode from \( y \) that is observable from \( u \). Moreover, it can be shown that the mode that corresponds to the pole of the controller which cancels the plant zero at \( z = 1 \) will be unobservable from the output but will be observable from the plant input in any implementation of the two degrees of freedom controller.

The following example shows that a cancellation does not necessarily imply a hidden mode neither from \( y \) nor from \( r \).

**Example 2**

Let \( P = 1/(s-1), G = (s^2+a+1)/(s(s+10)), \) \( R = 1 \) and \( H = 2(s+10)(s+1)/(s^2+a+1) \). Observe that the pole of \( G \) at \( s = -10 \) cancels in \( GH \) but the closed loop transfer function is given by \( T = 1/(s+10) \) so that the pole of \( G \) at \( s = -10 \) does not correspond to an uncontrollable mode from \( r \) nor to an unobservable mode from \( y \). On the other hand the poles of \( H \) are uncontrollable from \( r \) and unobservable from \( y \). Furthermore, the poles of \( (I+PGH)^{-1} \), given by the zeros of \( (s^2+a+1) \), will be uncontrollable from \( r \) and unobservable from \( u \).

The characterizations given in Theorems 1-3 can be used by the control systems designer to avoid hidden modes which can introduce undesirable effects.

**References**


