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Passivity-Based Output Synchronization of Networked Euler-Lagrange Systems Subject To Nonholonomic Constraints

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***Abstract*— In this paper, we study the output synchronization problem of networked Euler-Lagrange(EL) systems subject to nonholonomic constraints. An EL system subject to nonholonomic constraints can be input-output linearized if a proper decoupling matrix can be found, and the linearized form is equivalent to a double integrator. Although a double integrator is not a passive system, with proper design of the control law and some coordinate transformation, we are able to obtain a new state-space representation of the EL system which is a linear passive system. The underlying assumption is that the communication graph is bidirectional and strongly connected. To deal with time-varying communication delays among the interconnected agents, we embed the scattering transformation into our proposed setup and show that output synchronization can be achieved in this case as well.**

I. Introduction

Recent advances in computing, communication, sensing and actuation have made it feasible to have large numbers of autonomous vehicles (air, ground, and water) working cooperatively to accomplish an objective. The problem of communication and control in multi-agent systems becomes more and more important in numerous practical applications, such as sensor networks, unmanned aerial vehicles, and robot networks. In most of these applications, we need a team of agents to communicate with neighbors and agree on key pieces of information that enable them to work together in a coordinated fashion. The problem is particularly challenging because communication channels have limited capacity and experience fading and dropout.

In the past 10 years, many papers have been published that study information flow, group consensus, multi-agent coordination and formation problems, see [14]-[27]. Many of the existing results in the literature model the agents as velocity-controlled particles, i.e., first order integrators, or model the agents as Lagrangian and Hamiltonian systems, such as n-degree-of-freedom robots. One should notice that most of these models satisfy a natural and well-studied passivity property. Group coordination and output synchronization problem based on passivity design framework and passivity based stability analysis have also be addressed in [1]-[5]. The reason why passivity has been used in the design and analysis of networked control system is that it allows one to analyze the system from a input-output perspective, which does not require detailed knowledge of the system behavior, and is therefore particularly well-suited to applications displaying high uncertainty on parameters and structure. However, by restricting ourself to systems that are passive, we are restricted ourself to systems that are stable and minimum phase and of low relative degree [13]. This is a serious drawback of some of the existing work in the literature which uses passivity framework to study networked control systems.

In this paper, we study the output synchronization problem of networked Euler-Lagrange (EL) systems subject to nonholonomic constraints, where the goal is to make the outputs of the agents converge to a common trajectory. An EL system subject to nonholonomic constraints can be input-output linearized if a proper decoupling matrix can be found, and the linearized form is equivalent to a double integrator, which is not a passive system. The results presented in the current paper solves the output synchronization problem with respect to particular outputs of networked non-passive systems by using passivity based analysis. The results developed not only suggest a way to solve real application problems but also suggest how to apply the passivity-based analysis to networked control systems where the subsystems are not passive.

The paper is organized as follows: Section II.A introduces some basic concepts of passivity, Section II.B provides some background on EL systems and nonholonomic constraints; Section III discusses how to derive state-space representation of an EL system subject to nonholonomic constraints which is used in this paper. Our proposed setup and the main results are given in Section IV, where we first address the output synchronization problem without considering the time-varying communication delays in the networks; then we employ scattering transformation and embed it in our setup to deal with the delays. In section V, an example of the path-following problem for a group of car-like robots is provided. Finally, conclusions are given in Section VI.

II. Background

A. Passivity

Definition 1 (Passive System, [13]) *The dynamic system*

$$\begin{cases} \dot{x} = f(x,u) \\ y = h(x,u) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$and $u, y \in \mathbb{R}^p$, is said to be ***passive*** *if there exists a C^1 storage function $V(x) \geq 0$ such that*

$$\dot{V} = \nabla V(x)^T f(x,u) \leq -S(x) + u^T y \tag{2}$$

for some function $S(x) \geq 0$, where x is the state in (1). We say it is ***strictly passive*** *if $S(x) > 0$.*

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Theorem 1 (Feedback Interconnection, [29]) *The negative feedback interconnection of two passive systems is passive.*

B. EL Systems and Nonholonomic Constraints

Euler-Lagrange (EL) systems are systems whose motion is described by Euler-Lagrange equations, such as robotics and mobile vehicles. Most of the physical systems, while they can be modeled by Euler-Lagrange equations, are subject to certain nonholonomic constraints, where the constraints are involving both the state x (i.e., position) and its derivative $\dot{x}$ (i.e., velocity). For example, a wheeled mobile vehicle is a nonholonomic system since its state (position) depends on the path and its velocity taken to achieve it.

Consider a mechanical system with n generalized coordinates q subject to k nonholonomic constraints whose equations of motion are described by the constrained EL equations [8]:

$$\begin{aligned} &\frac{d}{dt}\frac{\partial L(q,\dot{q})}{\partial \dot{q}} - \frac{\partial L(q,\dot{q})}{\partial q} = A^T(q)\lambda + Q \\ &A(q)\dot{q} = 0, \end{aligned} \tag{3}$$

where $q, Q \in R^n$ are the generalized coordinates and the external forces respectively; $L(q,\dot{q})$ is the Lagrangian function and λ is the Lagrange multiplier. The first equation in (3) is the EL equation which describes the motion of the system, while the second equation in (3) defines the equations of nonholonomic constraints, which impose restrictions on q and $\dot{q}$ in the EL equation; $A(q)$ is a $k \times n$ matrix.

An EL system subject to nonholonomic constraints (3) can be transformed into the following state-space representation (refer to [8] for detailed derivations):

$$\begin{cases} \dot{q} = G(q)\nu \\ \dot{\nu} = \beta \end{cases} \tag{4}$$

where the columns of $G(q)$ are a basis for $ker(A(q))$. Let $x = [q^T \;\; \nu^T]^T$, then we can rewrite (4) as

$$\begin{cases} \dot{x} = f(x) + g(x)\beta \\ y = \nu = h(x) \end{cases} \tag{5}$$

where $f(x) = [\nu^T G^T(q) \;\; 0]^T$, $g(x) = [0 \;\; I_{m\times m}]^T$ and $h(x) = \nu$. It has been shown that a system with nonholonomic constraints in general is not input-state linearizable, but it is input-output linearizable if a proper set of output equations are chosen. The necessary and sufficient condition for input-output linearization is that the “decoupling matrix”below has full rank [9]. Suppose that the motion of the dynamic system is subject to k nonholonomic constraints, then we may have at most $n-k$ independent position output equations:

$$y = h(q) = [h_1(q), \ldots, h_{n-k}(q)]^T \tag{6}$$

the decoupling matrix $\Phi(q)$ for the nonlinear system (5) is the $(n-k)\times(n-k)$ matrix given by [8]:

$$\Phi(q) = J_h(q)G(q) \tag{7}$$

where $J_h(q)$ is the $(n-k)\times n$ Jacobian matrix of $h(q)$. To characterize the zero dynamics and achieve input-output linearization, we can introduce a new state vector defined as [8]:

$$Z = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} h(q) \\ L_f h(q) \\ \tilde{h}(q) \end{pmatrix} = \begin{pmatrix} h(q) \\ \Phi(q)\nu \\ \tilde{h}(q) \end{pmatrix} \tag{8}$$

where $L_f h(q) = [f,h] = \frac{\partial h}{\partial x} f - \frac{\partial f}{\partial x} h$, and $\tilde{h}(q)$ is an m-dimensional function such that $[\frac{\partial h}{\partial q}^T \;\; \frac{\partial \tilde{h}(q)}{\partial q}^T]$ has full rank. The system under the new state Z is characterized by

$$\begin{aligned} \dot{z}_1 &= \frac{\partial h}{\partial q}\dot{q} = z_2 \\ \dot{z}_2 &= \dot{\Phi}(q)\nu + \Phi(q)\beta \\ \dot{z}_3 &= \frac{\partial \tilde{h}(q)}{\partial q} G(q)\nu = \frac{\partial \tilde{h}}{\partial q}(\frac{\partial h}{\partial q})^{-1} z_2. \end{aligned} \tag{9}$$

If we use the following state feedback with $\Phi^{-1}(q)$ exists

$$\beta = \Phi^{-1}(q)(\mu - \dot{\Phi}(q)\nu) \tag{10}$$

then we could achieve the input-output linearization:

$$\dot{z}_1 = z_2 \qquad \dot{z}_2 = \mu \qquad y = z_1 \tag{11}$$

where the unobservable zero dynamics of the system is

$$\dot{z}_3 = 0. \tag{12}$$

One should notice that (11) can be re-written as

$$\ddot{z}_1 = \mu \qquad y = z_1 \tag{13}$$

which is a double integrator.

III. State-Space Representation Of Agents

Up to this point, it has been shown that a single Euler-Lagrange system with nonholonomic constraints could be input-output linearized into a linear system which is equivalent to a double integrator . Now assume there is a network of N agents where each agent is described by:

$$\ddot{z}_i(t) = \mu_i(t) \qquad y_i(t) = z_i(t), \quad i = 1, \ldots, N, \tag{14}$$

where $z_i(t), \mu_i, y_i(t) \in \mathbb{R}^n$. Consider the control law given by [1]:

$$\mu_i(t) = -\alpha_i[\dot{z}_i(t) - \upsilon_i(t)] + \dot{\upsilon}_i(t) + u_i(t), \quad \alpha_i > 0, \tag{15}$$

where $\upsilon_i(t) \in \mathbb{R}^n$ is some external reference point, $u_i(t) \in \mathbb{R}^n$ is the control input. Let $\xi_i(t) = \dot{z}_i(t) - \upsilon_i(t)$, then the state-space representation in terms of new variable $\xi_i(t)$ is given by:

$$H_i : \begin{cases} \dot{\xi}_i(t) = -\alpha_i \xi_i(t) + u_i(t) \\ p_i(t) = \xi_i(t) \end{cases} \tag{16}$$

where $p_i(t) = \xi_i(t) \in \mathbb{R}^n$ is the new outputs for the transformed system. One can verify that the transformed system (16) is passive. We will use this state-space representation for our subsystems in the following sections.

IV. Proposed Setup And Main Theorems

We first present our output synchronization results without considering time-varying communication delays in Section

A; our results for the case with delays are presented in Section B.

A. Synchronization in absence of communication delays

Theorem 2 . *Consider the feedback configuration shown in Figure 1, where $\upsilon(t) \in \mathbb{R}^n$ denotes the bounded and piecewise continuous external reference input to each agent, and $\upsilon_i(t) = \upsilon(t)$, for $i = 1,\ldots,N$; H_i is the state-space representation of agent i as given in (16); $L \in \mathbb{R}^{N\times N}$ is the graph Laplacian that represents the underlying communication graph of the interconnected agents; $P = [p_1^T,\ldots,p_N^T]^T$ is the concatenation vector, where $p_i \in \mathbb{R}^n$ are the outputs of agent i; $Z = [z_1^T,\ldots,z_N^T]^T$ and $\dot{Z} = [\dot{z}_1^T,\ldots,\dot{z}_N^T]^T$, where $\dot{z}_i = \upsilon(t) + p_i$; $U = (L\otimes I_n)Z) = [u_1^T,\ldots,u_N^T]^T$, where u_i is the control input to agent i. Assume that the underlying communication graph is bidirectional and strongly connected and $\upsilon(t)$=0. Then as $t \to \infty$, $z_i(t) = z_j(t)$, $\forall i, j = 1,\ldots,N$.*

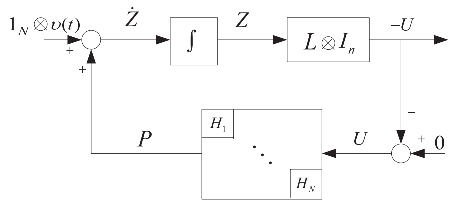

Fig. 1: Feedback Configuration

Proof. 1) Consider the storage function for the feed-forward path from $\dot{Z}$ to $-U$ given by:

$$V_f(Z) = \frac{1}{2}Z^T(L\otimes I_p)Z, \tag{17}$$

we have

$$\dot{V}_f(Z) = ((L\otimes I_p)Z)^T\dot{Z} = (-U)^T\dot{Z} \tag{18}$$

which implies that the feedforward path from $\dot{Z}$ to $-U$ is passive. Moreover, since

$$\dot{Z} = 1_N \otimes \upsilon(t) + P \tag{19}$$

we have

$$\begin{aligned}\dot{V}_f(Z) &= ((L\otimes I_p)Z)^T\dot{Z} = Z^T(L^T\otimes I_p)(1_N\otimes\upsilon(t)+P)\\ &= Z^T(L^T\otimes I_p)P = (-U)^T P.\end{aligned} \tag{20}$$

Here $(L^T\otimes I_p)(1_N\otimes\upsilon(t)) = 0$ follows from L being graph Laplacian, and since we assume that the underlying communication graph is bidirectional and strongly connected, L is symmetric with zero row sums and zero column sums. So (20) shows that the feed-forward path from P to -U is also passive.
2) We can also prove that the feedback path from U to P is passive. First we need to show that each subsystem H_i is passive. Consider the storage function for each subsystem given by:

$$V_{bi}(p_i) = \frac{1}{2}p_i^T p_i \tag{21}$$

then we have

$$\dot{V}_{bi}(p_i) = p_i^T\dot{p}_i = p_i^T(-\alpha_i p_i + u_i) = -\alpha_i p_i^T p_i + p_i^T u_i \le u_i^T p_i \tag{22}$$

This shows that each subsystem H_i is passive. Let the storage function for the feedback path be given by:

$$V_b = \sum_{i=1}^{N} V_{bi}. \tag{23}$$

It follows that

$$\dot{V}_b = \sum_{i=1}^{N}\dot{V}_{bi} = -P^T\Gamma P + U^T P \le U^T P, \tag{24}$$

where $\Gamma = diag\{\alpha_1,\ldots,\alpha_N\} > 0$. This shows that the feedback path from U to P is passive.
3) Now if we choose the storage function $V(Z,P) = V_f(Z) + V_b(P)$ as the candidate Lyapunov like function for the closed loop system, we can obtain

$$\dot{V}(Z,P) = \dot{V}_f(Z) + \dot{V}_b(P) = -P^T\Gamma P < 0, \tag{25}$$

according to LaSalle's principle[28], the largest invariant set for $\dot{V}(Z,P) = 0$ is given by $P = 0$, and this implies that as $t\to\infty$, $P(t)\to 0$, or $\lim_{t\to\infty}\dot{p}_i(t) = 0$, for $i = 1,\ldots,N$. In view of (19), since $\upsilon(t) = 0$, we can conclude that $\lim_{t\to\infty}\dot{Z}(t) = 0$. Since $U(t) = (L\otimes I_n)Z(t)$, we can obtain $\lim_{t\to\infty}\dot{U}(t) = 0$, or $\lim_{t\to\infty}\dot{u}_i(t) = 0$, for $i = 1,\ldots,N$. Let's re-examine the state-space representation for H_i:

$$\dot{p}_i(t) = -\alpha_i p_i(t) + u_i(t), \tag{26}$$

and thus $\ddot{p}_i(t) = -\alpha_i\dot{p}_i(t) + \dot{u}_i(t)$. Since $\lim_{t\to\infty}\dot{u}_i(t) = 0$ and $\alpha_i > 0$ we can obtain that $\lim_{t\to\infty}\dot{p}_i(t) = 0$, and in view of (26), $\lim_{t\to\infty}\dot{p}_i(t) = 0$ and $\lim_{t\to\infty}p_i(t) = 0$ indicate that $\lim_{t\to\infty}u_i(t) = 0$, for $i = 1,\ldots,N$. Since $U(t) = -(L\otimes I_n)Z(t)$, it follows that $\lim_{t\to\infty}(L\otimes I_n)Z(t) = 0$, this implies that $\lim_{t\to\infty}z_i(t) = \lim_{t\to\infty}z_j(t)$, $\forall(i,j)\in\varepsilon(G)$, where ε denotes the set of edges of the underlying communication graph G. So as long as the underlying communication graph is strongly connected, we will have $\lim_{t\to\infty}z_i(t) = \lim_{t\to\infty}z_j(t)$, $\forall i, j = 1,\ldots,N$. ∎

B. Scattering Transformation and Passivation Scheme

In the case of time-varying communication delays, the interconnected systems may not be able to preserve the passivity property because of the extra energy produced by communication due to increasing time delays in the communication network [10]. One way to solve this problem is by using the scattering transformation(wave variables)[30]as the passivation scheme, which has been reported in [2] and [10]-[12]. In this section, we propose a way to achieve output synchronization in the presence of time-varying communication delays by embedding a scattering transformation into each communication link.

The passivation configuration for agent i and agent j is illustrated schematically in Figure 2: the symbol "ST"denotes the scattering transformation; the superscript +,- for the scattering variables is a convention for the direction of the power flow; $T_{ij}(t)$ denotes the delay from agent i to agent j while $T_{ji}(t)$ denotes the delay from agent j to agent i. Each agent transmits its scattering variables in the communication channel, for example, agent i transmits its scattering variable $S^+_{ij}(t)$ to agent j, who receives it as the scattering variable $f_{ij}S^{d+}_{ij}(t)$, where $S^{d+}_{ij}(t) = S^+_{ij}(t-T_{ij}(t))$. The scattering transformation is given by:

$$\begin{aligned} S^+_{ij} &= \tfrac{1}{2}[-K(z_{js}-z_i)+p_{js}] \quad & f_{ij}S^{d+}_{ij} &= \tfrac{1}{2}[K(z_{is}-z_j)+p_{is}] \\ S^-_{ji} &= \tfrac{1}{2}[K(z_{is}-z_j)-p_{is}] \quad & f_{ji}S^{d-}_{ji} &= \tfrac{1}{2}[-K(z_{js}-z_i)-p_{js}], \end{aligned} \tag{27}$$

where $K > 0$ is a constant, $p_{js} = \dot{z}_{js}$, $p_{is} = \dot{z}_{is}$, $S^{d-}_{ji}(t) = S^-_{ji}(t-T_{ji}(t))$ and $S^{d+}_{ij}(t) = S^+_{ij}(t-T_{ij}(t))$.

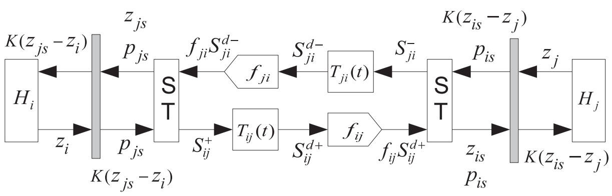

Fig. 2: Scattering Transformation

We can calculate the energy stored in the communication channel between agent i and agent j for any time interval:

$$\begin{aligned} V^{ij}(t) &= \frac{1}{4}\int_{t_0}^{t}\Big(\| S^+_{ij}(\tau)\|^2 - f^2_{ij}\| S^{d+}_{ij}(\tau)\|^2 \\ &\quad + \| S^-_{ji}(\tau)\|^2 - f^2_{ji}\| S^{d-}_{ji}(\tau)\|^2 \Big)d\tau \\ &= \frac{1}{4}\Big(\int_{t-T_{ij}(t)}^{t} \| S^+_{ij}(\tau)\|^2\, d\tau + \int_{t-T_{ji}(t)}^{t} \| S^-_{ji}(\tau)\|^2\, d\tau \\ &\quad + \int_{t_0}^{t-T_{ij}(t)} \| S^+_{ij}(\tau)\|^2\, d\tau + \int_{t_0}^{t-T_{ji}(t)} \| S^-_{ji}(\tau)\|^2\, d\tau \\ &\quad - \int_{t_0}^{t} f^2_{ij}\| S^+_{ij}(\tau - T_{ij}(\tau))\|^2\, d\tau \\ &\quad - \int_{t_0}^{t} f^2_{ji}\| S^-_{ji}(\tau - T_{ji}(\tau))\|^2\, d\tau\Big) \\ &= \frac{1}{4}\Big(\int_{t-T_{ij}(t)}^{t} \| S^+_{ij}(\tau)\|^2\, d\tau + \int_{t-T_{ji}(t)}^{t} \| S^-_{ji}(\tau)\|^2\, d\tau \\ &\quad + \int_{t_0}^{t-T_{ij}(t)} \frac{1-T'_{ij}(\sigma_1)-f^2_{ij}}{1-T'_{ij}(\sigma_1)} \| S^+_{ij}(\sigma_1)\|^2\, d\sigma_1 \\ &\quad + \int_{t_0}^{t-T_{ji}(t)} \frac{1-T'_{ji}(\sigma_2)-f^2_{ji}}{1-T'_{ji}(\sigma_2)} \| S^-_{ji}(\sigma_2)\|^2\, d\sigma_2\Big) \end{aligned} \tag{28}$$

where $\sigma_1 = \tau - T_{ij}(\tau)$, $\sigma_2 = \tau - T_{ji}(\tau)$. From (28) we can see that as long as $\frac{1-T'_{ij}(t)-f^2_{ij}}{1-T'_{ij}(t)} \geq 0,\ \forall (i,j)$, we will have $V^{ij}(t) \geq 0$. Also it is easy to verify that

$$\dot{V}^{ij}(t) = -K(z_{js}-z_i)^T p_{js} - K(z_{is}-z_j)^T p_{is}. \tag{29}$$

After introducing the proposed scattering transformation setup, we are ready to present the following theorem.

Theorem 3. *Consider a group of N networked agents $\{H_1,\ldots,H_N\}$, where each agent's state-space representation is given by (16). Assume that:*

1) *the underlying communication graph is bidirectional and strongly connected;*
2) *the scattering transformation is embedded into each communication link as shown in Figure 2;*
3) $\frac{1-T'_{ij}(t)-f^2_{ij}}{1-T'_{ij}(t)} \geq 0,\ \forall (i,j) \in \varepsilon(G)$*;*
4) *the external reference $\upsilon_i(t)$ to each agent i is equal to zero.*

Then under the control law given by:

$$u_i = \sum_{\forall (i,j)\in\varepsilon(G)} K(z_{js}-z_i) \qquad i = 1,\ldots,N, \tag{30}$$

where $z_{js} = z_j(t-T_{ji}(t))$, $\lim_{t\to\infty} z_i(t) = \lim_{t\to\infty} z_j(t)$, for $i, j = 1,\ldots,N$.

Proof. Since $\frac{1-T'_{ij}(t)-f^2_{ij}}{1-T'_{ij}(t)} \geq 0$, we have $V^{ij}(t) \geq 0$, $\forall (i,j) \in \varepsilon(G)$, so we can choose the following storage function for the entire interconnection:

$$V = \sum_{i=1}^{N} V_{b_i} + \sum_{\forall (i,j)\in\varepsilon(G)} V^{ij} + \frac{1}{2}\sum_{i=1}^{N}\sum_{\forall (i,j)\in\varepsilon(G)} K(z_{js}-z_i)^T(z_{js}-z_i), \tag{31}$$

where V_{b_i} is given in (21). Then we have

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N} \dot{V}_{b_i} + \sum_{\forall (i,j)\in\varepsilon(G)} \dot{V}^{ij} + \sum_{i=1}^{N}\sum_{\forall (i,j)\in\varepsilon(G)} K(z_{js}-z_i)^T(p_{js}-p_i) \\ &= -P^T\Gamma P + U^T P + \sum_{\forall (i,j)\in\varepsilon(G)} \dot{V}^{ij} \\ &\quad + \sum_{i=1}^{N}\sum_{\forall (i,j)\in\varepsilon(G)} K(z_{js}-z_i)^T(p_{js}-p_i). \end{aligned} \tag{32}$$

Since

$$\sum_{\forall (i,j)\in\varepsilon(G)} \dot{V}^{ij} = \sum_{\forall (i,j)\in\varepsilon(G)} [-K(z_{js}-z_i)^T p_{js} - K(z_{is}-z_j)^T p_{is}], \tag{33}$$

$$U^T P = \sum_{i=1}^{N}\sum_{\forall (i,j)\in\varepsilon(G)} K(z_{js}-z_i)^T p_i, \tag{34}$$

and the underlying communication graph is bidirectional, it is easy to verify that

$$U^T P + \sum_{\forall (i,j)\in\varepsilon(G)} \dot{V}^{ij} + \sum_{i=1}^{N}\sum_{\forall (i,j)\in\varepsilon(G)} K(z_{js}-z_i)^T(p_{js}-p_i) = 0. \tag{35}$$

So again we can get

$$\dot{V} = -P^T\Gamma P < 0, \tag{36}$$

and by using LaSalle's principle, we can conclude that $\lim_{t\to\infty} p_i(t) = 0$ for $i = 1,\ldots,N$. Since $\dot{z}_i(t) = \upsilon_i(t) + p_i(t)$ and $\upsilon_i(t) = 0$, we can obtain $\lim_{t\to\infty} \dot{z}_i(t) = 0$, $\forall i$, and since $z_{is}(t) = z_i(t-T_{ij}(t))$, we can conclude that $\lim_{t\to\infty} \dot{z}_{is}(t) = 0$, $\forall i$. In view of (30), this implies that $\lim_{t\to\infty} \dot{u}_i(t) = 0$, $\forall i$; and since

$\ddot{p}_i(t) = -\alpha_i \dot{p}_i(t) + \dot{u}_i(t)$, $\lim_{t\to\infty} \dot{u}_i(t) = 0$ and $\alpha_i > 0$, we can obtain that $\lim_{t\to\infty} \dot{p}_i(t) = 0$, and again with $\lim_{t\to\infty} p_i(t) = 0$, we can get $\lim_{t\to\infty} u_i(t) = 0$, $\forall i$. In view of (30), we have

$$\lim_{t\to\infty} z_{js}(t) = \lim_{t\to\infty} z_i(t), \ \forall (i,j) \in \varepsilon(G). \tag{37}$$

Since $z_{js}(t) = z_j(t - T_{ji}(t))$, $\forall j$, then as long as the underlying communication graph is bidirectional and strongly connected, $\lim_{t\to\infty} z_i(t) = \lim_{t\to\infty} z_j(t)$, for $i, j = 1, \ldots, N$. ■

V. Example

To illustrate the results above, we solve the path-following problem for a group of car-like robots.

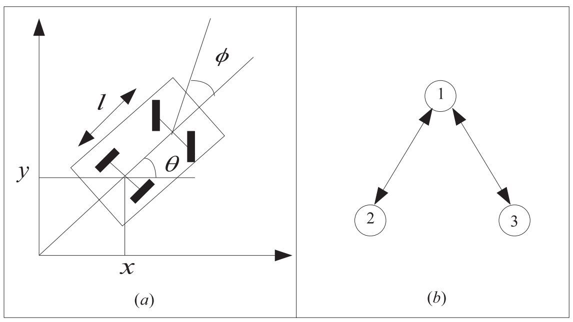

Fig. 3: (a) Car-Like Robot subject to Nonholonomic Constraints; (b) Underlying Communication Graph.

- here, the generalized coordinates are $q = (x, y, \theta, \phi)$, where θ is the heading angle, ϕ is the steering angle and (x, y) denotes the position of the real-wheal at the axle midpoint ;
- nonholonomic constraints are:

$$\begin{aligned} \dot{x}_f sin(\theta+\phi) - \dot{y}_f cos(\theta+\phi) = 0 \ (front\ wheel) \\ \dot{x} sin(\theta) - \dot{y} cos(\theta) = 0 \ (rear\ wheel) \end{aligned} \tag{38}$$

- where the front wheel position is:

$$x_f = x + lcos(\theta) \quad y_f = y + lsin(\theta) \tag{39}$$

From (38) and (39), we can see that the constraint matrix is:

$$A^T(q) = \begin{pmatrix} sin(\theta+\phi) & -cos(\theta+\phi) & -lcos(\phi) & 0 \\ sin(\theta) & -cos(\theta) & 0 & 0 \end{pmatrix} \tag{40}$$

If we choose

$$G(q) = \begin{pmatrix} cos(\theta)cos(\phi) & 0 \\ sin(\theta)sin(\phi) & 0 \\ \frac{1}{l} sin(\phi) & 0 \\ 0 & 1 \end{pmatrix} = [g_1(q) \quad g_2(q)] \tag{41}$$

then according to (4), we can get

$$\dot{q} = g_1(q)v_1 + g_2(q)v_2 \tag{42}$$

where v_1 = forward velocity, v_2 = steering velocity. The state-space representation for the closed-loop system can be written as:

$$\begin{aligned} \dot{X} &= \begin{pmatrix} \dot{q} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} G(q)v \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} \beta \\ y &= v \end{aligned} \tag{43}$$

where $v = [v_1^T, v_2^T]^T$. In our case, in order to use the input-output linearization technique, we need to define two new independent outputs since the robot has 2 degrees of freedom. We can define any point $(\hat{x}, \hat{y})$ with an arbitrarily distance d away from the position of the front wheel of the robot as the new independent outputs. Then the equation of $(\hat{x}, \hat{y})$ is given by:

$$h(q) = [h_1^T(q) \quad h_2^T(q)]^T = \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} x + lcos(\theta) + dcos(\theta+\phi) \\ y + lsin(\theta) + dsin(\theta+\phi) \end{pmatrix} \tag{44}$$

and we obtain

$$J_h(q) = \begin{pmatrix} \frac{\partial \hat{x}}{\partial q} \\ \frac{\partial \hat{y}}{\partial q} \end{pmatrix} = \begin{pmatrix} 1 & 0 & -lsin(\theta) - dsin(\theta+\phi) & -dsin(\theta+\phi) \\ 0 & 1 & lcos(\theta) + dcos(\theta+\phi) & dcos(\theta+\phi) \end{pmatrix} \tag{45}$$

The decoupling matrix $\Phi(q) is$

$$\Phi(q) = \begin{pmatrix} cos(\theta+\phi) - \frac{d}{l} sin(\theta+\phi)sin(\phi) & -dsin(\theta+\phi) \\ sin(\theta+\phi) + \frac{d}{l} cos(\theta+\phi)sin(\phi) & dcos(\theta+\phi) \end{pmatrix} \tag{46}$$

and according to (8)-(11), we obtain the new state-space representation for the car-like robot :

$$\dot{z}_1 = z_2 \qquad \dot{z}_2 = \mu \qquad p = z_1 \tag{47}$$

where $z_1 = [\hat{x}^T \ \hat{y}^T]^T$.

Now assume that we have three car-like robots to complete the task, and the underlying communication graph is bidirectional and strongly connected as shown in Fig.3(b)(for the path-following task, we assign a virtual-leader to one of these agents, i.e., agent 1). We embed scattering transformation into each communication link as shown in Fig.2. Assume that the change rate of time-varying delays are bounded by $T'_{ij}(t) \le 0.2$, $\forall (i,j) \in \varepsilon(G)$, and we set $\upsilon_i(t) = [0, 0]^T$, for $i = 1, \ldots, 3$. The simulations which verify our results for path-following of a circle are shown in Figure 4-5.

VI. Conclusions

In this paper, we have shown how to use passivity as the design and analysis tool for the output synchronization problem of networked EL systems subject to nonholonomic constraints. We also propose a scattering transformation set-up to solve the outputs synchronization problem in the presence of time-varying communication delays.

VII. Acknowledgments

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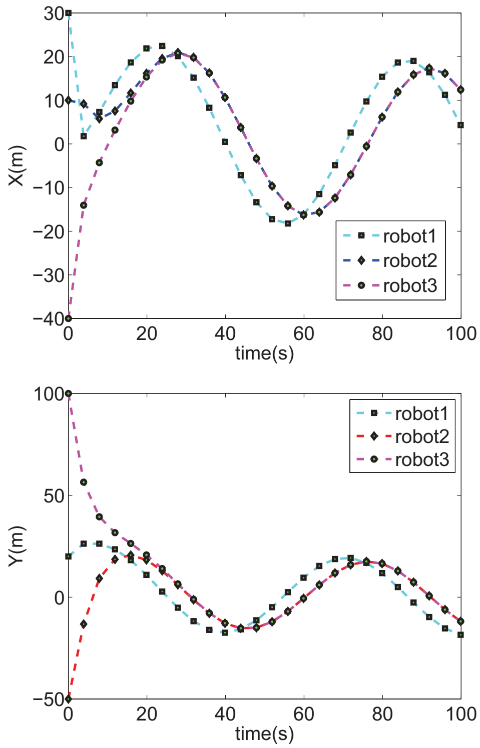

Fig. 4: Path following of the Car-Like Robots

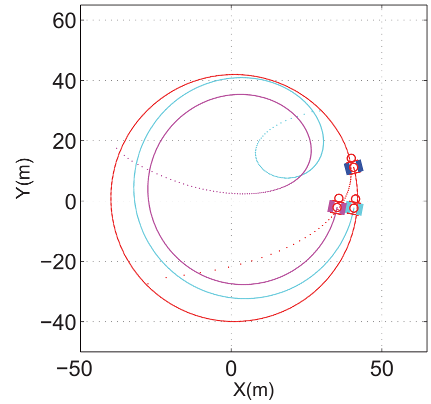

Fig. 5: Path following of the Car-Like Robots

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