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HIDDEN MODES IN TWO DEGREES OF FREEDOM DESIGN: {R;G,H} CONTROLLER

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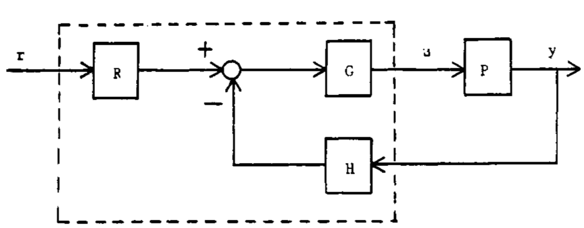
Abstract

The stable and unstable hidden modes of the {R;G,H} controlled system are characterized by considering cancellations in products of transfer matrices. This characterization is complete because of the assumption that the plant description is controllable and observable, and that R, G and H are implemented via irreducible realizations. Under these assumptions the hidden modes of the controlled system are due exclusively to the system interconnection. The characterizations given show how hidden modes are introduced in the controlled system. This information can be used by the control systems designer to avoid hidden modes if possible; otherwise, the effect of the hidden modes on the system performance can be determined.

The hidden modes of a compensated system correspond to the closed-loop eigenvalues which are uncontrollable and/or unobservable from certain inputs or outputs, respectively. The hidden modes for single degree of freedom controllers have been studied in the literature [1-4]. In this paper we study the hidden modes of a two degrees of freedom controller; these controllers are useful in design as they address control problems with multiple objectives. Consider a linear time-invariant and finite dimensional plant described by y = Pu and the general two degrees of freedom controller C described by $u = -C_y y + C_r r$ where r is the command input. In particular we consider the typical {R;G,H} control configuration:



{R;G,H} controller

Assume that the plant description is controllable and observable. Let the controller {R;G,H} be designed to internally stabilize the controlled system. The controller {R;G,H} could be designed, for example, using the methods in [2] and setting R to be stable. Assume that R, G and H are implemented via irreducible realizations. Under these assumptions the hidden modes of the controlled system are due exclusively to the system interconnection. Furthermore, the hidden modes of the controlled system if any will be stable. Stable hidden modes need to be studied because they can degrade the performance and they can increase the necessary order of the compensators (see example below). Consequently, a complete understanding of stable hidden modes will lead to better control design algorithms.

Historical note: before internal stability in a feedback system became well understood, unstable hidden modes were of primary concern in early control designs using transfer functions. Under the assumption of internal stability in the controlled system only stable hidden modes will be present. Nevertheless, the characterization of hidden modes which follows is complete and includes also unstable hidden modes if internal stability is not present.

The hidden modes can be characterized using internal descriptions [4-10]. In this paper they will be characterized by considering "cancellations" in products of transfer matrices see for example [10], [11]. Hidden modes can be characterized in terms of transfer matrices since it is assumed that each subsystem is completely described by its input-output description. By considering products of transfer matrices, that is, cascade connections, the interconnection that gives rise to the hidden modes is specified. This information can be used by the control systems designer to avoid hidden modes if possible; otherwise, the effect of the hidden modes on the system performance can be determined.

Theorems 1 and 2 characterize the hidden modes which correspond to the uncontrollable modes from r and the unobservable modes from y, respectively; they appear as modes of the controlled system but not as poles of the closed loop transfer matrix. It is also of interest to study hidden modes from other inputs and outputs. For example, Theorem 3 characterizes the unobservable modes from u, the control signal. These theorems have been proven using internal operator descriptions.

Theorem 1: The uncontrollable modes from r in the controlled {R;G,H} system correspond to:

(i) poles of P which cancel in PG, and
(ii) poles of H which cancel in H(PG), and
(iii) poles of G which cancel in G[H R], and
(iv) poles of $(I+GHP)^{-1}$ which cancel in $(I+GHP)^{-1}GR$.

Theorem 2: The unobservable modes from y in the controlled {R;G,H} system correspond to:

(i) poles of G which cancel in PG, and
(ii) poles of H which cancel in (PG)H, and
(iii) poles of R which cancel in $P(I+GHP)^{-1}GR$, and
(iv) poles of (I+GHP)-1 which cancel in $P(I+GHP)^{-1}$.

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Theorem 3: The unobservable modes from u in the controlled {R;G,H} system correspond to:

(i) poles of H which cancel in GH, and
(ii) poles of P which cancel in (GH)P, and
(iii) poles of R which cancel in $(I+GHP)^{-1}GR$. and
(iv) poles of $(I+GHP)^{-1}$ which cancel in $(I+GHP)^{-1}G$.

Theorem 2 shows that only poles of the controller can correspond to unobservable modes from the output y. This is intuitively pleasing because the controlled system can be thought of as a cascade connection of P and M with $M = G(I+PGH)^{-1}R$. In this setup if any poles of P cancel in PM they will correspond to uncontrollable modes from r (systems in cascade); so only the poles of M and those that cancel in M can be unobservable from y as seen in Theorem 2.

Discussion: Theorems 1-3 give insight as to how the system interconnection introduces hidden modes. Consider the {R;G,H} controller as a separate system. This system consists of the parallel connection of R and -H in cascade with G with input [r' v']' (' denotes transposition) and output y_c. The unobservable modes from y_c correspond to the poles of R which cancel in GR and the poles of H which cancel in GH. The controlled system is put together by connecting y = Pu with the controller by setting $u = y_c$ and v = y. From Theorems 2 and 3 the unobservable modes of the controller remain unobservable from u and from y in the controlled system together with additional modes introduced by the interconnection. The uncontrollable modes from [r' v']' of the controller correspond to the poles of G which cancel in G[H,R]. These modes remain also uncontrollable from r in the controlled system as seen from Theorem 1 (iii). Therefore, the unobservable modes from y_c and the uncontrollable modes from [r' v']' of the controller remain unobservable from y and uncontrollable from r in the controlled system.

Uncontrollable and unobservable closed loop eigenvalues can affect the closed loop system in different ways as shown in the examples below.

Example 1 [5, p. 232]

Astrom and Wittenmark show how a stable hidden mode can degrade performance. They consider the plant $P = K(z-b)/[(z-1)(z-a)]$ where b<0 and the general two degrees of freedom controller $C = [-C_y\ C_r]$. It can be easily shown that their control law can be implemented via $G = t_0z/(z-b)$, $H = (s_0z+s_1)/(t_0z)$ and R=1 in the {R;G,H} control configuration. Note that s_0 and s_1 are chosen so that the compensated system is internally stable and t_0, K and a are real constants defined in [5]. Note that the particular implementation of the controller used here does not affect the following remarks: A simulation shows that the step response of the system contains an undesirable "ripple" or "ringing" in the control signal u while the output signal y is well behaved at the sampling instants. It is pointed out in [5] that the "ringing" is caused by the cancellation of the (z-b) factor. From Theorems 2 and 3 we can see that the reason for "ringing" is that the pole of G at b which cancels in PG corresponds to an unobservable mode from y that is observable from u. Moreover, it can be shown that the mode that corresponds to the pole of the controller which cancels the plant zero at b will be unobservable from the output but will be observable from the plant input in any implementation of the two degrees of freedom controller.

The following example shown that a cancellation does not necessarily imply a hidden mode neither from y nor from r.

Example 2

Let $P = 1/(s-1)$, $G = (s^2+s+1)/[s(s+10)]$, R = 1 and $H = 2(s+10)(s+1/2)/(s^2+s+1)$. Observe that the pole of G at s = -10 cancels in GH but the closed loop transfer function is given by $T = 1/(s+10)$ so that the pole of G at s = -10 does not correspond to an uncontrollable mode from r nor to an unobservable mode from y. On the other hand the poles of H are uncontrollable from r and unobservable from y and u. Furthermore, the poles of $(I+PGH)^{-1}$, given by the zeros of (s^2+s+1), will be uncontrollable from r and unobservable from u.

The characterizations given in Theorems 1-3 can be used by the control systems designer to avoid hidden modes which can introduce undesirable effects.

References

[1] P.J. Antsaklis and M.K. Sain, "Unity Feedback compensation of Unstable Plants," Proceedings 20th IEEE Conference on Decision and Control, pp. 305-308, December 1981.
[2] P.J. Antsaklis and M.K. Sain, "Feedback Synthesis with Two Degrees of Freedom: {G;H,P} Controller," Proceedings of the 9th World Congress of the International Federation of Automatic Control, Hungary, July 1984.
[3] P.J. Antsaklis and M.K. Sain, "Feedback Controller Parameterizations: Finite Hidden Modes and Causality," in Multivariable Control: New Concepts and Tools, S.G. Tzafestas, Editor, Dordrecht, Holland: D. Reidel Publisher, 1984.
[4] F.M. Callier and C.A. Desoer, Multivariable Feedback Systems, New York: Springer Verlag, 1982.
[5] K.J. Astrom and B. Wittenmark, Computer Controlled Systems Theory and Design, Prentice-Hall Information and Systems Science Series, 1984.
[6] H.H. Rosenbrock, State Space and Multivariable Theory, Nelson, London, 1970.
[7] W.A. Wolovich, Linear Multivariable Systems, New York: Springer Verlag, 1974.
[8] T. Kailath, Linear Systems, Englewood Cliffs, New Jersey, Prentice Hall, 1980.
[9] C.T. Chen, Linear System Theory and Design, Holt, Rinehart and Winston, 1984.
[10] E.G. Gilbert, "Controllability and Observability in Multivariable Control Systems," SIAM Journal of Control, Vol. 2, No. 1, pp. 128-151, 1963.
[11] B.D.O. Anderson and M.R. Gevers, "On Multivariable Pole-Zero Cancellations and the Stability of Feedback Systems," IEEE Transactions on Circuits and Systems, Vol. CAS-28, pp. 830-833, August 1981.